

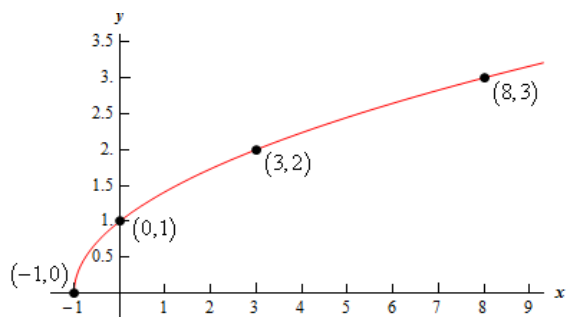
Chapter 1

Answer 1:

Here is the table of points we'll use for this problem.

x	$f(x)$	(x, y)
-1	0	$(-1, 0)$
0	1	$(0, 1)$
3	2	$(3, 2)$
8	3	$(8, 3)$

Here is a sketch of the function.



Answer 2:

Given that

$$x = \sqrt[5]{2y + 11}$$

Solve the equation from above for y .

$$x^5 = 2y + 11$$

$$x^5 - 11 = 2y$$

$$\frac{x^5 - 11}{2} = y$$

Replace y with $A^{-1}(x)$.

$$A^{-1}(x) = \frac{x^5 - 11}{2}$$

Checking once,

$$\begin{aligned}(A \circ A^{-1})(x) &= A[A^{-1}(x)] = A\left[\frac{x^5 - 11}{2}\right] \\ &= \sqrt[5]{2\left(\frac{x^5 - 11}{2}\right) + 11} = \sqrt[5]{x^5 - 11 + 11} = \sqrt[5]{x^5} = x\end{aligned}$$

The check works out so we know we did the work correctly and have inverse.

Answer 3:

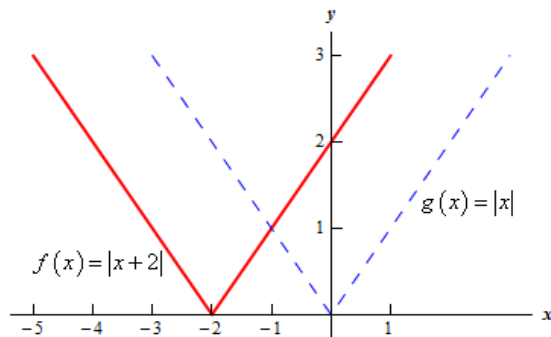
Let's first identify the "base" function (*i.e.* the function we are transforming). In this case it looks like we are transforming $g(x) = |x|$

The function that have here looks like it can be written as,

$$f(x) = |x + 2| = g(x + 2)$$

Therefore, we can see that the graph of $f(x)$ is simply going to be the graph of $g(x)$ shifted left by 2.

Here is a sketch of both the base function (blue dashed curve) and the function we were asked to graph (red solid curve).



Answer 4:

a)

$$(f - g)(2) = f(2) - g(2) = 14 - (-4) = \boxed{18}$$

b)

$$(g - f)(2) = g(2) - f(2) = -4 - 14 = \boxed{-18}$$

c)

$$fg = f(x)g(x) = (6x + 2)(10 - 7x) = \boxed{-42x^2 + 46x + 20}$$

d)

$$\left(\frac{f}{g}\right)(x) = \frac{f(x)}{g(x)} = \boxed{\frac{6x + 2}{10 - 7x}}$$

Answer 5:

We have a piecewise function here so we need to make sure and pick points from each range of x when building our table of points.

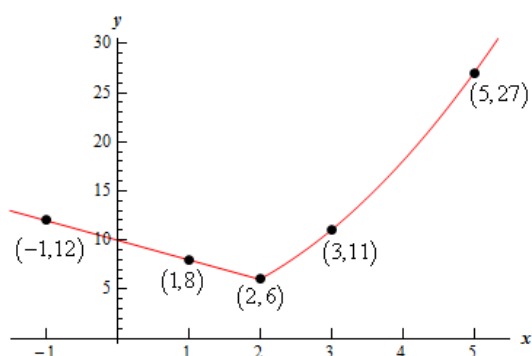
Also, we'll want to plug the "cutoff point", $x=2$, into the top equation as well so we know where it will be at the end of the range of x 's for which it is valid.

