Chapter 1 Unit 1

Q1.

An investment bank owns equities with a current market value of $\$1m$. The fund manager wishes to estimate the Value at Risk over a 10-day period at the 5% lower tail level. She does so assuming that the investment returns on the equities over a 10-day period are normally distributed with a standard deviation of 5%. The mean return is assumed to be zero, which assumption is often made in practice when calculating the Value at Risk over such a short time period.

Calculate the 10-Day VaR.

Q2.

Consider a new hedge fund that is about to invest $\$100k$ in high-risk information technology (IT) shares. Based on the available historical performance data, the annual investment return on similar existing hedge funds investing in IT shares is assumed to follow a normal distribution with mean 10% *pa* and standard deviation 20% *pa*.

(i) Use the above information to calculate the hedge fund's 1-year Value at Risk based on the 5% lower tail. [2]

(ii) Comment in detail on the validity of your calculation in (i). [5]

(iii) The hedge fund now decides to leverage its returns by borrowing $\$100k$ at a fixed rate of 5% *pa*, the new money being invested entirely in additional IT shares. Calculate the increase in the hedge fund's estimated 1-year Value at Risk based on the 5% lower tail due to leveraging. [2]

[Total 9]

Q3.

(i) Define Value at Risk. [1]

(ii) Calculate the Value at Risk at the 5% significance level over a 30-day period for a portfolio of £500*m* equity shares and £300*m* of bonds. You should assume the following:

- the annual return on equities $R_e \sim N\left(10\%,(14\%)^2\right)$
- the annual return on bonds $R_b \sim N\left(5\%,(8\%)^2\right)$
- the correlation between bonds and equities is 0.6. [4]

(iii) Discuss the drawbacks of using this calculation to measure the risk of the portfolio. [5]

Q4.

i. What are the Drawbacks of Value at Risk?
ii. Compare Historical VaR, Parametric VaR and Monte Carlo VaR

Q5. Calculate the 5% Daily VaR on the portfolio. Assume 252 Trading days in a year.

		Annualized	
	Allocation	**Return**	**Standard Deviation**
SPY	80%	10.5%	20.0%
SPLB	20%	6.0%	8.5%

Note: The correlation of SPLB and SPY = −0.06.

Q6

1. The parameters of normal distribution required to estimate parametric VaR are:

A. expected value and standard deviation.

B. skewness and kurtosis.

C. standard deviation and skewness.

Show Solution

2. Assuming a daily expected return of 0.0384% and daily standard deviation of 1.0112% (as in the example in the text), which of the following is *closest* to the 1% VaR for a $150 million portfolio? Express your answer in dollars.

A. $3.5 million

B. $2.4 million

C. $1.4 million

Show Solution

3. Assuming a daily expected return of 0.0384% and daily standard deviation of 1.0112% (as in the example in the text), the daily 5% parametric VaR is $2,445,150. Rounding the VaR to $2.4 million, which of the following values is *closest* to the annual 5% parametric VaR? Express your answer in dollars.

A. $38 million

B. $25 million

C. $600 million

Q7

Historical Simulation VaR

1. Which of the following statements about the historical simulation method of estimating VaR is *most* correct?

A. A 5% historical simulation VaR is the value that is 5% to the left of the expected value.

B. A 5% historical simulation VaR is the value that is 1.65 standard deviations to the left of the expected value.

C. A 5% historical simulation VaR is the fifth percentile, meaning the point on the distribution beyond which 5% of the outcomes result in larger losses.

Show Solution

2. Which of the following is a limitation of the historical simulation method?

A. The past may not repeat itself.

B. There is a reliance on the normal distribution.

C. Estimates of the mean and variance could be biased.

Show Solution

Q8

Monte Carlo Simulation VaR

1. When will the Monte Carlo method of estimating VaR produce virtually the same results as the parametric method?

A. When the Monte Carlo method assumes a non-normal distribution.

B. When the Monte Carlo method uses the historical return and distribution parameters.

C. When the parameters and the distribution used in the parametric method are the same as those used in the Monte Carlo method and the Monte Carlo method uses a sufficiently large sample.

Show Solution

2. Which of the following is an advantage of the Monte Carlo method?

A. The VaR is easy to calculate with a simple formula.

B. It is flexible enough to accommodate many types of distributions.

C. The number of necessary simulations is determined by the parameters.

Show Solution

Q9

1. Which of the following is **not** an advantage of VaR?

   A. It is a simple concept to communicate.

   B. There is widespread agreement on how to calculate it.

   C. It can be used to compare risk across portfolios or trading units.

   Show Solution

2. Which of the following is a limitation of VaR?

   A. It requires the use of the normal distribution.

   B. The maximum VaR is prescribed by federal securities regulators.

   C. It focuses exclusively on potential losses, without considering potential gains.

   Show Solution

Q10

1. Conditional VaR measures the:

   A. VaR over all possible losses.

   B. VaR under normal market conditions.

   C. average loss, given that VaR is exceeded.

   Show Solution

2. Which of the following correctly identifies incremental VaR?

   A. The change in VaR from increasing a position in an asset.

   B. The increase in VaR that might occur during extremely volatile markets.

   C. The difference between the asset with the highest VaR and the asset with the second highest VaR.

   Show Solution

3. Which of the following statements is correct about marginal VaR?

   A. The marginal VaR is the same as the incremental VaR.

   B. The marginal VaR is the VaR required to meet margin calls.

   C. Marginal VaR estimates the change in VaR for a small change in a given portfolio holding.

   Show Solution

Q11

Randy Gorver, chief risk officer at Eastern Regional Bank, and John Abell, assistant risk officer, are currently conducting a risk assessment of several of the bank's independent investment functions. These reviews include the bank's fixed-income investment portfolio and an equity fund managed by the bank's trust department. Gorver and Abell are also assessing Eastern Regional's overall risk exposure.

