

Class: M.Sc Sem 3

Subject : Actuarial Practice 1

Chapter: Unit 3 Chapter 14

Chapter Name: Solving Actuarial Problem

Today's Agenda

1. Producing a solution to the problem
2. Understanding the characteristics of an Actuarial Model
3. Understanding the concept of deterministic and stochastic models
4. Using models for pricing
5. Using models for future financing strategies
6. Using models for risk management
7. Valuing provisions on an individual basis
8. Valuing options and guarantees
9. Sensitivity analysis

1 Producing a Solution

- **Approaches to Solving Actuarial or Financial Problems**

There are various approaches that can be taken to produce the solution to an actuarial or financial problem. The approach adopted largely depends on the problem to be solved. For example a lot more detailing will be required when complying with statutory regulations laid by the government, than that required in self regulation.

Simple problems can have a simple solution that is arrived at by some straightforward mathematics, for example calculating the yield on a fixed-interest asset, or the present value of a series of known cashflows.

- **The Need to Develop a Model**

However, most problems that require actuarial skills involve taking a view on uncertain future events. It is possible to take a view on various parameters, such as future economic conditions, future mortality rates, or the amount of business that a provider might write in future, and produce a single answer that is appropriate in these best estimate conditions. If this is done then the communication of the solution to the client needs care, because of the uncertainties in the underlying assumptions.

1 Producing a Solution

In these circumstances the client is likely to wish to know the variability of the answer provided, should circumstances not be as estimated. To assess the effects of varying the assumptions used in producing the answer, it is normally necessary to use an actuarial model of future events.



What is a model ?

A model can be defined as a 'cut down, simplified version of reality that captures the essential features of a problem and aids understanding'. The final phrase in this definition recognises the importance of being able to communicate the results effectively. Modelling requires a balance to be struck between realism (hence complexity) and simplicity (for ease of application, verification and interpretation of results).

1 Producing a Solution

- Finding a Model

When faced with an actuarial or financial problem, there are several approaches to modelling:

1. A commercial modelling product could be purchased
2. An existing model could be reused, after modifications
3. A new model could be developed

However, the suitability of each of these approaches depends on the following :

- The level of accuracy required – for greater accuracy, a commercial model could be purchased which would be tried and tested or a new model could be developed specifically for that problem
- The in-house expertise available – if the developers are skilled and are highly experienced in developing a model, a new model can easily be developed
- The number of times the model is to be used

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- The number of times the model is to be used
- The desired flexibility of the model
- The cost of each option – not all organisations can afford to develop new models for every problem due to lack of funds. They may hence prefer using an existing model subjecting it to certain modifications

Existing Models

Deterministic or stochastic models can be used. There are now many stochastic asset models in existence, in both the public and private domains.

2 Construction of an Actuarial Model

- Key Objective

A model should be fit for the purpose for which it is being used

This is particularly relevant when a model is being purchased from an external provider or when an existing model is being reused for a different purpose, possibly after modification.

Even with new purpose-built models, the potential for model error remains – a model that replicates past results may still prove unreliable in projecting future results, as assumptions made on the basis of past experience may not always support future scenarios.

2 Construction of an Actuarial Model

- Operational Issues

Along with technical areas, there are several operational issues which need to be considered:

- The model being used should be adequately documented – this is to ensure that the assumptions used are easily understood by other members apart from the developer, and hence the model can be used even by first timers
- The workings of the model should be easy to appreciate and communicate. The results should be displayed clearly. Not every employee understands the modelling jargon and hence the results should be displayed in layman's language
- The model should exhibit sensible joint behaviour of model variables – when model variables are correlated, the model needs to account for their correlation. The assumptions used also need to be consistent with the links between variables.
- The outputs from the model need to be capable of independent verification for reasonableness and should be communicable to those to whom advice will be given.