Here is the table of points we'll use for this problem.

x	$10 - 2x$	(x, y)
-1	12	$(-1, 12)$
1	8	$(1, 8)$
2	6	$(2, 6)$

x	$x^2 + 2$	(x, y)
2	6	$(2, 6)$
3	11	$(3, 11)$
5	27	$(5, 27)$

Here is a sketch of the function.



Note that in this case the two pieces “met” at the point $(2, 6)$. Sometimes this will happen with these problems and sometimes it won't. This is the reason we always evaluate both equations at the “cutoff point”. Without doing that we wouldn't know if the pieces meet or not.

Answer 6:

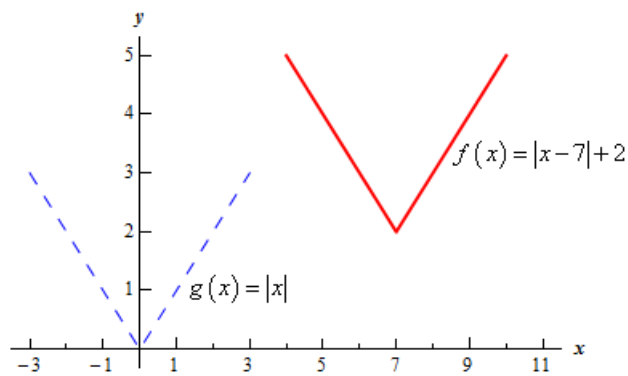
Let's first identify the “base” function (*i.e.* the function we are transforming). In this case it looks like we are transforming $g(x) = |x|$.

The function that have here looks like it can be written as,

$$f(x) = |x - 7| + 2 = g(x - 7) + 2$$

Therefore, we can see that the graph of $f(x)$ is simply going to be the graph of $g(x)$ shifted right by 7 and up by 2.

Here is a sketch of both the base function (blue dashed curve) and the function we were asked to graph (red solid curve).



Answer 7:

Solution:

Take $f(x)$ as the original function and observe how it is moving/transforming to give $g(x)$. Observe the **vertex** of both graphs to get an idea. It is very clear that

- it moved 6 units to the left and so the function is $f(x + 6)$.
- it then reflected with respect to the x-axis, so the function is $-f(x + 6)$.

Answer: $g(x) = -f(x + 6)$.

Answer 8:

☐ $f(x - 3) - 4$

☐ $f(x - 3) + 4$

☒ $f(x + 3) + 4$ ✓

☐ $f(x + 3) - 4$

Answer 9:

The given function is $f(x) = 5x + 4$

we rewrite it as $y = 5x + 4$ and simplify it to find the value of x .

$$y = 5x + 4$$

$$y - 4 = 5x$$

$$x = (y - 4)/5$$

$$f^{-1}(x) = (x - 4)/5$$

Answer: Therefore the inverse function is $f^{-1}(x) = (x - 4)/5$

Answer 10: B

Answer 11:

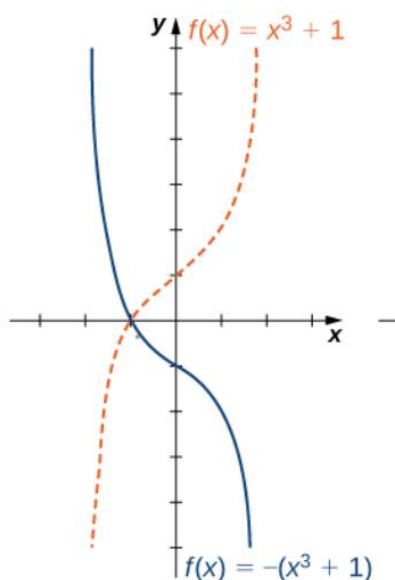
1. $f(-3) = 3 - 5(-3) - 2(-3)^2 = 0$

2.

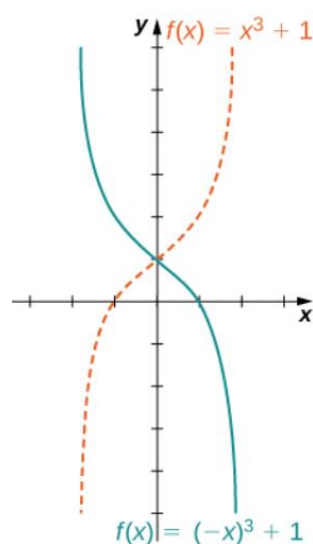
$$\begin{aligned} f(7-4x) &= 3 - 5(7-4x) - 2(7-4x)^2 \\ &= 3 - 5(7-4x) - 2(49 - 56x + 16x^2) \\ &= 3 - 35 + 20x - 98 + 112x - 32x^2 \\ &= -130 + 132x - 32x^2 \end{aligned}$$