Eastern Regional Bank Fixed-Income Investment Portfolio

The bank's proprietary fixed-income portfolio is structured as a barbell portfolio: About half of the portfolio is invested in zero-coupon Treasuries with maturities in the 3- to 5-year range (Portfolio P_1), and the remainder is invested in zero-coupon Treasuries with maturities in the 10- to 15-year range (Portfolio P_2). Georges Montes, the portfolio manager, has discretion to allocate between 40% and 60% of the assets to each maturity "bucket." He must remain fully invested at all times. Exhibit 1 shows details of this portfolio.

Exhibit 1: US Treasury Barbell Portfolio

	Maturity	
	P_1	P_2
	3-5 Years	10-15 Years
Average duration	3.30	11.07
Average yield to maturity	1.45%	2.23%
Market value	$50.3 million	$58.7 million

Trust Department's Equity Fund

a. **Use of Options**: The trust department of Eastern Regional Bank manages an equity fund called the Index Plus Fund, with $325 million in assets. This fund’s objective is to track the S&P 500 Index price return while producing an income return 1.5 times that of the S&P 500. The bank’s chief investment officer (CIO) uses put and call options on S&P 500 stock index futures to adjust the risk exposure of certain client accounts that have an investment in this fund. The portfolio of a 60-year-old widow with a below-average risk tolerance has an investment in this fund, and the CIO has asked his assistant, Janet Ferrell, to propose an options strategy to bring the portfolio’s delta to 0.90.
b. **Value at Risk**: The Index Plus Fund has a value at risk (VaR) of $6.5 million at 5% for one day. Gorver asks Abell to write a brief summary of the portfolio VaR for the report he is preparing on the fund’s risk position.

Combined Bank Risk Exposures

The bank has adopted a new risk policy, which requires forward-looking risk assessments in addition to the measures that look at historical risk characteristics. Management has also become very focused on tail risk since the subprime crisis and is evaluating the bank's capital allocation to certain higher-risk lines of business. Gorver must determine what additional risk metrics to include in his risk reporting to address the new policy. He asks Abell to draft a section of the risk report that will address the risk measures' adequacy for capital allocation decisions.

Q11 a

Practice Problem

Q. To comply with the new bank policy on risk assessment, which of the following is the *best* set of risk measures to add to the chief risk officer’s risk reporting?

A. Conditional VaR, stress test, and scenario analysis
B. Monte Carlo VaR, incremental VaR, and stress test
C. Parametric VaR, marginal VaR, and scenario analysis

Q11 b.

Practice Problem

Q. Based on Exhibit 1, Flusk's portfolio is expected to experience:

A. a minimum daily loss of $1.10 million over the next year.
B. a loss over one month equal to or exceeding $5.37 million 5% of the time.
C. an average daily loss of $1.10 million 5% of the time during the next 250 trading days.

Q12

Tina Ming is a senior portfolio manager at Flusk Pension Fund (Flusk). Flusk's portfolio is composed of fixed-income instruments structured to match Flusk's liabilities. Ming works with Shrikant McKee, Flusk's risk analyst.

Ming and McKee discuss the latest risk report. McKee calculated value at risk (VaR) for the entire portfolio using the historical method and assuming a lookback period of five years and 250 trading days per year. McKee presents VaR measures in Exhibit 1.

Exhibit 1: Flusk Portfolio VaR (in $ millions)

Confidence Interval	Daily VaR	Monthly VaR
95%	1.10	5.37

After reading McKee's report, Ming asks why the number of daily VaR breaches over the last year is zero even though the portfolio has accumulated a substantial loss.

Practice Problem

Q. The number of Flusk's VaR breaches *most likely* resulted from:

A. using a standard normal distribution in the VaR model.
B. using a 95% confidence interval instead of a 99% confidence interval.
C. lower market volatility during the last year compared with the lookback period.

Q13
a

Ming recommends purchasing newly issued emerging market corporate bonds that have embedded options. Prior to buying the bonds, Ming wants McKee to estimate the effect of the purchase on Flusk's VaR. McKee suggests running a stress test using a historical period specific to emerging markets that encompassed an extreme change in credit spreads.

At the conclusion of their conversation, Ming asks the following question about risk management tools: "What are the advantages of VaR compared with other risk measures?"

Practice Problem

Q. Which measure should McKee use to estimate the effect on Flusk's VaR from Ming's portfolio recommendation?

A. Relative VaR
B. Incremental VaR
C. Conditional VaR

b.

Practice Problem

Q. When measuring the portfolio impact of the stress test suggested by McKee, which of the following is *most likely* to produce an accurate result?

