

Subject: Introduction to Actuarial Models

Principles of Actuarial Modelling

Introduction

In this chapter we discuss

- why we want to model
- how to model
- the benefits and limitations of modelling
- testing the suitability of the model
- analyzing the output
- communicating the results.

Why Models are used?

A model is an imitation of a real-world system or process.

Models of many activities can be developed, for example:

- the economy of a country
- the workings of the human heart, and
- the future cashflows of the broker distribution channel of a life insurance company

Suppose we wished to 'predict' the effect that a real-world change would have on these three models. In some cases it might be too risky, or too expensive or too slow, to try a proposed change in the real world even on a sample basis. Trying out the change first without the benefit of a model could have serious consequences. The economy might go into recession costing a government the next election, the patient might die and the life office could suffer a surge in new business but at highly unprofitable premium rates.

Parameters

A model enables the possible consequences to be investigated. The effect of changing certain input parameters can be studied before a decision is made to implement the plans in the real world.

- To build a model of a system or process, a set of mathematical or logical assumptions about how it works needs to be developed.
- The complexity of a model is determined by the complexity of the relationships between the various model parameters.
- For example, in modelling a life office, consideration must be given to issues such as regulations, taxation and cancellation terms.
- Future events affecting investment returns, inflation, new business, lapses, mortality and expenses also affect these relationships.

Data

In order to produce the model and determine suitable parameters, data need to be considered and judgements need to be made as to the relevance of the observed data to the future environment. Such data may result from past observations, from current observations (such as the rate of inflation) or from expectations of future changes (e.g. tax change by government)

Where observed data are considered to be suitable for producing the parameters for a chosen model, statistical methods can be used to fit the data.



Objectives

Before finalizing the choice of model and parameters, it is important to consider the objectives for creation and use of the model. For example, in many cases there may not be a desire to create the most accurate model, but instead to create a model that will not understate costs or other risks that may be involved.



How models are used?

While in reality a modelling process does not follow a rigid pattern of prescribed steps, it is helpful in introducing the topic to imagine a set of key steps. In practice, actuaries who build and use models move back and forth between these key steps continuously to improve the model.

Key Steps in a modelling process

- Develop a well-defined set of objectives that need to be met by the modelling process.
- Plan the modelling process and how the model will be validated.
- Collect and analyze the necessary data for the model.
- Define the parameters for the model and consider appropriate parameter values.
- Define the model initially by capturing the essence of the real-world system. Refining the level of detail in the model can come at a later stage.
- Involve the experts on the real-world system you are trying to model in order to get feedback on the validity of the conceptual model.
- Decide on whether a simulation package or general-purpose language is appropriate for the implementation of the model. If necessary, choose a statistically reliable random number generator that will perform adequately in the context and complexity of the model.

Key Steps in a modelling process (contd...)

- Write the computer program for the model.
- Debug the program to make sure it performs the intended operations in the model definition.
- Test the reasonableness of the output from the model.
- Review and carefully consider the appropriateness of the model in the light of small changes to the input parameters.
- Analyse the output from the model.
- Ensure that any relevant professional guidance has been complied with. For example, the Financial Reporting Council has issued a Technical Actuarial Standard on the principles for Technical Actuarial Work (TAS100), which includes principles for models used in technical actuarial work.
- Communicate and document the results and the model.

Advantages of Modelling

- Systems with long time frames such as the operation of a pension fund can be studied in compressed time
- Different future policies or possible actions can be compared to see which best suits the requirements or constraints of a user.
- In a model of a complex system we can usually get control over the experimental conditions so that we can reduce the variance of the results output from the model without upsetting their mean values.
- Complex systems with stochastic elements, such as the operation of a life insurance company, cannot be properly described by a mathematical or logical model that is capable of producing results that are easy to interpret. Simulation modelling is a way of studying the operation of a life insurance company.

Limitations of Modelling

- Model development requires a considerable investment of time, and expertise. The
 financial costs of development can be quite large given the need to check the validity of
 the model's assumptions, the computer code, the reasonableness of results and the way in
 which results can be interpreted in plain language by the target audience e.g. Human
 resource Directors, Life Insurance Policy-holders etc.
- In a stochastic model, for any given set of inputs each run gives only estimates of a model's outputs. So, to study the outputs for any given set of inputs, several independent runs of the model are needed.
- As a rule, models are more useful for comparing the results of input variations than for optimizing outputs i.e. easier to predict outputs than what is required as inputs to get particular output.



Limitations of Modelling (contd..)

