

Class: MSc Sem 4

Subject : Actuarial Practice 2

Chapter: Unit 2 Chapter 5

Chapter Name: Quantifying risk

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1 Risk Quantification

Introduction

For all risk events there are two key features to be assessed:

- the **probability** of the event occurring i.e. the **frequency** of the event
- the **expected loss** if the event occurs i.e. the **severity** of the event.

Each of these is normally a **random variable**, although there are some situations where the loss if the event occurs is a fixed amount rather than a random variable. This situation would normally arise in a single event insurance risk, for example an individual purchasing a term assurance policy that pays a fixed sum assured if the individual dies within the specified term.

A **single event insurance risk** is one where only one claim payment is made for each policy, unlike general insurance policies, where several claims can be made within the period of the cover.

1 Risk Quantification

Subjective assessment

A common approach to **risk assessment** used by financial institutions is to extend the risk identification 'brainstorming' approach discussed earlier so that the **probability** and **cost** or impact of the risk event are each estimated.

These estimates would be on a **five-point scale** (or three-point scale). For a five-point scale the assessments would be based on: 5 = high, 4 = medium-high, 3 = medium, 2 = medium-low, 1 = low.

The product of the **probability** assessment and the **impact** assessment gives a scale of 1 to 25 (or 1 to 9 for the three-point scale) as an assessment of the risk. This **risk-scoring** approach provides a method for ranking risk events.

The organization could then **prioritize** its methods of dealing with risks with a score of higher than a certain amount. The assessment would be carried out with and without possible **risk controls**, to generate a figure for the effectiveness of proposed controls. This will enable the efficiency of risk controls to be assessed against their cost.

1 Risk Quantification

Using a model

A risk event may be assessed by **developing a model** in which the **probability** of loss and the **amount** of loss are both treated as **random variables**.

To use a mathematical model, the first need is to assign a **distribution** both to the probability of the risk event occurring, and to the loss if the event occurs.

All the considerations need to be taken into account in designing an appropriate model – as to whether a **stochastic** or **deterministic** model is appropriate.

Obtaining the **data** to parameterize the model will be a crucial issue, and the availability of data may influence the decision as to what, or whether, a model is used. This is particularly important when rare events are considered.

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2 Evaluation of Risks

Scenario analysis

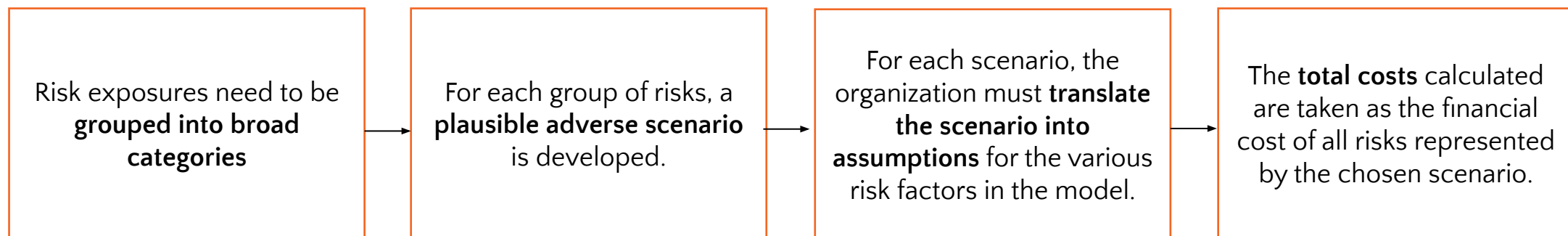
Scenario analysis is a **deterministic** method of evaluating risk. It is useful where it is difficult to fit full probability distributions to risk events (and hence where a stochastic model would be inappropriate). This could be because the risks are not suitable for **mathematical modelling**, or because the distribution would need so many **subjective parameters** that the value of using it is eroded.

Scenario analysis is frequently used when evaluating **operational risks** but can also be used to assess the impact of financial risks such as a global recession.

2 Evaluation of Risks

Scenario analysis

It involves several steps:



One **drawback** to scenario analysis is that it quantifies the **severity** of the scenario but not the **probability** of it occurring.

Organizations often use their **capital models** to determine the probability of a particular scenario occurring.

If capital requirements can be modelled stochastically, then the probability distributions can be used to identify a confidence level for a particular outcome.

2 Evaluation of Risks

Stress testing

Stress testing is also a **deterministic method** of modelling risks, where the **risk events are extreme**. It is commonly used to model extreme market movements but also has applications in modelling **credit** and **liquidity risks**.

For example, in relation to market risk, this involves subjecting an asset portfolio to extreme market moves by radically changing the underlying assumptions and characteristics, in order to gain insight into the portfolio's sensitivities to predefined risk factors. In particular, both asset correlations and volatilities are often observed to increase during extreme market events.