A. Marginal VaR
B. Full revaluation of securities
C. The use of sensitivity risk measures

c.

Practice Problem

Q. The risk management tool referenced in Ming's question:

A. is widely accepted by regulators.
B. takes into account asset liquidity.
C. usually incorporates right-tail events.

Q14 a

Carol Kynnersley is the chief risk officer at Investment Management Advisers (IMA). Kynnersley meets with IMA's portfolio management team and investment advisers to discuss the methods used to measure and manage market risk and how risk metrics are presented in client reports.

The three most popular investment funds offered by IMA are the Equity Opportunities, the Diversified Fixed Income, and the Alpha Core Equity. The Equity Opportunities Fund is composed of two exchange-traded funds: a broadly diversified large-cap equity product and one devoted to energy stocks. Kynnersley makes the following statements regarding the risk management policies established for the Equity Opportunities portfolio:

Statement 1 IMA's preferred approach to model value at risk (VaR) is to estimate expected returns, volatilities, and correlations under the assumption of a normal distribution.

Statement 2 In last year's annual client performance report, IMA stated that a hypothetical $6 million Equity Opportunities Fund account had a daily 5% VaR of approximately 1.5% of portfolio value.

Practice Problem

Q. Based on Statement 1, IMA's VaR estimation approach is *best* described as the:

A. parametric method.
B. historical simulation method.
C. Monte Carlo simulation method.

b.

Practice Problem

Q. In Statement 2, Kynnersley implies that the portfolio:

A. is at risk of losing $4,500 each trading day.
B. value is expected to decline by $90,000 or more once in 20 trading days.
C. has a 5% chance of falling in value by a maximum of $90,000 on a single trading day.

Q15

Kynnersley informs the investment advisers that the risk management department recently updated the model for estimating the Equity Opportunities Fund VaR based on the information presented in Exhibit 1.

Exhibit 1: Equity Opportunities Fund—VaR Model Input Assumptions

	Large-Cap ETF	Energy ETF	Total Portfolio
Portfolio weight	65.0%	35.0%	100.0%
Expected annual return	12.0%	18.0%	14.1%
Standard deviation	20.0%	40.0%	26.3%

Correlation between ETFs: 0.90
Number of trading days/year: 250

Practice Problem

Q. Based on Exhibit 1, the daily 5% VaR estimate is *closest* to:

A. 1.61%.
B. 2.42%.
C. 2.69%.

Q16

Hamilton develops the assumptions shown in Exhibit 2, which will be used for estimating the portfolio VaR.

Exhibit 2: VaR Input Assumptions for Proposed CAD260 Million Portfolio

Method	Average Return Assumption	Standard Deviation Assumption
Monte Carlo simulation	0.026%	0.501%
Parametric approach	0.026%	0.501%
Historical simulation	0.023%	0.490%

Hamilton elects to apply a one-day, 5% VaR limit of CAD2 million in her risk assessment of LICIA's portfolio. This limit is consistent with the risk tolerance the committee has specified for the Hiram portfolio.

Practice Problem

Q. Using the data in Exhibit 2, the portfolio's annual 1% parametric VaR is *closest* to:

A. CAD17 million.
B. CAD31 million.
C. CAD48 million.

Q17

Later that day, Woolridge receives an email from Alexander asking her to estimate the dollar VaR at the 5% level for the firm's developing market equity fund. Woolridge estimates VaR using the parametric method (assuming normal distribution) with the inputs in Exhibit 1.

Exhibit 1 Data for Developing Market Equity Fund

Portfolio value	$500 million
Daily expected return	0.04%
Daily expected volatility	1.30%

Q. Using the inputs in Exhibit 1, Woolridge's estimate of VaR is *most likely* closest to:

A. $6.5 million.

B. $18.9 million.

C. $10.5 million.

Q18.
How is Marginal VaR different from incremental VaR?

Q19.

Consider a portfolio M which has three equally weighted securities 1, 2, and 3. Each security has a value of $100,000. Assume that the portfolio VaR is $20,000.

Given that security 2 has a beta of 1.5, calculate the marginal VaR of security 2.

Q20 Calculate the Incremental VaR using full revaluation

Consider a portfolio composed of two foreign currencies, namely USD and EUR. Assume that these currencies are uncorrelated and have volatility against the dollar of 5% and 10%, respectively. The portfolio has US$4 million invested in the USD and US$3 million in EUR.

At a 95% confidence level, find the incremental VaR given that the USD currency increases by US$15,000.

Solution

1.

The distribution of the estimated fund value at the end of the 10-day period is thus assumed to have a mean of $\$1m$ and a 5% lower tail given by:

$$1m - 5\% \times 1m \times 1.645 = \$917,750$$

The Value at Risk is therefore:

$$1m - 917,750 = \$82,250$$

,

2.