2 Construction of an Actuarial Model

- The model, however must not be overly complex so that either the results become difficult to interpret and communicate or the model becomes too long or expensive to run, unless this is required by the purpose of the model. It is important to avoid the impression that everything can be modelled.
- The model should be capable of development and refinement – nothing complex can be successfully designed and built in a single attempt. There must be room for trial and error
- A range of methods of implementation should be available to facilitate testing, parameterisation and focus of results
- The more frequently the cashflows are calculated the more reliable the output from the model, although there is a danger of spurious accuracy
- The less frequently the cashflows are calculated the faster the model can be run and results obtained

Cashflows can be calculated on a monthly, quarterly or yearly basis. In the initial period when parameters are known with the highest degree of certainty, giving more meaningful results, and hence cashflows must be calculated more frequently than that in the later years to avoid spurious accuracy.

3 Deterministic or Stochastic Model

➤ **Deterministic Model**

In a deterministic model, parameters are fixed and known in advance before the model is run. As a result we get a single outcome when the model is run. Sensitivity analysis and scenario testing can be carried out to allow for variability in results. No matter how many times the model is updated, it gives the same results for a given set of inputs

- **Merits of a Deterministic Model**

1. A deterministic model is more readily explicable to a non-technical audience, who has no knowledge of the concept of variables or probability distribution.
2. It is clearer what economic scenarios have been tested
3. It is easy to resolve any errors occurring while implementing this type of model
4. The model is cost effective, easier to design and quicker to run.

3 Deterministic or Stochastic Model

- Demerits of a Deterministic Model
 1. It requires thought as to the range of economic scenarios that should be tested – a limitation of the deterministic model is that it can be used in modelling only a certain type of scenarios and other scenarios may go unnoticed
 2. Users can get 'blinded by science' by complex models, and may assume that deterministic models must be working correctly, without verifying the test results
 3. They may struggle with complicated, non linear relationships in the data since the rules are unbending making it difficult for them to modify effectively to changing data



- For example if 10000 individuals have a 95% chance of surviving 1 year, then we can be reasonable certain that 9500 of them will indeed survive
- Rolling a fair die: each number on a six sided die has the same odds ($1/6$) of coming up
- In a simple linear regression, if the response and explanatory variables have an exact relationship, then that relationship is deterministic, where the value of y can be predicted with 100% certainty based on the value of x

3 Deterministic or Stochastic Model

➤ **Stochastic Model**

A stochastic model is where one or more parameters are derived from a probability distribution. The model is then run a multiple times, each time with a different parameter value derived from the distribution. This gives us a range values for the outcome, and helps in establishing its likely distribution.

- Merits of a Stochastic Model
 1. It tests a wider range of economic scenarios. The programming is more complex and the run time longer, but the benefit is in the quality of the result. It does depend on the parameters that are used in standard investment model.
 2. Decision needs to be taken, whether the additional information provided by the model is worth the time taken by the model and its additional complexities

3 Deterministic or Stochastic Model

- Demerits of a Stochastic Model
 1. There are chances of spurious accuracy,
 2. Interpreting and communicating the results is tedious
 3. The accuracy of the distribution assigned is questionable

Stochastic models are particularly important in assessing the impact of financial guarantees or to allow for investment mismatching risks.

➤ A Combination of Deterministic and Stochastic Modelling

In many cases the problem can be solved by a combination of stochastic and deterministic modelling. Variables whose performance is unknown and where the risk associated with them is high might be modelled stochastically, while other variables can sensibly be modelled deterministically.

For example, a model for pricing an investment guarantee attached to a life insurance policy might use a stochastic investment model, but would be unlikely to model fluctuations in mortality rates other than deterministically. This is because it is normally self-evident which direction of movement in mortality rates would give rise to financial difficulties.

3 Deterministic or Stochastic Model

Consider modelling for a general insurance claims experience. We may have two approaches here, the first one where the number of claims are modelled stochastically and the average claim amount for a homogeneous group of policies is found using a deterministic model. The second approach being that claim amounts are found using a stochastic model, where the average number of claims is found deterministically.

