INSTITUTE OF ACTUARIAL & QUANTITATIVE STUDIES

Class: M.Sc. Sem 3

Subject : Actuarial Practice 1

Chapter: Unit 3 Chapter 5

Chapter Name: Modelling

Table of contents

Table of contents

1 Producing a Solution

Approaches to Solving Actuarial or Financial Problems

There are various approaches that can be taken to produce the solution to an actuarial or financial problem

The approach adopted largely depends on the problem to be solved. For example, a lot more detailing will be required when complying with statutory regulations laid by the government, than that required in self-regulation

Simple problems can have a simple solution that is arrived at by some straightforward mathematics, for example calculating the yield on a fixed-interest asset, or the present value of a series of known cashflows.

1 Producing a Solution

The need to develop a model

However, most problems that require actuarial skills involve taking a view on **uncertain future events**.

It is possible to take a view on various parameters, such as **future economic conditions**, **future mortality rates**, or the **amount of business** that a provider might write in future and produce a single answer that is appropriate in these best estimate conditions.

If this is done then the communication of the solution to the client needs care, because of the uncertainties in the underlying assumptions.

In these circumstances the client is likely to wish to know the variability of the answer provided, should circumstances not be as estimated. To assess the effects of varying the assumptions used in producing the answer, it is normally necessary to use an actuarial model of future events.

1 Producing a Solution

What is a model ?
A model can be defined as a 'cut down, simplified version of reality that captures the essential features of a problem and aids understanding'. The final phrase in this definition recognises the importance of being able to communicate the results effectively. Modelling requires a balance to be struck between realism (hence complexity) and simplicity (for ease of application, verification and interpretation of results).

- Finding a Model

When faced with an actuarial or financial problem, there are several approaches to modelling:

1. A commercial modelling product could be **purchased**
2. An existing model could be **reused**, after modifications
3. A new model could be **developed**

1 Producing a Solution

However, the suitability of each of these approaches depends on the following :

- The level of **accuracy** required – for greater accuracy, a commercial model could be purchased which would be tried and tested or a new model could be developed specifically for that problem
- The in-house **expertise** available – if the developers are skilled and are highly experienced in developing a model, a new model can easily be developed
- The number of **times** the model is to be used
- The desired **flexibility** of the model
- The **cost** of each option – not all organisations can afford to develop new models for every problem due to lack of funds. They may hence prefer using an existing model subjecting it to certain modifications

Existing Models
Deterministic or stochastic models can be used. There are now many stochastic asset models in existence, in both the public and private domains.

Table of contents

2 Construction of an Actuarial Model

- Key Objective

A model should be **fit for the purpose** for which it is being used

This is particularly relevant when a model is being **purchased from an external provider** or when an **existing model is being reused** for a different purpose, possibly after modification.

Even with new purpose-built models, the potential for **model error** remains – a model that replicates past results may still prove unreliable in projecting future results, as assumptions made on the basis of past experience may not always support future scenarios.

2 Construction of an Actuarial Model

- Operational Issues

Along with **technical areas**, there are several operational issues which need to be considered:

- The model being used should be adequately **documented** – this is to ensure that the assumptions used are easily understood by other members apart from the developer, and hence the model can be used even by first timers

- The **workings** of the model should be easy to **appreciate and communicate**. The results should be displayed clearly. Not every employee understands the modelling jargon and hence the results should be displayed in layman's language

- The model should exhibit **sensible joint behaviour** of model variables – when model variables are correlated, the model needs to account for their correlation. The assumptions used also need to be consistent with the links between variables.

- The outputs from the model need to be capable of **independent verification for reasonableness** and should be communicable to those to whom advice will be given.

2 Construction of an Actuarial Model

- The model must not be **overly complex** so that either the results become difficult to interpret and communicate or the model becomes too long or expensive to run, unless this is required by the purpose of the model. It is important to avoid the impression that everything can be modelled.

- The model should be capable of **development and refinement** – nothing complex can be successfully designed and built in a single attempt. There must be room for trial and error

- A range of **methods of implementation** should be available to facilitate testing, parameterisation and focus of results

- The more **frequently the cashflows** are calculated the more reliable the output from the model, although there is a danger of spurious accuracy

- The less frequently the cashflows are calculated the faster the model can be **run** and results obtained

Cashflows can be calculated on a **monthly, quarterly or yearly basis**. In the initial period when parameters are known with the highest degree of certainty, giving more meaningful results, and hence cashflows must be calculated more frequently than that in the later years to avoid spurious accuracy.