(i) ***Hedge fund's 1-year Value at Risk***

The distribution of the annual investment returns from existing IT hedge funds (R) is assumed to be:

$$R \sim N\left[10, 20^2\right]$$

Thus, the 5% lower tail value for the investment return over the next year is:

$$+10\% - 1.645 \times 20\% = -22.9\%$$ [1]

The 1-year Value at Risk at the 5% lower tail is therefore given by:

$$100k - 77.1k = \$22.9k$$ [1]

[Total 2]

(ii) ***Comment in detail on the validity of your calculation***

This may well be an under-estimate of the true Value at Risk because the assumption of normality of investment returns is unlikely to be true for hedge funds investing in high-risk information technology (IT) shares. [½]

In particular, the actual distribution of returns is likely to exhibit both negative skew and fat tails, due to the high possibility of company failure amongst such shares. [½]

In addition, the estimated figure for the historical average return on IT-based hedge funds is likely to be an over-estimate, as it is likely to exclude some such hedge funds that have been wound up due to poor performance. In other words, the estimate is likely to be biased upwards due to *survivorship bias*. [1]

It may also be an overstatement of the true average return if surviving IT-based hedge funds with poor performance records are not included in the database from which the mean has been estimated. In other words, the estimate of the mean is likely to be biased upwards due to *selection bias*. [1]

Likewise, the estimate of the standard deviation of returns is likely to be too small. [½]

This may be due to:

- *marking to market bias* – reflecting the fact that some of the IT shares may not be publicly quoted, and so the hedge fund performance will be based on occasional and subjective estimates of the value of then underlying assets [½]
- *survivorship bias* – resulting from the exclusion of wound-up hedge funds, which are likely to be those that perform very poorly. [½]

Finally, the "similar" hedge funds on whose performance the parameter estimates are based may actually be very different in terms of the shares in which they invest, *eg* start-up and/or more mature IT shares. [½]

Likewise, the "similar" funds may differ greatly in terms of their use of short selling, leveraging and derivatives. [½]

[Maximum 5]

(iii) ***Hedge fund's 1-year Value at Risk allowing for leverage***

Recall that the 5% lower tail for the annual return on the IT shares is –22.9%.

Based on this, and remembering that the leverage allows the fund to increase its current IT share exposure to \$200*k*, the 5% lower tail value for the value of the IT share portfolio in a year's time is:

$$200k \times (1-0.229) = \$154.2k \quad [½]$$

And so allowing for the cost of the \$100*k* debt at 5% *pa*, the corresponding 5% lower tail value for the overall portfolio value is:

$$154.2k - 100k \times 1.05 = \$49.2k \quad [½]$$

The 1-year Value at Risk at the 5% lower tail is therefore given by:

$$100k - 49.2k = \$50.8k \quad [½]$$

So the *increase* in the estimated Value at Risk due to leveraging is:

$$50.8k - 22.9k = \$27.9k \quad [½]$$

[Total 2]

In this case, leveraging to double the exposure of the fund has more than doubled the estimated Value at Risk.

3.

(i) ***Value at Risk definition***

Value at Risk (VaR) the potential loss that a portfolio is likely to suffer, over a given future time period to a specified degree of confidence. [1]

(ii) ***Value at Risk calculation***

The annual variance of the portfolio that combines the equities and the bonds is:

$$\sigma_{portf}^2 = x_e^2 \times \sigma_e^2 + x_b^2 \times \sigma_b^2 + 2 \times x_e \times x_b \times \sigma_e \times \sigma_b \times \rho \quad [½]$$

$$\sigma_{portf}^2 = \left(\frac{5}{8}\right)^2 \times (14\%)^2 + \left(\frac{3}{8}\right)^2 \times (8\%)^2$$

$$+2 \times \left(\frac{5}{8}\right) \times \left(\frac{3}{8}\right) \times 14\% \times 8\% \times 0.6 = (10.82\%)^2 \quad [1]$$

Therefore the 30-day standard deviation of the portfolio is:

$$\sigma_{portf}^{30-day} = \sigma_{portf}^{year} \times \frac{\sqrt{30}}{\sqrt{365}} = 10.82 \times \frac{\sqrt{30}}{\sqrt{365}} = 3.10\% \quad [1]$$

Markers allow the use of 252 business days in a year and 21 business days in a month, giving $\sigma_{portf}^{30-day} = 3.12\%$

The mean return for the portfolio in 30 days is:

$$\left(\frac{5}{8}\right) \times 10\% \times \frac{30}{365} + \left(\frac{3}{8}\right) \times 5\% \times \frac{30}{365} = 0.668\% \quad [1]$$

So, the 30-day return on the portfolio, $R_P \sim N\left(0.668\%, (3.10\%)^2\right)$

The 5% lower tail of this is:

$$0.668\% - 1.645 \times 3.10\% = -4.43\% \quad [1]$$

So, the VaR at the 5% significance level, on this £800*m* portfolio is:

$$-4.43\% \times £800m = £35.44m \quad [1]$$

[Maximum 5]

(iii) ***Drawbacks of VaR***

VaR is normally calculated using a normal distribution. [½]

Historical data suggests that fund returns are not normally distributed but are fat-tailed. If we are using the tails of these distributions, the accuracy of the estimates can be considerably reduced by using the wrong distribution. [1]

The fat-tailed problem becomes worse if the risk being measured has systematic bias or (in the case of an equity portfolio) has significant derivative exposure. [½]

Such cases need the use of a distribution such as the Gumbel, Frechet or Weibull distributions. [½]

The further out in the tail we get, the more sensitive the results are to the assumptions and the modelling. [1]

The measure relies on historical data, which may not be a reliable guide to future performance if the data was taken when the fund was run differently. [1]

To perform a forward-looking VaR calculation requires Monte Carlo modelling techniques. [1]

If VaR is used to estimate extreme events, but the data is measured during normal market circumstances, then the results may be flawed. [½]

In extreme circumstances the correlations between shares and sectors and indeed markets do not behave as they have done in the past. [1]

VaR only gives information regarding the underperformance at the level of probability chosen – no information is obtained about the magnitude of extreme events at other points in the tail. [½]
[Maximum 5]

4.

i. Same as 3 iii

ii.