➤ Dynamism of a Model

When talking about dynamism, we imply that all the various components of the model, all its assumptions must interact with one another as they would in real life.

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4 Using Models for Pricing

A model could be developed to determine a premium or charging structure for a new or existing product that will meet an insurance company's profit requirement.

During its initial years, an insurance company will develop a model to establish its premium rates. The model will then be used at regular intervals to ensure that :

- The premium rate charged makes the business profitable
- The rates are appropriate for all groups, and that policyholders are neither undercharged or overcharged
- The rates are competitive with reference to other investment companies

4 Using Models for Pricing

➤ The Use of Model Points

The underlying business being modelled will typically comprise a very wide range of different policies, and these will need to be brought together into a manageable number of relatively homogeneous groups. The groupings need to be made in a way that each policy in a group is expected to produce similar results when the model is run. It is then sufficient for a representative single policy in each group to be run through the model, the result to be found, and for this result to be scaled up to give the result of the total set of policies in the group.

The representative single policy in a group is termed a 'model point' and a set of such model points can then be used to represent the whole of the underlying business.



Suggest important features that need to be captured by the model points that would be used when modelling a with-profit endowment assurance product in order to set premiums.

4 Using Models for Pricing

➤ Choosing Model Points

A set of model points will be chosen to represent the expected new business under the product. In the case of an existing product, the profile of the existing business, modified to allow for any expected changes in future, can be used to obtain the model points. For a new product, the profile of any similar existing product combined with advice from the company's marketing department would be used.

1. The number of model points acceptable, will depend on how robust the model is. This will in turn depend on :
2. The computing power of the model
3. Any time constraints
4. The business mix
5. How sensitive the results are to using different model points
6. The purpose of the exercise

4 Using Models for Pricing

➤ Rate for Discounting Cashflows

When projecting cashflows for model points we allow for the products reserving and solvency margin requirements, on the basis of a set of base values for the parameters in the model.

The net projected cashflows will then be discounted at a rate of interest, the risk discount rate.

This could be a rate that allows for:

- The return required by the company, and
- The level of statistical risk attaching to the cashflows under the particular contract, ie their variation about the mean as represented by the cashflows themselves.

Statistical risk here refers to a broad term, that covers model risk, parameter risk and random fluctuation risk. It is then allowed for in the discounting rate

4 Using Models for Pricing

The level of statistical risk could be assessed:

- in some situations, analytically – by considering the variances of the individual parameter values used
- by using sensitivity analysis, as described below, with deterministically assessed variations in the parameter values
- by using stochastic models for some, or all, of the parameter values and simulation
- by comparison with any available market data.

The idea behind using a stochastic model for pricing is to get a range of scenarios, by varying the parameter values in the model according to their probability distribution function and recalculating the rate of return for each new scenario. As more and more simulations are run, an almost accurate variance of the rate of return can be achieved.

In theory, a separate risk discount rate should be applied to each separate component of the cashflows, as the statistical risk associated with each component will be different.

In practice a single risk discount rate is commonly used, bearing in mind the 'average' risk of the product.

4 Using Models for Pricing

➤ Meeting Profit Requirements

The premium for the model points can be set so as to produce the profit required by the company. For small policies, where the sum assured is less, and hence the level of premium charged, achieving profitability for all model points is difficult. This happens because the expenses incurred in relation to each policy will be fixed, and allowing for fixed costs reduces their profitability. Instead, focusing on the total profitability resulting from a group of model points will give more positive results. The group of model points chosen, aims at representing the new business mix and volume, leaving room for a certain degree of cross subsidy

4 Using Models for Pricing

➤ Competitive Premiums

The premiums, or charges, produced need to be considered for marketability. This might lead to a reconsideration of:

- the design of the product, so as either to remove features that increase the risks within the net cashflows, or to include features that will differentiate the product from those of competing companies for example providing cover to one more person related to the policyholder upto a certain amount
- the distribution channel to be used, if that would permit either a revision of the assumptions to be used in the model, or a higher premium or charges to be used without loss of marketability
- the company's profit requirement
- the size of the market
- whether to proceed with marketing the product.