2 Deterministic or Stochastic Model

Deterministic model

In a deterministic model, parameters are **fixed** and **known in advance** before the model is run.

As a result, we get a **single outcome** when the model is run. **Sensitivity analysis** and **scenario testing** can be carried out to allow for **variability** in results. No matter how many times the model is updated, it gives the same results for a given set of inputs

- Merits of a Deterministic Model

1. A deterministic model is more readily **explicable** to a non-technical audience, who has no knowledge of the concept of variables or probability distribution.
2. It is clearer what **economic scenarios** have been tested
3. It is easy to **resolve any errors** occurring while implementing this type of model
4. The model is **cost effective**, easier to design and quicker to run.

2 Deterministic or Stochastic Model

- Demerits of a Deterministic Model

1. It requires thought as to the range of **economic scenarios** that should be tested – a limitation of the deterministic model is that it can be used in modelling only a certain type of scenarios and other scenarios may go unnoticed

2. Users can get '**blinded by science**' by complex models, and may assume that deterministic models must be working correctly, without verifying the test results

3. They may struggle with **complicated**, **non-linear relationships** in the data since the rules are unbending making it difficult for them to modify effectively to changing data

- For example, if 10000 individuals have a 95% chance of surviving 1 year, then we can be reasonably certain that 9500nof them will indeed survive
- Rolling a fair die: each number on a six-sided die has the same odds (1/6) of coming up
- In a simple linear regression, if the response and explanatory variables have an exact relationship, then that relationship is deterministic, where the value of y can be predicted with 100% certainty based on the value of x

2 Deterministic or Stochastic Model

Stochastic model

A stochastic model is where **one or more parameters** are derived from a **probability distribution**.

The model is then run **multiple times**, each time with a **different parameter value** derived from the distribution. This gives us a range values for the outcome and helps in establishing its likely distribution.

- Merits of a Stochastic Model

1. It tests a **wider range of economic scenarios**. The programming is more **complex** and the run time **longer**, but the benefit is in the quality of the result. It does depend on the parameters that are used in standard investment model.

2. Decision needs to be taken, whether the additional information provided by the model is worth the time taken by the model and its additional complexities

2 Deterministic or Stochastic Model

- Demerits of a Stochastic Model

1. There are chances of **spurious accuracy**,.
2. **Interpreting** and **communicating** the results is tedious
3. The **accuracy of the distribution** assigned is questionable

Stochastic models are particularly important in assessing the impact of **financial guarantees** or to allow for investment mismatching risks.

2 Deterministic or Stochastic Model

A combination of deterministic and stochastic modelling

In many cases the problem can be solved by a **combination of stochastic and deterministic modelling**.

Variables whose performance is unknown and where the **risk associated** with them is high might be modelled **stochastically**, while other variables can sensibly be modelled **deterministically**.

For example, a model for pricing an investment guarantee attached to a life insurance policy might use a stochastic investment model but would be unlikely to model fluctuations in mortality rates other than deterministically. This is because it is normally self-evident which direction of movement in mortality rates would give rise to financial difficulties.

2 Deterministic or Stochastic Model

Dynamism of a model

When talking about **dynamism**, we imply that all the various components of the model, all its assumptions must interact with one another as they would in real life.

Consider modelling for a general insurance claims experience. We may have two approaches here, the first one where the number of claims are modelled stochastically and the average claim amount for a homogeneous group of policies is found using a deterministic model. The second approach being that claim amounts are found using a stochastic model, where the average number of claims is found deterministically. Considerable actuarial judgement may be required in choosing and using the model and in setting the parameters and interactions between the different features.

Table of contents

3 Using Models for Pricing

A model could be developed to **determine a premium or charging structure** for a new or existing product that will meet an insurance company's profit requirement.

During its initial years, an insurance company will develop a model to establish its **premium rates**. The model will then be used at **regular intervals** to ensure that:

- The premium rate charged makes the **business profitable**
- The rates are appropriate for all **groups**, and that policyholders are neither **undercharged or overcharged**
- The rates are **competitive** with reference to other investment companies

3 Using Models for Pricing

Use of model points

The underlying business being modelled will typically comprise a very wide range of **different policies**, and these will need to be brought together into a **manageable number of relatively homogeneous groups**.