SNo	Monte Carlo	Historical	Parametric
1	Simulates return data to calculate VaR in similar manner as Historical method later	Raw return data is used to calculate Var by calculating probabilities – hence no probability distribution is assumed	Assumes the data to be normally or lognormally distributed and uses mean and SD into a formula
2	Costly and time consuming	Easy and cheap if data is correct and whole – no distribution	Easy and cheap if mean and SD can be correctly estimated
3	Does not assume a single distribution or rely on historical data	Relies on historical data that might not predict future	While not using historical data directly, the mean and SD calculations are

		well or might not be same in future	done using historical data for a period called lookback period
4	Better to calculate stressed VaR (in case of stress scenarios)	Cannot be used as it is for stress testing or otherwise	Better than historical method for stress testing – but normal assumption might not hold if the data available is not enough

5.

$$E\left(R_p\right) = 0.8\,(0.105) + 0.2\,(0.06) = 0.096000$$

$$\sigma_p = \sqrt{(0.8)^2(0.2)^2 + (0.2)^2(0.085)^2 + 2\,(0.8)\,(0.2)\,(-0.06)\,(0.2)\,(0.085)}$$

=0.159883.

Thus, our portfolio, consisting of an 80% position in SPY and a 20% position in SPLB, is estimated to have an expected return of 9.6% and a volatility of approximately 15.99%.

But these inputs are based on annual returns. If we want a one-day VaR, we should adjust the expected returns and volatilities to their daily counterparts. Assuming 250 trading days in a year, the expected return is adjusted by dividing by 250 and the standard deviation is adjusted by dividing by the square root of 250. (Note that the variance is converted by dividing by time, 250 days; thus, the standard deviation must be adjusted by using the square root of time, 250 days.) Thus, the daily expected return and volatility are

$$E\left(R_p\right) = \frac{0.096}{250} = 0.000384 \quad (3)$$

and

$$\sigma_p = \frac{0.159883}{\sqrt{250}} = 0.010112. \quad (4)$$

It is important to note that we cannot estimate a daily VaR and annualize it to arrive at an annual VaR estimate. First, to assume that a daily distribution of returns can be extrapolated to an annual distribution is a bold assumption. Second, annualizing the daily VaR is not the same as adjusting the expected return and the standard deviation to annual numbers and then calculating the annual VaR. The expected return is annualized by multiplying the daily return by 250, and the standard deviation is annualized by multiplying the daily standard deviation by the square root of 250. Thus, we can annualize the data and estimate an annual VaR, but we cannot estimate a daily VaR and annualize it without assuming a zero expected return.

Having calculated the daily expected return and volatility, the parametric VaR is now easily obtained. With the distribution centered at the expected return of 0.0384% and a one standard deviation move equal to 0.996%, a 5% VaR is obtained by identifying the point on the distribution that lies 1.65 standard deviations to the left of the mean. It is now easy to see why parametric VaR is so named: The expected values, standard deviations, and covariances are the *parameters* of the distributions.

The following step-by-step procedure shows how the VaR is derived:

$\{[E(Rp) - 1.65\sigma_p](-1)\}(\$150{,}000{,}000)$

Step 1 Multiply the portfolio standard deviation by 1.65.

$0.010112 \times 1.65 = 0.016685$

Step 2 Subtract the answer obtained in Step 1 from the expected return.

$0.000384 - 0.016685 = -0.016301$

Step 3 Because VaR is expressed as an absolute number (despite representing an expected loss), change the sign of the value obtained in Step 2.

Change -0.016301 to 0.016301

Step 4 Multiply the result in Step 3 by the value of the portfolio.

$\$150{,}000{,}000 \times 0.016301 = \$2{,}445{,}150$

6.

1. The parameters of normal distribution required to estimate parametric VaR are:

A. expected value and standard deviation.

B. skewness and kurtosis.

C. standard deviation and skewness.

Hide Solution

Solution to 1:

A is correct. The parameters of a normal distribution are the expected value and standard deviation. Skewness, as mentioned in B and C, and kurtosis, as mentioned in B, are characteristics used to describe a *non*-normal distribution.

2. Assuming a daily expected return of 0.0384% and daily standard deviation of 1.0112% (as in the example in the text), which of the following is *closest* to the 1% VaR for a $150 million portfolio? Express your answer in dollars.

A. $3.5 million

B. $2.4 million

C. $1.4 million

Hide Solution

Solution to 2:

A is correct and is obtained as follows:

Step 1 $2.33 \times 0.010112 = 0.023561$

Step 2 $0.000384 - 0.023561 = -0.023177$

Step 3 Convert -0.023177 to 0.023177

Step 4 $0.023177 \times \$150$ million = $3,476,550

B is the estimated VaR at a 5% threshold, and C is the estimated VaR using a one standard deviation threshold.