There are **two types** of stress test:

- to identify '**weak areas**' in the portfolio and investigate the effects of localized stress situations by looking at the effect of different combinations of correlations and volatilities
- to gauge the impact of **major market turmoil** affecting all model parameters, while ensuring consistency between correlations while they are 'stressed'.

2 Evaluation of Risks

Combining stress and scenario testing

The **principle of stress testing** can be coupled with **scenario testing** to determine a stress scenario.

Scenario analysis outlines the factors that may be affected under a **given scenario**, on which **stress testing** may be later applied. The overall stress scenario test combines the **individual factor stress tests** simultaneously to allow for any **inter-relationships**.

When constructing a stress scenario, decisions need to be made as to how other **aspects of the business** will react if a stress event occurs.

2 Evaluation of Risks

Combining stress and scenario testing

For example, for a provider of unit-linked investment bonds, a sustained reduction in market values will affect:

- income received from fund management charges
- persistency of existing investment bonds
- new business volumes
- the provider's regulatory capital requirements
- the value of the shareholders' interests
- the probability of any guarantees biting.

All these factors need to be **built into the model**.

The scenarios should be tailored to reveal **weaknesses** in terms of **risk exposure** and **sensitivity** and should thus focus on the risk factors to which the business is most **exposed**.

2 Evaluation of Risks

Reverse stress testing

A requirement of **regulatory bodies** is often that regulated firms carry out a **reverse stress test**.

This is the construction of a **severe stress scenario** that just allows the firm to be able to continue to operate its **business plan**. In layman's language it means formulating a scenario which impacts the business in such a manner that it is just **capable of fulfilling its strategic business plan** and nothing more.

Business plan failure needs to be determined by the firm and needs to consider both the **short-term** and the **long-term** plan. This might, for example, occur if the firm has insufficient capital to meet statutory requirements, or to cover its minimum risk appetite.

It might also occur from a non-financial external event that causes the firm to cease having access to its major market. For well-capitalized firms, a reverse stress test may be an extreme event, but it nevertheless needs to be a plausible scenario

2 Evaluation of Risks

Stochastic modelling

An obvious extension of **stress testing** is a **full stochastic model** with all the variables that give rise to risk being incorporated as probability distributions, and a full set of **dynamic interactions** between the variables specified. The model can then determine the **capital** necessary to (just) avoid ruin at any desired probability level.

Not only is such a model extremely **complex** to specify and build, but the run times also that result from having more than one, or possibly two, variables simulated by stochastic methods become **impractical** with even the most modern computing power.

2 Evaluation of Risks

Stochastic modelling

It is therefore necessary to limit the ideal scope of the model by one or more of the following approaches:

- **Restrict the duration** of the model to two years if the risk criterion is expressed as a one-year ruin probability. Some parts of the model, such as calculation of basic policy reserves for life assurance contracts, will still require projections to run-off.
- **Limit the number of risk variables** that are modelled stochastically. Deterministic approaches can be used for other risk variables. Variables that only have an adverse effect when they move in one direction can be modelled using deterministic scenario analysis. For example, in a benefit scheme it is increasing longevity that will put the scheme under stress, rather than deteriorating mortality.
- Carry out **a number of runs** with a different single stochastic variable, followed by a single deterministic run using all the worst-case scenarios together. This will determine the effect of interactions between the various variables.

It is important to remember that the **results are only as good as the model used**.

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3 Aggregating Risks

Capital requirements and relationship between risks

In managing risk, attention needs to be paid to all risks, though the methods outlined in the previous sections indicate the key risks that merit most management involvement.

In many regulatory regimes for financial product providers, the capital requirement is set in respect of an event occurring within 12 months with a probability 0.5%.

This is frequently called the capital requirement for a '1 in 200-year event'. This phrase can be misleading to non-experts, as it implies that if an event has just occurred, it will be another 200 years before they need to worry about the next one. In practice, rare events, such as stock market crashes and extreme weather events, appear to be happening more frequently than the assumed probability indicates.

The technique of stochastic modelling can be used to **determine capital requirements** for a firm for all risks to which it is exposed. **Stochastic modelling** can provide a complete distribution of outcomes to calculate capital required at a pre-determined probability level.

A stochastic model should allow for **correlations between risk events** for every simulation.

3 Aggregating Risks

Capital requirements and relationship between risks

Fully dependent risk events

If events are fully dependent, then the capital requirement to cover the aggregation of all risks is simply the sum of the capital required for each risk at a pre-determined probability level.