The groupings need to be made in a way that **each policy** in a group is expected to **produce similar results** when the model is run. It is then sufficient for a **representative single policy** in each group to be run through the model, the result to be found, and for this result to be scaled up to give the result of the total set of policies in the group.

The representative single policy in a group is termed a '**model point**' and a set of such model points can then be used to represent the whole of the underlying business.

Suggest important features that need to be captured by the model points that would be used when modelling a with-profit endowment assurance product in order to set premiums.

3 Using Models for Pricing

Choosing model points

A set of model points will be chosen to **represent the expected new business** under the product.

In the case of an existing product, the profile of the existing business, modified to allow for any expected changes in future, can be used to obtain the model points.

For a new product, the profile of any similar existing product combined with advice from the company's marketing department would be used.

The number of model points **acceptable**, will depend on how **robust** the model is. This will in turn depend on :

1. The **computing power** of the model
2. Any **time constraints**
3. The **business mix**
4. How **sensitive** the results are to using different model points
5. The **purpose** of the exercise

3 Using Models for Pricing

Rate for discounting cashflows

When projecting cashflows for model points we allow for the **products reserving and solvency margin** requirements, on the basis of a set of base values for the parameters in the model.

The net **projected cashflows** will then be discounted at a rate of interest, **the risk discount rate.**

This could be a rate that allows for:

- The **return** required by the company, and
- The level of **statistical risk** attaching to the cashflows under the particular contract, ie their variation about the mean as represented by the cashflows themselves.

Statistical risk here refers to a broad term, that covers model risk, parameter risk and random fluctuation risk. It is then allowed for in the discounting rate

3 Using Models for Pricing

The level of statistical risk could be assessed:

- in some situations, **analytically** – by considering the variances of the individual parameter values used
- by using **sensitivity analysis**, as described below, with deterministically assessed variations in the parameter values
- by using **stochastic models** for some, or all, of the parameter values and simulation
- by **comparison** with any available market data.

The idea behind using a stochastic model for pricing is to get a range of scenarios, by varying the parameter values in the model according to their probability distribution function and recalculating the rate of return for each new scenario.

As more and more simulations are run, an almost accurate variance of the rate of return can be achieved.

In theory, a **separate risk discount rate** should be applied to each separate component of the cashflows, as the statistical risk associated with each component will be different.

In practice a single risk discount rate is commonly used, bearing in mind the **'average' risk of the product**.

3 Using Models for Pricing

Meeting profit requirements

The **premium** for the model points can be set so as to produce the **profit required** by the company.

For small policies, where the **sum assured is less**, and hence the level of premium charged, achieving profitability for all model points is difficult. This happens because the **expenses** incurred in relation to each policy will be **fixed**, and allowing for fixed costs reduces their profitability.

Instead, focusing on the total profitability resulting from a group of model points will give more positive results.

The group of model points chosen, aims at representing the **new business mix and volume**, leaving room for a certain degree of **cross subsidy**

3 Using Models for Pricing

Competitive premiums

The premiums, or charges, produced need to be considered for **marketability**. This might lead to a reconsideration of:

- the **design** of the product, so as either to remove features that increase the risks within the net cashflows, or to include features that will differentiate the product from those of competing companies for example providing cover to one more person related to the policyholder upto a certain amount
- the **distribution channel** to be used, if that would permit either a revision of the assumptions to be used in the model, or a higher premium or charges to be used without loss of marketability
- the company's **profit requirement**
- the **size** of the market
- whether to proceed with **marketing** the product.

3 Using Models for Pricing

Business strategy

The **net cashflows** in respect of the model points, appropriately **scaled up** for the expected new business under the product, will be incorporated into a model of the business of the whole company.

The **desired level of profitability** can be considered on an **aggregate level** instead on an individual model point level, where it is not required for every single model point to be profitable, if the overall profitability is achieved.

However, if certain model points are **unprofitable**, then the aggregate profitability of the business is exposed to changes in **mix** and **volume** of the contracts sold.

The actuary can assess the impact on **capital management** of writing the product, by observing the modelled amount and timing of cashflows. If capital is a problem, this may lead to a reconsideration of the design of the product to reduce or amend the timing of its financing requirement.

Once acceptable premiums or charges have been determined for the model points, premiums or charges for all contract variations can be determined.