3. Assuming a daily expected return of 0.0384% and daily standard deviation of 1.0112% (as in the example in the text), the daily 5% parametric VaR is $2,445,150. Rounding the VaR to $2.4 million, which of the following values is *closest* to the annual 5% parametric VaR? Express your answer in dollars.

A. $38 million

B. $25 million

C. $600 million

Hide Solution

Solution to 3:

B is correct. It is found by annualizing the daily return and standard deviation and using these figures in the calculation. The annual return and standard deviation are, respectively, 0.096000 (0.000384 × 250) and 0.159885 (0.010112 × $\sqrt{250}$).

Step 1	0.159885 × 1.65 = 0.263810
Step 2	0.096000 − 0.263810 = −0.167810
Step 3	Convert −0.167810 to 0.167810
Step 4	0.167810 × $150 million = $25,171,500

A incorrectly multiplies the daily VaR by the square root of the number of trading days in a year ($\sqrt{250}$), and C incorrectly multiplies the daily VaR by the approximate number of trading days in a year (250). Neither A nor C make the appropriate adjustment to annualize the standard deviation.

7.

1. Which of the following statements about the historical simulation method of estimating VaR is *most* correct?

A. A 5% historical simulation VaR is the value that is 5% to the left of the expected value.

B. A 5% historical simulation VaR is the value that is 1.65 standard deviations to the left of the expected value.

C. A 5% historical simulation VaR is the fifth percentile, meaning the point on the distribution beyond which 5% of the outcomes result in larger losses.

Hide Solution

Solution to 1:

C is correct. In the historical method, the portfolio returns are arrayed lowest to highest and the observation at the fifth percentile (95% of the outcomes are better than this outcome) is the VaR. A is not correct because it draws a point on the distribution relative to the expected value rather than using the 5% of the outcomes that are in the left-most of the distribution. B confuses the parametric and historical methods. In the parametric method, the 5% VaR lies 1.65 standard deviations below the mean.

2. Which of the following is a limitation of the historical simulation method?

A. The past may not repeat itself.

B. There is a reliance on the normal distribution.

C. Estimates of the mean and variance could be biased.

Hide Solution

Solution to 2:

A is correct. The historical simulation method estimates VaR based on the historical distribution of the risk factors. B is not correct; the historical simulation method does not rely on any particular distribution because it simply uses whatever distribution applied in the past. C is not correct because the historical distribution does not formally estimate the mean and variance.

8.

1. When will the Monte Carlo method of estimating VaR produce virtually the same results as the parametric method?

A. When the Monte Carlo method assumes a non-normal distribution.

B. When the Monte Carlo method uses the historical return and distribution parameters.

C. When the parameters and the distribution used in the parametric method are the same as those used in the Monte Carlo method and the Monte Carlo method uses a sufficiently large sample.

Hide Solution

Solution to 1:

C is correct. The Monte Carlo method simulates outcomes using whatever distribution is specified by the user. *If* a normal distribution is used *and* a sufficiently large number of simulations are run, the parameters of the Monte Carlo sample will converge with those used in the parametric method and the overall VaR should be very close to that of the parametric method. A is incorrect because the parametric method is not well-adapted to a non-normal distribution. B is incorrect because neither the Monte Carlo method nor the parametric method focuses on historical outcomes.

2. Which of the following is an advantage of the Monte Carlo method?

A. The VaR is easy to calculate with a simple formula.

B. It is flexible enough to accommodate many types of distributions.

C. The number of necessary simulations is determined by the parameters.

Hide Solution

Solution to 2:

B is correct. The method can handle any distribution. A is incorrect because Monte Carlo simulation is not a simple formula. C is incorrect; there is no industry-wide agreement as to the necessary number of simulations.

9.

1. Which of the following is **not** an advantage of VaR?

A. It is a simple concept to communicate.

B. There is widespread agreement on how to calculate it.

C. It can be used to compare risk across portfolios or trading units.

Hide Solution

Solution to 1:

B is correct. There is no consensus on how to calculate VaR. A and C are both advantages of VaR, as we noted that VaR is fairly simple to communicate and it can show the contribution of each unit to the overall VaR.

2. Which of the following is a limitation of VaR?

A. It requires the use of the normal distribution.

B. The maximum VaR is prescribed by federal securities regulators.

C. It focuses exclusively on potential losses, without considering potential gains.

Hide Solution

Solution to 2:

C is correct. VaR deals exclusively with left-tail or adverse events. A is wrong because although parametric VaR does generally use the normal distribution, the historical simulation method uses whatever distribution occurred in the past and Monte Carlo simulation uses whatever distribution the user chooses. B is incorrect because regulators do not specify maximum VaRs, although they may encourage and require that the measure be used.

10.

Extensions of VaR

1. Conditional VaR measures the:

A. VaR over all possible losses.

B. VaR under normal market conditions.

C. average loss, given that VaR is exceeded.

Hide Solution

Solution to 1:

C is correct. Conditional VaR is the average loss conditional on exceeding the VaR. A is not correct because CVaR is not concerned with losses that do not exceed the VaR threshold, and B is incorrect because VaR does not distinguish between normal and non-normal markets.