Answer 12:

(a) $f(x) = -(x^3 + 1)$



(b) $f(x) = (-x^3) + 1$



Answer 13:

Starting with $y = 2x^3 + 1$ we solve for x as follows:

$$y - 1 = 2x^3 \quad \rightarrow \quad \frac{y - 1}{2} = x^3 \quad \rightarrow \quad x = \sqrt[3]{\frac{y - 1}{2}}.$$

Therefore, $f^{-1}(x) = \sqrt[3]{\frac{x - 1}{2}}.$

Answer 14:

$$f(x) = \frac{x+2}{x-2}$$

we can find $f^{-1}(x)$ as follows. First, let $y = f(x)$. Then,

$$\begin{aligned} y &= \frac{x+2}{x-2} \\ y(x-2) &= x+2 \\ xy - x &= 2 + 2y \\ x(y-1) &= 2(1+y) \\ x &= \frac{2(y+1)}{(y-1)} \end{aligned}$$

Therefore, $f^{-1}(x) = \frac{2(x+1)}{(x-1)}$.

Answer 15:

Let us first notice that the function,

$$h(x) = e^x + x + 1,$$

is one-to-one. Since we only need to find $h^{-1}(2)$, there is no need to solve for the general case (which would be difficult for this example). Instead, notice that

$$h(0) = 1 + 1 = 2.$$

Thus, $h^{-1}(2) = 0$.

Chapter 2

Answer 1:

We know that the first thing that we should try to do is simply plug in the value and see if we can compute the limit.

$$\lim_{x \rightarrow 2} (8 - 3x + 12x^2) = 8 - 3(2) + 12(4) = \boxed{50}$$

Answer 2:

$$\lim_{t \rightarrow -3} \frac{6 + 4t}{t^2 + 1} = \frac{-6}{10} = \boxed{-\frac{3}{5}}$$

Answer 3:

In this case we see that if we plug in the value we get 0/0. Recall that this DOES NOT mean that the limit doesn't exist. We'll need to do some more work before we make that conclusion. All we need to do here is some simplification and then we'll reach a point where we can plug in the value.

$$\lim_{z \rightarrow 8} \frac{2z^2 - 17z + 8}{8 - z} = \lim_{z \rightarrow 8} \frac{(2z - 1)(z - 8)}{-(z - 8)} = \lim_{z \rightarrow 8} \frac{2z - 1}{-1} = \boxed{-15}$$

Answer 4:

$$\lim_{h \rightarrow 0} \frac{(6 + h)^2 - 36}{h} = \lim_{h \rightarrow 0} \frac{36 + 12h + h^2 - 36}{h} = \lim_{h \rightarrow 0} \frac{h(12 + h)}{h} = \lim_{h \rightarrow 0} (12 + h) = \boxed{12}$$

Answer 5:

a) For this part we know that $-6 < 1$ and so there will be values of x on both sides of -6 in the range $x < 1$ and so we can assume that, in the limit, we will have $x < 1$. This will allow us to use the piece of the function in that range and then just use standard limit techniques to compute the limit.

$$\lim_{x \rightarrow -6} f(x) = \lim_{x \rightarrow -6} (7 - 4x) = \boxed{31}$$

b) This part is going to be different from the previous part. We are looking at the limit at $x=1$ and that is the "cut-off" point in the piecewise functions. Recall from the discussion in the section, that this means that we are going to have to look at the two one sided limits.

$$\lim_{x \rightarrow 1^-} f(x) = \lim_{x \rightarrow 1^-} (7 - 4x) = \underline{3} \quad \text{because } x \rightarrow 1^- \text{ implies that } x < 1$$

$$\lim_{x \rightarrow 1^+} f(x) = \lim_{x \rightarrow 1^+} (x^2 + 2) = \underline{3} \quad \text{because } x \rightarrow 1^+ \text{ implies that } x > 1$$

So, in this case, we can see that,

$$\lim_{x \rightarrow 1^-} f(x) = \lim_{x \rightarrow 1^+} f(x) = 3$$

and so we know that the overall limit must exist and,

$$\lim_{x \rightarrow 1} f(x) = \boxed{3}$$

Answer 6:

a)

From the graph we can see that,

$$\boxed{f(-4) = 3}$$

because the closed dot is at the value of $y = 3$.