3 Using Models for Pricing

Once **premiums** for the most **basic contracts** have been calculated, the premiums for other products that differ due to certain variations can be calculated be **adjusting the basic premium** and allowing for the variations. They can also be found by **interpolating** between the premiums for the various model points.

Assessing the capital requirements and the return on capital

The **net cashflows** for the model points can be **grossed up** for the expected new business and used to assess the **amount of capital** that will be required to write the product, either on a regulatory or an economic basis.

Any **one-off development costs** can be added, to the extent that they are not amortized and included in the cashflows used.

This gives the **total capital requirement** and can be compared with the profits expected to emerge from the product so as to determine the expected return on that capital.

Table of contents

4 Using models for setting future financing strategies

- For a benefit scheme, the equivalent to determining the **price for a product** is setting the **future financing strategy**, and similar modelling techniques can be used. The existing membership can be divided into **categories** and represented by a set of **model points (**example – active, deferred and current pensioners).

- A **potential financing strategy** is determined, in terms of both the **amount** and **timing** of future contributions. The cashflows from the existing assets and future contributions can be modelled, as can the liability cashflows, taking all the **possible decrements** into account.

- Unlike an insurance company, a benefit scheme can show a **deficit** at a point in time (ie the value of accumulated assets does not exceed the value of accrued liabilities), provided that there is a **sponsor with a good enough covenant** to make good the shortfall.

- However, the scheme does need to be **solvent** to the extent that it has sufficient assets to meet benefit outgo as it falls due. A well-designed model will check this feature as well as determining the discounted value of asset and liability cashflows.

5 Using models for risk management

- **Cashflow models** are used in risk management to determine the **amount of capital** that it is necessary to hold to support the risks retained by a financial institution

- As well as the full corporate model to assess capital requirements, **models of specific risks** can be used to determine the extent of a risk event that will occur at a **given probability**, even if a full stochastic model is too slow, too complex, or otherwise not used.

- For example, a company that is targeting being able to withstand a 0.1% probability of ruin needs to know what equity market fall to test in a deterministic scenario. A standard **equity market stochastic model** can be used and calibrated to historical performance of the market being considered. By running the model several thousand times and ranking the results, the equity fall that gives the one in a thousand worst result can be found.

Table of contents

6 Valuing Provisions on an Individual Basis

The normal procedure for **determining life assurance** or **pension scheme liabilities** is to value the benefits for each policy or scheme member individually. In many territories this may be required by legislation or regulation. This is more so to ensure that **insurance companies have sufficient provisions** to meet their claim liabilities.

An **approximate matching** using **model points** may lead to **under-reserving**.

Consequently, for published results there is little scope for using model points. However, before finalizing a published basis, many '**what if**' questions might be asked. These could be answered by running a model of the business.

These questions might include:

- The insurance company's ability in **meeting regulatory requirements**, in case of a fall in returns
- The impact on **finances** in case of worsening of mortality experience
- In case of a **reinsurer defaulting** on his obligations or a policyholder lapsing on his premium payments

7 Pricing and Valuing Options and Guarantees

Investment related options and guarantees

In most cases the **options** and **guarantees** that give a provider of benefits on future financial events cause for concern are those that are **dependent on future investment returns**, or an investment value (yield or capital value) at some future point in time.

Because of the uncertainty, a **stochastic investment model** should be used to assess the provisions necessary for such guarantees, as they can establish the **likelihood of the option or guarantee** applying together with the **associated cost**.

If future returns exceed a certain level, or if a value or index is above (or below) a fixed value at some future point, there will be no cost to the company. But if future returns are below that level, there will be a cost, which increases as returns reduce. Hence a range of future investment scenarios should be tested

7 Pricing and Valuing Options and Guarantees

Other options

Insurance products exist where options are not dependent on investment outcomes, but more on death and sickness. For example, a term product may provide coverage upon the death of a spouse along with the main policyholder.

Table of contents

8 Sensitivity Analysis

Sensitivity analysis is a financial model that determines how **target variables** are affected based on **changes** in other **variables** known as **input variables**. It is a way to predict the outcome of a decision given a certain range of variables

- Reliability of the Results

The results from the model **depend on the model** itself and the values assigned to the parameters in the model. Models should not be treated as **black boxes**, the output of which is assumed to be correct. one must leave room for a certain degree of **uncertainty** credited to the model, and there may be a slight chance

8 Sensitivity Analysis

- Understanding the Potential Variability of Experience

The use of a **stochastic model** goes some way to illustrating the **potential variability** of the experience, but the results that it produces are still dependent on the **accuracy of the model** and its parameter values.

