2. PORTFOLIO VaR

Return of a portfolio = $\sum_{i=1}^{N} W_i R_{i,t+1}$ ——— (1)

i.e.

$R_{p,t+1}$

$$R_p = W_1 R_1 + W_2 R_2 + W_3 R_3 + \dots W_N R_N$$

$$= \begin{bmatrix} W_1 & W_2 & W_3 & \dots & W_N \end{bmatrix} \begin{bmatrix} R_1 \\ R_2 \\ \vdots \\ R_N \end{bmatrix}$$

$$= W' R$$

W' = transposed vector (i.e) horizontal of weights

R = Vertical vector containing individual asset returns.

Expected Return of a Portfolio

$$E(R_p) = \mu_p = \sum^{N} W_i \mu_i$$

$$V(R_p) = \sigma^2_p$$

$$= \sum_{i=1}^{N} W_i^2 \sigma_i^2 + \sum_{i=1}^{N} \sum_{j=1, j\neq i}^{N} W_i W_j \sigma$$

$$\frac{N(N-1)}{2} = \frac{10(10-1)}{2}$$

$= 45$ different terms

$$\sigma^2_p = [W_1 \; W_2 \; W_3 \; W_4 \; \cdots \; W_N]$$

$$\begin{bmatrix} \sigma_1^2 & \sigma_{12} & \sigma_{13} & \sigma_{14} & \cdots & \sigma_{1N} \\ \sigma_{21} & \sigma_2^2 & \sigma_{23} & \sigma_{24} & \cdots & \sigma_{2N} \\ \sigma_{N1} & \sigma_{N2} & \sigma_{N3} & \cdots & & \sigma^2_N \end{bmatrix}$$

$$\begin{bmatrix} W_1 \\ W_2 \\ W_3 \\ \vdots \\ W_N \end{bmatrix}$$

$$M_1 = [1 \quad 2 \quad 3]$$

$$\begin{bmatrix} 4 & 7 & 10 \\ 5 & 8 & 11 \\ 6 & 9 & 12 \end{bmatrix}$$

$$\begin{bmatrix} 13 \\ 14 \\ 15 \end{bmatrix}$$

$$\left[1\times10 + 2\times11 + 3\times12\right]\begin{bmatrix}13\\14\\15\end{bmatrix}$$

$$\left[\begin{matrix}32 & 1\times4+2\times5+3\times6 & 1\times7+2\times8+3\times9\\ & 50 & \\ & 68 & \end{matrix}\right]\begin{bmatrix}13\\14\\15\end{bmatrix}$$

$$32\times13 + 50\times14 + 68\times15 = 1636 - 2136$$

Eg -

$$\sigma_p^2 = [w_1\ w_2\ \dots\ w_N]$$

$$\begin{bmatrix}\sigma_1^2 & \sigma_{12} & \sigma_{13} & \dots & \sigma_{1N}\\ \sigma_{21} & \sigma_{22} & \sigma_{23} & \dots & \sigma_{2N}\\ \sigma_{N1} & \sigma_{N2} & \sigma_{N3} & \dots & \sigma_N^2\end{bmatrix}\begin{bmatrix}w_1\\w_2\\w_3\\ \vdots\\w_N\end{bmatrix}$$

$$= \left[w_1\times\sigma_1^2 + w_2\times\sigma_{21} + w_3\times\sigma_{31} + \dots + w_N\times\sigma_{N1}\right.$$

$$\left.w_1\times\sigma_{12} + w_2\times\sigma_2^2 + w_3\times\sigma_{32} + \dots\right]$$

$$\begin{bmatrix}w_1\\w_2\\ \vdots\\w_N\end{bmatrix}$$

$$= \left[w_1\times\sigma_1^2 + w_2\times\sigma_{21} + \dots + w_N\times\sigma_{N1}\right)\times w_1$$

$$= \left[w_1^2\times\sigma_1^2 + w_1 w_2\,\sigma_{21} + \dots + w_1\times w_N\times\sigma_{N1}\right.$$

$$\sigma^2_p = w' \Sigma w$$

$W \rightarrow$ Initial Investment / Total Amount of a portfolio

$w_i \rightarrow$ Weights

$$W_i = W \times w_i$$

$$X_i = W \times w_i$$

$$\sigma^2_p \times W^2 = x' \Sigma x$$

Variance of a portfolio

$VaVaR_p = \alpha \sigma_p W$

$\alpha \rightarrow$ Confidence interval C translate

S.

.D α such that probability of

loss is low

$\alpha =$ Confidence interval C translate

into S.D α such that

probability of a loss worse than $-\alpha$ is c

$$VaR_p = \alpha \sqrt{x' \Sigma x}$$

EXAMPLE:

Asset 1 $\sigma_1 = 6\%$

Asset 2 $\sigma_2 = 14\%$

$W_1 = 4m$ $W_2 = 2m$