We can also see that as we approach $x = -4$ from the left the graph is approaching a value of 3 and as we approach from the right the graph is approaching a value of -2. Therefore, we get,

$$\lim_{x \rightarrow -4^-} f(x) = 3 \quad \& \quad \lim_{x \rightarrow -4^+} f(x) = -2$$

Now, because the two one-sided limits are different we know that,

$$\boxed{\lim_{x \rightarrow -4} f(x) \text{ does not exist}}$$

b)

From the graph we can see that,

$$\boxed{f(-1) = 4}$$

because the closed dot is at the value of $y = 4$.

We can also see that as we approach $x = -1$ from both sides the graph is approaching the same value, 4, and so we get,

$$\lim_{x \rightarrow -1^-} f(x) = 4 \quad \& \quad \lim_{x \rightarrow -1^+} f(x) = 4$$

The two one-sided limits are the same so we know,

$$\boxed{\lim_{x \rightarrow -1} f(x) = 4}$$

c)

From the graph we can see that,

$$\boxed{f(2) = -1}$$

because the closed dot is at the value of $y = -1$.

We can also see that as we approach $x = 2$ from the left the graph is approaching a value of -1 and as we approach from the right the graph is approaching a value of 5. Therefore, we get,

$$\lim_{x \rightarrow 2^-} f(x) = -1 \quad \& \quad \lim_{x \rightarrow 2^+} f(x) = 5$$

Now, because the two one-sided limits are different we know that,

$$\boxed{\lim_{x \rightarrow 2} f(x) \text{ does not exist}}$$

d)

Because there is no closed dot for $x = 4$ we can see that,

$$f(4) \text{ does not exist}$$

We can also see that as we approach $x = 4$ from both sides the graph is approaching the same value, 2, and so we get,

$$\lim_{x \rightarrow 4^-} f(x) = 2 \quad \& \quad \lim_{x \rightarrow 4^+} f(x) = 2$$

The two one-sided limits are the same so we know,

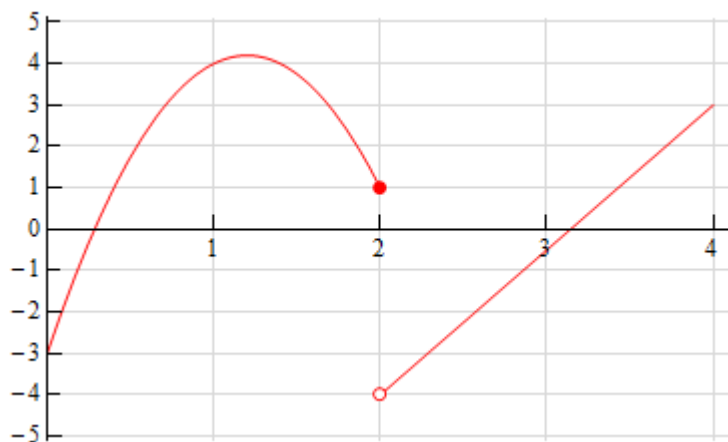
$$\lim_{x \rightarrow 4} f(x) = 2$$

Always recall that the value of a limit (including one-sided limits) does not actually depend upon the value of the function at the point in question. The value of a limit only depends on the values of the function around the point in question. Therefore, even though the function doesn't exist at this point the limit and one-sided limits can still have a value.

Answer 7:

There are literally an infinite number of possible graphs that we could give here for an answer. However, all of them must have a closed dot on the graph at the point (2,1), the graph must be approaching a value of 1 as it approaches $x=2$ from the left (as indicated by the left-hand limit) and it must be approaching a value of -4 as it approaches $x=2$ from the right (as indicated by the right-hand limit).

Here is a sketch of one possible graph that meets these conditions.