2. Which of the following correctly identifies incremental VaR?

A. The change in VaR from increasing a position in an asset.

B. The increase in VaR that might occur during extremely volatile markets.

C. The difference between the asset with the highest VaR and the asset with the second highest VaR.

Hide Solution

Solution to 2:

A correctly defines incremental VaR. Incremental VaR is the change in VaR from increasing a position in an asset, not a change in VaR from an increase in volatility. B is not correct because incremental volatility reflects the results of intentional changes in exposure, not uncontrollable market volatility. C is not correct because incremental VaR is not the difference in the VaRs of the assets with the greatest and second greatest VaRs.

3. Which of the following statements is correct about marginal VaR?

A. The marginal VaR is the same as the incremental VaR.

B. The marginal VaR is the VaR required to meet margin calls.

C. Marginal VaR estimates the change in VaR for a small change in a given portfolio holding.

Hide Solution

Solution to 3:

C is correct. In A, marginal VaR is a similar concept to incremental VaR in that they both deal with the effect of changes in VaR, but they are not the same concept. B is incorrect because marginal VaR has nothing to do with margin calls.

Q11 a

Solution

A is correct. The bank policy requires the addition of forward-looking risk assessments, and management is focused on tail risk. Conditional VaR measures tail risk, and stress tests and scenario analysis subject current portfolio holdings to historical or hypothetical stress events.

Q11 b.

Solution

B is correct. VaR is the minimum loss that would be expected a certain percentage of the time over a specified period of time given the assumed market conditions. A 5% VaR is often expressed as its complement—a 95% level of confidence. Therefore, the monthly VaR in Exhibit 5 indicates that $5.37 million is the minimum loss that would be expected to occur over one month 5% of the time. Alternatively, 95% of the time, a loss of more than $5.37 million would not be expected.

Q12

Solution

C is correct. Flusk experienced zero daily VaR breaches over the last year yet incurred a substantial loss. A limitation of VaR is its vulnerability to different volatility regimes. A portfolio might remain under its VaR limit every day but lose an amount approaching this limit each day. If market volatility during the last year is lower than in the lookback period, the portfolio could accumulate a substantial loss without technically breaching the VaR constraint.

A is incorrect because VaR was calculated using historical simulation, so the distribution used was based on actual historical changes in the key risk factors experienced during the lookback period. Thus, the distribution is not characterized using estimates of the mean return, the standard deviation, or the correlations among the risk factors in the portfolio. In contrast, the parametric method of estimating VaR generally assumes that the distribution of returns for the risk factors is normal.

B is incorrect because a specification with a higher confidence level will produce a higher VaR. If a 99% confidence interval was used to calculate historical VaR, the VaR would be larger (larger expected minimum loss). During the last year, none of Flusk's losses were substantial enough to breach the 5% VaR number (95% confidence interval); therefore, if McKee used a 1% VaR (99% confidence interval), the number of VaR breaches would not change.

Q13
a

Solution

B is correct. Incremental VaR measures the change in a portfolio's VaR as a result of adding or removing a position from the portfolio or if a position size is changed relative to the remaining positions.

b.

Solution

B is correct. McKee suggests running a stress test using a historical scenario specific to emerging markets that includes an extreme change in credit spreads. Stress tests, which apply extreme negative stress to a particular portfolio exposure, are closely related to scenario risk measures. A scenario risk measure estimates the portfolio return that would result from a hypothetical change in markets (hypothetical scenario) or a repeat of a historical event (historical scenario). When the historical simulation fully revalues securities under rate and price changes that occurred during the scenario period, the results should be highly accurate.

A is incorrect because marginal VaR measures the change in portfolio VaR given a very small change in a portfolio position (e.g., change in VaR for a $1 or 1% change in the position). Therefore, marginal VaR would not allow McKee to estimate how much the value of the option-embedded bonds would change under an extreme change in credit spreads.

C is incorrect because sensitivity risk measures use sensitivity exposure measures, such as first-order (delta, duration) and second-order (gamma, convexity) sensitivity, to assess the change in the value of a financial instrument. Although gamma and convexity can be used with delta and duration to estimate the impact of extreme market movements, they are not suited for scenario analysis related to option-embedded bonds.

c.

Solution

A is correct. VaR has emerged as one of the most popular risk measures because global banking regulators require or encourage the use of it. VaR is also frequently found in annual reports of financial firms and can be used for comparisons.

Q14 a

Solution

A is correct. VaR is an estimate of the loss that is expected to be exceeded with a given level of probability over a specified time period. The parametric method typically assumes that the return distributions for the risk factors in the portfolio are normal. It then uses the expected return and standard deviation of return for each risk factor and correlations to estimate VaR.

b.

Solution

B is correct. Value at risk is the minimum loss that would be expected a certain percentage of the time over a certain period of time. Statement 2 implies that there is a 5% chance the portfolio will fall in value by $90,000 (= $6,000,000 × 1.5%) or more in a single day. If VaR is measured on a daily basis and a typical month has 20–22 business days, then 5% of the days equates to about 1 day per month or once in 20 trading days.