4 Using Models for Pricing

➤ Business Strategy

The net cashflows in respect of the model points, appropriately scaled up for the expected new business under the product, will be incorporated into a model of the business of the whole company.

The desired level of profitability can be considered on an aggregate level instead on an individual model point level, where it is not required for every single model point to be profitable, if the overall profitability is achieved. However, if certain model points are unprofitable, then the aggregate profitability of the business is exposed to changes in mix and volume of the contracts sold.

The actuary can assess the impact on capital management of writing the product, by observing the modelled amount and timing of cashflows. If capital is a problem, this may lead to a reconsideration of the design of the product to reduce or amend the timing of its financing requirement.

Once acceptable premiums or charges have been determined for the model points, premiums or charges for all contract variations can be determined.

4 Using Models for Pricing

Once premiums for the most basic contracts have been calculated, the premiums for other products that differ due to certain variations can be calculated by adjusting the basic premium and allowing for the variations. They can also be found by interpolating between the premiums for the various model points.

➤ Assessing the Capital Requirements and the Return on Capital

The net cashflows for the model points can be grossed up for the expected new business and used to assess the amount of capital that will be required to write the product, either on a regulatory or an economic basis. Any one-off development costs can be added, to the extent that they are not amortised and included in the cashflows used.

This gives the total capital requirement and can be compared with the profits expected to emerge from the product so as to determine the expected return on that capital.

5 Valuing Provisions on an Individual Basis

The normal procedure for determining life assurance or pension scheme liabilities is to value the benefits for each policy or scheme member individually. In many territories this may be required by legislation or regulation. This is more so to ensure that insurance companies have sufficient provisions to meet their claim liabilities.

An approximate matching using model points may lead to under-reserving.

Consequently, for published results there is little scope for using model points. However, before finalising a published basis, many 'what if' questions might be asked. These could be answered by running a model of the business.

These questions might include:

- The insurance company's ability in meeting regulatory requirements, in case of a fall in returns
- The impact on finances in case of worsening of mortality experience
- In case of a reinsurer defaulting on his obligations or a policyholder lapsing on his premium payments

6 Pricing and Valuing Options and Guarantees

➤ Investment related Options and Guarantees

In most cases the options and guarantees that give a provider of benefits on future financial events cause for concern are those that are dependent on future investment returns, or an investment value (yield or capital value) at some future point in time.

Because of the uncertainty, a stochastic investment model should be used to assess the provisions necessary for such guarantees, as they can establish the likelihood of the option or guarantee applying together with the associated cost.

If future returns exceed a certain level, or if a value or index is above (or below) a fixed value at some future point, there will be no cost to the company. But if future returns are below that level, there will be a cost, which increases as returns reduce. Hence a range of future investment scenarios should be tested

➤ Other Options

Insurance products exist where options are not dependent on investment outcomes, but more on death and sickness. For example a term product may provide coverage upon the death of a spouse along with the main policyholder.

7 Sensitivity Analysis

Sensitivity analysis is a financial model that determines how target variables are affected based on changes in other variables known as input variables. It is a way to predict the outcome of a decision given a certain range of variables

- Reliability of the Results

The results from the model depend on the model itself and the values assigned to the parameters in the model. Models should not be treated as black boxes, the output of which is assumed to be correct. one must leave room for a certain degree of uncertainty credited to the model, and there may be a slight chance

- Understanding the Potential Variability of Experience

The use of a stochastic model goes some way to illustrating the potential variability of the experience, but the results that it produces are still dependent on the accuracy of the model and its parameter values. In the case of a deterministic model, the potential uncertainty of the results is greater, because fewer scenarios are tested,, using the same parameter values for each scenario

The re-running of a model (deterministic or stochastic) with different, but feasible, parameter values will produce alternative results and hence help to illustrate the potential deviations.