In the case of a **deterministic model**, the **potential uncertainty** of the results is greater, because **fewer scenarios are tested**, using the same parameter values for each scenario

The re-running with a series of different sets of parameter values, perhaps chosen from a **probability distribution** for such values, will help to illustrate the likely range in which actual experience may lie, perhaps as far as creating a probability distribution for this experience.

For example, consideration of the effect of a change in the membership profile of a funded pension scheme may be needed to illustrate the extent of potential variability in future contributions if the model used is based on a stable membership profile.

8 Sensitivity Analysis

- Model Error

There is a possibility of **model error** if the model developed is not appropriate for the financial products, schemes, contracts or transactions being modelled.

Checks of **goodness of fit** will be needed to assess the suitability of the model but taking account of expected changes in experience into the future.

The model's accuracy depends on the **quality of assumptions** built in the model. Erroneous assumptions lead to inaccurate outputs.

Also, the model may not always account for **interdependencies** among input variables.

When assumptions are based on historical data, they cannot always be relied upon to predict future results.

8 Sensitivity Analysis

- Parameter Error

The effect of **mis-estimation of parameter values** can also be investigated by carrying out a **sensitivity analysis**. When doing this, any **correlation** between different parameters should be allowed for.

In the case of a model used for pricing, the results from the sensitivity analysis will help to assess the margins that need to be incorporated into the parameter values.

Where any product appears to be highly sensitive to any of the input factors, i.e., a slight change in any of the factors, largely affects the product's profitability, then we might consider redesigning the product, or increase the profitability margins in the assumptions used to calculate premium.

High sensitivity towards increasing withdrawal rates, calls for a reduction in surrender values, high sensitivity towards mortality results in the revision of reinsurance terms.

8 Sensitivity Analysis

- Alternative Ways of Allowing for Risk

The **statistical risk** associated with the parameter values can be allowed through the risk element of the **risk discount rate**.

Here we first establish the **risk-free rate** of return expected by the shareholders from their investment. Any deviation or variation from this risk-free rate is then calculated, and the variation is allowed for in the actual rate of return. This would give us the **risk margin** to be added to the risk-free rate.

An alternative would be to use a predetermined discount rate and then assess the effect on the results of the models of statistical risk.

Under this approach, we find the present value of the cashflows using the risk-free rate, but instead of using the best estimate parameter values, we use more **prudent** or **pessimistic** parameter values. The extent of prudence or pessimism used, corresponds to the risk margin introduced.

Where a probability distribution can be assigned to a parameter, it may be possible to derive the variance of the profit or return on capital analytically.

9 Example

A unit-linked life assurance policy guarantees to pay a maturity value equal to the sum of premiums on the chosen maturity date, or the value of units allocated if greater. At all other times the surrender value is based on the value of units.
Describe the steps involved in assessing the provision to be made for the cost of this guarantee.

The steps involved are:

- Choose a stochastic asset model – a complex model gives better results but takes longer to run.
- Determine assumptions – particularly unit growth rate mean and volatility.
- Determine consistent deterministic assumptions for mortality and surrender rates and future expenses.
- Consider dynamic links between assumptions – eg lapse rates to unit values. □ Choose a time period – probably annual for efficient running.
- Determine appropriate model points for the portfolio.
- The model will project the unit values to maturity, allowing for future premiums and all decrements.
- This will be done for a large number of randomly generated investment scenarios – say between 1,000 and 5,000.

9 Example

- For each scenario and each model point, the projected unit value will be compared with the guaranteed maturity value, and the cost for that particular scenario and model point determined.
- The projected costs are discounted to the present, scaled up by the appropriate factors and summed across all model points.
- The average across all scenarios is the expected cost of the guarantee.
- The variability should be assessed by looking at the quartiles and 5th/95th percentiles, when the results are ranked.
- For reserving purposes, an appropriate ruin probability needs to be chosen. Perhaps 1 in 100, in which case the reserve is the 99th percentile.

Topics covered

- Producing a solution
- Construction of an actuarial model – Deterministic or stochastic model
- Uses of models for pricing
- Uses of models for setting future financing strategies
- Uses of models for risk management
- Valuing provisions on an individual basis
- Pricing and valuing options and guarantees
- Sensitivity analysis