Answer 8:

$$\lim_{x \rightarrow \infty} \frac{x}{\sqrt{1+2x^2}} \quad [\text{Form } \frac{\infty}{\infty}]$$

This limit is of the form $\frac{\infty}{\infty}$, Here, We can cancel a factor going to ∞ out of the numerator and denominator.

$$\begin{aligned} \lim_{x \rightarrow \infty} \frac{x}{\sqrt{1+2x^2}} \\ = \lim_{x \rightarrow \infty} \frac{x}{x\sqrt{\frac{1}{x^2}+2}} \end{aligned}$$

Factor x becomes ∞ at x tends to ∞ , So we need to cancel this factor from numerator and denominator.

$$\begin{aligned} &= \lim_{x \rightarrow \infty} \frac{1}{\sqrt{\frac{1}{x^2}+2}} \\ &= \frac{1}{\sqrt{\frac{1}{\infty^2}+2}} = \frac{1}{\sqrt{0+2}} = \frac{1}{\sqrt{2}} \end{aligned}$$

Answer 9:

Before starting the solution recall that in order for a function to be continuous at $x = a$ both $f(a)$ and $\lim_{x \rightarrow a} f(x)$ must exist and we must have,

$$\lim_{x \rightarrow a} f(x) = f(a)$$

Using this idea it should be fairly clear where the function is not continuous.

First notice that at $x = -4$ we have,

$$\lim_{x \rightarrow -4} f(x) = 3 \neq -2 = \lim_{x \rightarrow -4^+} f(x)$$

and therefore, we also know that $\lim_{x \rightarrow -4} f(x)$ doesn't exist. We can therefore conclude that $f(x)$ is **discontinuous** at $x = -4$ because the limit does not exist.

Likewise, at $x = 2$ we have,

$$\lim_{x \rightarrow 2^-} f(x) = -1 \neq 5 = \lim_{x \rightarrow 2^+} f(x)$$

and therefore, we also know that

$$\lim_{x \rightarrow 2} f(x)$$

doesn't exist. So again, because the limit does not exist, we can see that $f(x)$ is **discontinuous** at $x = 2$.

Finally let's take a look at $x = 4$. Here we can see that,

$$\lim_{x \rightarrow 4^-} f(x) = 2 = \lim_{x \rightarrow 4^+} f(x)$$

and therefore, we also know that $\lim_{x \rightarrow 4} f(x) = 2$. However, we can also see that $f(4)$ doesn't exist and so once again $f(x)$ is **discontinuous** at $x = 4$ because this time the function does not exist at $x = 4$.

All other points on this graph will have both the function and limit exist and we'll have $\lim_{x \rightarrow a} f(x) = f(a)$ and so will be continuous.

In summary then the points of discontinuity for this graph are : $x = -4$, $x = 2$ and $x = 4$.

Answer 10:

(a)

So, here we go.

$$\lim_{x \rightarrow -1} f(x) = \lim_{x \rightarrow -1} \frac{4x + 5}{9 - 3x} = \frac{\lim_{x \rightarrow -1} (4x + 5)}{\lim_{x \rightarrow -1} (9 - 3x)} = \frac{4 \lim_{x \rightarrow -1} x + \lim_{x \rightarrow -1} 5}{\lim_{x \rightarrow -1} 9 - 3 \lim_{x \rightarrow -1} x} = \frac{4(-1) + 5}{9 - 3(-1)} = f(-1)$$

So, we can see that $\lim_{x \rightarrow -1} f(x) = f(-1)$ and so the function **is continuous** at $x = -1$.

(b)

For justification on why we can't just plug in the number here check out the comment at the beginning of the solution to (a).

Here is the work for this part.

$$\lim_{x \rightarrow 0} f(x) = \lim_{x \rightarrow 0} \frac{4x + 5}{9 - 3x} = \frac{\lim_{x \rightarrow 0} (4x + 5)}{\lim_{x \rightarrow 0} (9 - 3x)} = \frac{4 \lim_{x \rightarrow 0} x + \lim_{x \rightarrow 0} 5}{\lim_{x \rightarrow 0} 9 - 3 \lim_{x \rightarrow 0} x} = \frac{4(0) + 5}{9 - 3(0)} = f(0)$$

So, we can see that $\lim_{x \rightarrow 0} f(x) = f(0)$ and so the function **is continuous** at $x = 0$.

(c)

For practice you might want to verify that,

$$\lim_{x \rightarrow 3^-} f(x) = \infty \qquad \lim_{x \rightarrow 3^+} f(x) = -\infty$$

and so $\lim_{x \rightarrow 3} f(x)$ also doesn't exist.

Answer 11:

I

Answer 12:

I. $f(1) = 1$

II. $\left[\lim_{x \rightarrow 1^+} f(x) = 1 \neq \lim_{x \rightarrow 1^-} f(x) = -2 \right]$
 $\lim_{x \rightarrow 1} \text{DNE}$

$\therefore f(x)$ is not continuous at $x = 1$

a