7 Sensitivity Analysis

The re-running with a series of different sets of parameter values, perhaps chosen from a probability distribution for such values, will help to illustrate the likely range in which actual experience may lie, perhaps as far as creating a probability distribution for this experience.

For example, consideration of the effect of a change in the membership profile of a funded pension scheme may be needed to illustrate the extent of potential variability in future contributions if the model used is based on a stable membership profile.

- Model Error

There is a possibility of model error if the model developed is not appropriate for the financial products, schemes, contracts or transactions being modelled.

Checks of goodness of fit will be needed to assess the suitability of the model, but taking account of expected changes in experience into the future.

The model's accuracy depends on the quality of assumptions built in the model. Erroneous assumptions lead to inaccurate outputs. Also the model may not always account for interdependencies among input variables. When assumptions are based on historical data, they cannot always be relied upon to predict future results.

7 Sensitivity Analysis

- Parameter Error

The effect of mis-estimation of parameter values can also be investigated by carrying out a sensitivity analysis. This involves assessing the effect on the output of the model of varying each of the parameter values. When doing this, any correlation between different parameters should be allowed for.

In the case of a model used for pricing, the results from the sensitivity analysis will help to assess the margins that need to be incorporated into the parameter values.

For example consider the steps taken when a product proves to be overly sensitive to withdrawal rates and mortality rates on carrying out sensitivity analysis.

Where the product appears to be highly sensitive to any of these factors, i.e. a slight change in any of these factors, affects the product's profitability to a great extent, then we might consider redesigning the product, or increase the profitability margins in the assumptions used to calculate premium.

High sensitivity towards increasing withdrawal rates, calls for a reduction in surrender values, high sensitivity towards mortality results in the revision of reinsurance terms.

7 Sensitivity Analysis

- Alternative Ways of Allowing for Risk

The statistical risk associated with the parameter values can be allowed through the risk element of the risk discount rate.

Here we first establish the risk free rate of return expected by the shareholders from their investment. Any deviation or variation from this risk free rate is then calculated, and the variation is allowed for in the actual rate of return. This would give us the risk margin to be added to the risk free rate.

An alternative would be to use a predetermined discount rate and then assess the effect on the results of the models of statistical risk.

Under this approach, we find the present value of the cashflows using the risk free rate, but instead of using the best estimate parameter values, we use more prudent or pessimistic parameter values. The extent of prudence or pessimism used, corresponds to the risk margin introduced.

Where a probability distribution can be assigned to a parameter, it may be possible to derive the variance of the profit or return on capital analytically.

7 Example



A unit-linked life assurance policy guarantees to pay a maturity value equal to the sum of premiums on the chosen maturity date, or the value of units allocated if greater. At all other times the surrender value is based on the value of units.

Describe the steps involved in assessing the provision to be made for the cost of this guarantee.

The steps involved are:

- Choose a stochastic asset model – a complex model gives better results but takes longer to run.
- Determine assumptions – particularly unit growth rate mean and volatility.
- Determine consistent deterministic assumptions for mortality and surrender rates and future expenses.
- Consider dynamic links between assumptions – eg lapse rates to unit values. • Choose a time period – probably annual for efficient running.
- Determine appropriate model points for the portfolio.
- The model will project the unit values to maturity, allowing for future premiums and all decrements.
- This will be done for a large number of randomly generated investment scenarios – say between 1,000 and 5,000.

7 Example



- For each scenario and each model point, the projected unit value will be compared with the guaranteed maturity value, and the cost for that particular scenario and model point determined.
- The projected costs are discounted to the present, scaled up by the appropriate factors and summed across all model points.
- The average across all scenarios is the expected cost of the guarantee.
- The variability should be assessed by looking at the quartiles and 5th/95th percentiles, when the results are ranked.
- For reserving purposes, an appropriate ruin probability needs to be chosen. Perhaps 1 in 100, in which case the reserve is the 99th percentile.