The formula for the resultant capital requirement for that probability level from n dependent risks each with a capital requirement R_j is given by:

$$\text{Capital requirement} = \sum_{j=1}^n R_j .$$

3 Aggregating Risks

Capital requirements and relationship between risks

Fully independent risk events

If the risks are fully independent (and hence uncorrelated) then the capital requirement for a combination of risks that occurs with a given probability is less than the sum of the individual capital requirements.

For example, if the joint distribution of risks demonstrates certain statistical properties, the formula for the resultant capital requirement for n fully independent risks could be:

$$\text{Capital requirement} = \sqrt{\sum_{j=1}^n R_j^2} .$$

3 Aggregating Risks

Capital requirements and relationship between risks

Partially dependent risk events

In most circumstances, there are **dependencies** between risks such that they are neither **fully independent nor fully dependent**.

Dependency between risks is also called **correlation**. If risks are partially dependent (i.e., not perfectly correlated), the capital requirement for a combination of risks that occurs with a given probability is again **less than the sum of the individual capital requirements**.

The extent to which the **overall capital requirement** is less than the sum of the individual capital requirements is called the **diversification benefit**. If the risks are all fully dependent on each other, there is no diversification benefit. The lower the correlation between risks, the higher the diversification benefit.

Diversification is maximized (and overall capital requirements minimized) if the correlations are **negative**.

3 Aggregating Risks

Correlations between risks

Some likely **correlations between risks** are:

- Inflation risk is heavily correlated with expense risk for most long-term financial products.
- Traditionally equity markets have moved in the opposite direction to interest rates, but in recent years this correlation has not been so obvious.
- Falling equity markets are likely to be correlated with increasing lapse rates on unit-linked savings products.
- Operational risk is likely to be weakly correlated with all other risks, because if management are concentrating on some other issue they may not be concentrating on routine operational matters.
- In life insurance the longevity risk on an annuity book is strongly negatively correlated with mortality risk on a term assurance book (not perfect negative correlation because the typical ages are different). An annuity writer can reduce its capital requirements for mortality / longevity by writing term assurances.

3 Aggregating Risks

Other aggregation methods

Other aggregation methods include:

- Correlation matrices
- Copulas

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4 Risk Measures

Liability risks

The most common way of **measuring liability risks** is the analysis of experience – in other words, the ratio of the actual occurrences of an event to the occurrences expected when the risk was accepted.

It is important to stress the need for consistent classification and measurement not only of the risk events, but also of the population exposed to risk.

Apart from analyzing mortality, expenses and withdrawal experience, correspondence in the exposed to risk analysis is also important.

Value at risk

Value at Risk (VaR) generalizes the likelihood of **underperforming** by providing a **statistical measure of downside risk**. VaR represents the maximum potential loss on a portfolio over a given future period with a given degree of confidence.

4 Risk Measures



A 99% one-day VaR is the **maximum loss on a portfolio over a one-day period with 99% confidence**, ie there is a 1% probability of a greater loss.

For example, if the 95% one-month VAR is \$1 million, there is 95% confidence that over the next month the portfolio will not lose more than \$1 million.

VaR can be measured either in **absolute terms** or **relative to a benchmark**.

VaR is based on assumptions that may not be immediately apparent. In particular it is frequently calculated assuming a **normal distribution** of returns. If the distribution of returns is '**fat-tailed**', or skewed, tracking error (with its focus on the standard deviations of returns) may be misleading.

4 Risk Measures



Unfortunately, portfolios exposed to credit risk, systematic bias or derivatives may exhibit **non-normal distributions**. The usefulness of VaR in these situations depends on modelling **skewed** or **fat-tailed distributions** of returns, either in the form of statistical distributions or via Monte Carlo simulations. Lack of sufficient data observations within the 'tails' of the distributions means there is increasing subjectivity in the choice of the underlying probability.

Another shortcoming of VaR is that it does not give a value to the loss that may occur. Hence, another measure known as **Tail VaR** is used. It gives the value of the **expected shortfall**, given that the shortfall has occurred. For example, if we believe that our average loss on the worst 5% of the possible outcomes for a portfolio is \$5 million, then the Tail VaR is \$5 million for the 5% tail.

5 Risk Portfolios

Risk categorization and quantification

It is advisable for an individual or company exposed to risk to establish a **risk portfolio** or **risk register**. The risk portfolio **categorizes** the various risks to which the business is exposed.

Against each risk would be recorded a quantification of:

- impact
- probability

The product of the impact and the probability measures gives an idea of the relative importance of the various risks.