- Models can look impressive when run on a computer so that there is a danger that one
 gets lulled into a false sense of confidence. If a model has not passed the tests of validity
 and verification, its impressive output is a poor substitute for its ability to imitate its
 corresponding real-world system.
- Models rely heavily on the data input. If the data quality is poor or lacks credibility, then the output from the model is likely to be flawed.
- It is important that the users of the model understand the model and the uses to which it can be safely put. There is a danger of using a model as a 'black box' from which it is assumed that all results are valid without considering the appropriateness of using that model for the data input and the output expected.
- It is not possible to include all future events in a model. A significant change might render the model obsolete.
- It may be difficult to interpret some of the outputs of the model. They may only be valid in relative rather than absolute terms, as when, for example, comparing the level of risk of the outputs associated with different inputs.

Stochastic and Deterministic Models

- If it is desired to represent reality as accurately as possible, the model needs to imitate the random nature of the variables. A stochastic model is one that recognizes the random nature of the input components. A model that does not contain any random component is deterministic in nature.
- In a deterministic model, the output is determined once the set of fixed inputs and the relationships between them have been defined.
- By contrast, in a stochastic model the output is random in nature like the inputs, which are random variables. The output is only a snapshot or an estimate of the characteristics of the model for a given set of inputs.
- Several independent runs are required for each set of inputs so that statistical theory can be used to help in the study of the implications of the set of inputs.
- A deterministic model is really just a special (simplified) case of a stochastic model.
- Whether one wishes to use a deterministic or a stochastic model depends on whether one is interested in the results of a single 'scenario' or in the distribution of results of possible 'scenarios'.

Discrete and continuous state spaces and time sets

- The state of a model is the set of variables that describe the system at a particular point in time taking into account the goals of the study. For e.g. Alive, Sick, Dead model.
- Discrete states are where the variables exhibit step function changes in time. For example, from a state of alive to dead, or an increase in the number of policies for an insurer.
- Continuous states are where the variables change continuously with respect to time. For example, real time changes in values of investments.
- The decision to use a discrete or continuous state model for a particular system is driven by the objectives of the study, rather than whether or not the system itself is of a discrete or continuous nature.
- A model may also consider time in a discrete or a continuous way. This may reflect the fact that outputs from the model are only required at discrete points in time, or may be to satisfy the objectives of the modelling. For e.g. modelling number of claims since start of the month or doing it all together at the end of the month.



Scenario-based and proxy models

- Most models depend on many input parameters. A scenario-based model would take into consideration a particular scenario; that is a series of input parameters based on this scenario. Different scenarios would be useful in decision analysis as one can evaluate the expected impact of a course of action.
- For e.g. different scenarios in a financial model i.e. neutral performance, underperformance, and outperformance
- A proxy model may be used to replace Monte Carlo simulation, providing faster but less
 accurate results. A simplified formula is developed that we believe predicts the result with
 reasonable accuracy, and this is then used as a substitute for running the full model.

Suitability of a model

In assessing the suitability of a model for a particular exercise it is important to consider the following:

- The objectives of the modelling exercise.
- The validity of the model for the purpose to which it is to be put.
- The validity of the data to be used.
- The validity of the assumptions.
- The possible errors associated with the model or parameters used not being a perfect representation of the real-world situation being modelled.
- The impact of correlations between the random variables that 'drive' the model.
- The extent of correlations between the various results produced from the model.
- The current relevance of models written and used in the past.
- The credibility of the data input and results output.
- The dangers of spurious accuracy.
- The ease with which the model and its results can be communicated.
- Regulatory requirements.

Analysing Model Output

- Statistical sampling techniques are needed to analyse the output of a model, as a simulation is just a computer-aided statistical sampling project.
- The actuary must exercise great care and judgement at this stage of the modelling process as the observations in the process are correlated with each other and the distributions of the successive observations change over time.
- The useless and fatally attractive temptation of assuming that the observations are independent and identically distributed is to be avoided.
- If there is a real-world system against which results can be compared, a 'Turing' test should be used. In a Turing test, experts on the real-world system are asked to compare several sets of real-world and model data without being told which are which.
- If these experts can differentiate between the real-world and model data, their techniques for doing so could be used to improve the model.

Sensitivity testing

- Where possible, it is important to test the reasonableness of the output from the model against the real world. To do this, an examination of the sensitivity of the outputs to small changes in the inputs or their statistical distributions should be carried out.
- The appropriateness of the model should then be reviewed, particularly if small changes in inputs or their statistical distributions give rise to large changes in the outputs.
- In this way, the key inputs and relationships to which particular attention should be given
 in designing and using the model can be determined.
- The model should be tested by designing appropriate simulation experiments. It is through this process that the model can be refined.

Communicating the results

- The final step in the modelling process is the communication and documentation of the results and the model itself to others.
- The communication must be such that it takes account of the knowledge of the target audience and their viewpoint. A key issue here is to make sure that the client accepts the model as being valid and a useful tool in decision making.
- It is important to ensure that any limitations on the use and validity of the model are fully appreciated.



Thank You