5 Risk Portfolios

Risk response

The risk portfolio can then be extended to indicate how the risk has been dealt with. Whether the risk has been :

- **avoided** – by not taking the risk at all, eg an insurance company rejecting a client for a term life assurance whose medical underwriting claims he is a chain smoker
- **retained** (and how much capital is needed to support it)
- **diversified** (and a revised assessment of the remaining combination of risks)
- **mitigated** (and a revised assessment of the remaining risk) by opting for a reinsurance policy

5 Risk Portfolios

Additional details

For risks that are **retained**, the risk portfolio becomes a more **detailed risk register**. It would also include:

- details of control measures
- reassessment of value and impact after controls
- the risk owner
- the Board committee or senior manager with oversight of the risk (key strategic risks overseen by the full Board)
- identification of concentrations of risk and the need for management action in these areas.

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6 Risk Reporting

Importance of risk reporting

The production of regular **risk reporting** is vital so that management can understand and successfully manage the risks within its business. The production of **regular risk reporting allows the management** of a business to:

- identify **any new risks** faced by the business
- obtain a **better understanding of the risks** faced by the business in terms of quantifying the materiality and financial impact of individual risks
- determine **appropriate risk and control systems** to manage specific risks
- proactively **monitor and manage the effectiveness of risk** and control systems within its business
- assess whether the risks faced by a business are **changing over time**
- assess the **interaction** between individual risks
- appropriately **price, reserve** and determine any **capital requirements** for its business.

6 Risk Reporting

Regular risk reporting is likely to be helpful for other stakeholders as well.

For example, it could:

- give **shareholders** or **potential shareholders** in a business a greater understanding of the attractiveness of that business for investment
- help **credit rating agencies** determine an appropriate rating for the business
- give a **regulator** a greater understanding of the areas within a business which could require more scrutiny.

6 Risk Reporting

Reporting at an enterprise level

Risk governance discusses the advantages of **managing** and **budgeting** for risk at the enterprise level. By budgeting for risk across the whole enterprise, maximum use can be made of **diversification benefits**, and thus the minimum capital required to support the risks undertaken.

One of the consequences of this approach is that it is necessary to have a system of **risk reporting** across the whole enterprise. It is important for the Chief Risk Officer to be aware of whether all business units are using the risk allocation that they have been given.

Risk allocation is the maximum level of risk exposure that each business is permitted to accept.

If two business units are allocated risk exposures that diversify away at the enterprise level, but one of the two units does not take on the risk exposure allocated, this could increase the capital requirements of the enterprise. Risk exposures will not be matched, and additional capital will need to be held to cover the unbalanced risks taken on.

6 Risk Reporting

Reporting at an enterprise level

Where **diversification** between business units is used to minimize group capital requirements, the individual business units will need to report data at a much more granular level than their **own total capital requirement** to the group.

Analyzing the data from diverse business units can be a costly task, especially for multinational operations. There is a trade-off between the **costs of the additional analysis** required to minimize capital requirements in this way and the **cost of holding additional capital** if risk diversification between business units is not assumed.

6 Risk Reporting

Issues relating to reporting risk externally

The usual way for a financial product provider to report on risk is by **quantifying the capital required** to protect against ruin at a given ruin probability.

The **annual report** may only address risk issues in a qualitative manner, leaving the quantification of risk capital and solvency requirements to be covered in a separate report.

This is normally carried out using a **combination of stochastic and deterministic modelling** techniques. A common approach is to use a stochastic model to determine the risk event at the required ruin probability, and then to run a deterministic projection using that risk event.

For example, a stochastic asset model might be used to determine that a fall in the domestic equity market of 45% in one year occurs with a 0.5% probability. In assessing a market risk capital requirement with a ruin probability of 0.5%, the company's projection models might be run assuming an equity fall of 45% on day one.

6 Risk Reporting

Issues relating to reporting risk externally

The main issues facing providers of financial benefits in completing the assessment are:

- Should the **ruin probability** be expressed over a single year or over the whole run-off of the business? In the latter case the ruin probability will be a much higher figure than in the former.
- A stochastic model with more than two stochastic variables will be impractical to run. Thus, a means of assessing the **correlation** between the risks assessed needs to be developed. The most common technique uses a **correlation matrix**. Populating the correlation matrix is a largely subjective exercise.
- **Interactions between risks** may mean that the effect of multiple risk events is greater or less than the sum of the individual risks. A practical technique needs to be developed to address this.
- Some risks, particularly **operational risk**, are still highly subjective in their assessment, particularly when it is necessary to construct a plausible adverse scenario that occurs at a very low probability. The temptation is to think only of risk events that have occurred, which are likely to be more common than the required ruin probability.

6 Risk Reporting

Issues relating to reporting risk externally

- Using past data to estimate **future consequences** of rare events needs to be undertaken with caution. For example, the 1918-1919 Spanish 'flu pandemic has been assessed as an event with a probability of between 0.5% and 1%. However, because of advances in medical science, particularly the discovery of antibiotics, it is estimated that the same number of deaths as in 1918-1919 would now occur only from a much rarer event, perhaps one with 0.1% to 0.2% probability.

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