Q15

Solution

C is correct. Measuring VaR at a 5% threshold produces an estimated value at risk of 2.69%.

From Exhibit 6, the expected annual portfolio return is 14.1% and the standard deviation is 26.3%. Annual values need to be adjusted to get their daily counterparts. Assuming 250 trading days in a year, the expected annual return is adjusted by dividing by 250 and the standard deviation is adjusted by dividing by the square root of 250.

Thus, the daily expected return is 0.141/250 = 0.000564, and volatility is $0.263/\sqrt{250}$ = 0.016634.

5% daily VaR = $E(R_p) - 1.65\sigma_p$ = 0.000564 − 1.65(0.016634) = −0.026882. The portfolio is expected to experience a potential minimum loss in percentage terms of 2.69% on 5% of trading days.

Q16

Solution

B is correct. The VaR is derived as follows:

$$\text{VaR} = \{[E(Rp) - 2.33\sigma_p](-1)\}(\text{Portfolio value}),$$

where

$E(R_p)$ = Annualized daily return = (0.00026 × 250) = 0.065

250 = Number of trading days annually

2.33 = Number of standard deviations to attain 1% VaR

σ_p = Annualized standard deviation = $\left(0.00501 \times \sqrt{250}\right)$ = 0.079215

Portfolio value = CAD260,000,000

VaR = −(0.065 − 0.184571) × CAD260,000,000

= CAD31,088,460.

Q17

Solution

C is correct. Use the parametric method to calculate the following:

Step 1 Multiply the portfolio standard deviation by 1.65 (the 5% VaR level translates to a point on the distribution curve that lies 1.65 standard deviations to the left of the mean).

0.013 × 1.65 = 0.0215

Step 2 Subtract the answer obtained in Step 1 from the expected return.

0.0004 – 0.0215 = –0.0211

Step 3 Because VaR is expressed as a positive number, change the sign of Step 2.

–0.0211 becomes 0.0211

Step 4 Multiply the result in Step 3 by the portfolio value.

$500,000,000 × 0.0211 = $10,550,000

A is incorrect. This is the answer derived by multiplying the portfolio value by the daily volatility ($500,000,000 × 1.3%).

B is incorrect. This is the answer derived by using 0.04 instead of 0.0004 in Step 2.

18. In notes

19.

Using the formula:

$$\text{Marginal VaR}\,(\Delta\text{VaR}_\text{i}) = \frac{\delta\text{VaR}}{\delta\text{x}_\text{i}}\alpha\,(\beta_\text{i} \times \sigma_\text{p}) = \frac{\text{VaR}}{\text{W}} \times \beta_\text{i}$$

W is the value of the portfolio = $100,000 \times 3 = \$300,000$

$$\beta_2 = 1.5$$

Thus,

$$\Delta\text{VaR}_2 = \frac{\text{VaR}}{\text{W}} \times \beta_\text{i} = \frac{20,000}{300,000} \times 1.5 = 0.1$$

This implies that a one-dollar change in the value of security 2 causes a 0.1 change in the portfolio VaR.

20.

Revaluation approach – calc both VaR and then subtract

Moreover, we can compare this with the incremental VAR obtained from a full revaluation of the portfolio risk. An addition of $15,000 to the first position leads to:

$$\sigma^2_{\text{p+a}}\text{w}^2_{\text{p+a}} = [\$4.015 \quad \$3]\begin{bmatrix} 0.05^2 & 0 \\ 0 & 0.1^2 \end{bmatrix}\begin{bmatrix} \$4.015 \\ \$3 \end{bmatrix} = [4.015 \quad 3]\begin{bmatrix} 0.01 \\ 0.03 \end{bmatrix}$$

$$0.04030056 + 0.09 = \$0.13030056 \text{ million}$$

$$\text{VaR}_{\text{p+a}} = 1.65 \times \sqrt{\$0.130301} = \$595,603$$

Finally, we can compute the exact increment by finding the difference of the initial $\mathrm{VaR_p}$ and $\mathrm{VaR_{p+a}}$, i.e.,

$$\begin{aligned} \mathrm{VaR_p} &= 1.65 \times \sqrt{0.13} = \$594,916 \\ \mathrm{VaR_{p+a}} - \mathrm{VaR_p} &= \$595,603 - \$594,916 = \$687 \end{aligned}$$

From the above calculations, we note that the $\Delta\mathrm{VaR}$ approximation of \$686 is very close to the true value, \$687.

Approximation/Short Cut method:

	SD	Value	Addition
USD	5%	4m	+ 0.015m
EUR	10%	3m	+ 0m

$$\text{Dollar variance} = \sigma_p^2 = \left[4^2 \times 5\%^2\right] + \left[3^2 \times 10\%^2\right]$$

$$= 0.13$$

$$\beta_i = \frac{\mathrm{cov}[R_i, R_p]}{\sigma_p^2}$$

$$\mathrm{cov}[R_i, R_p] = \frac{\text{Variance}_i \times \text{Weight of value}_i}{\sigma_p^2}$$

$$\underset{\text{(matrix)}}{\mathrm{MVaR}_i} = Z_c \times \beta_i \times \sigma_p$$

$$= Z_c \times \frac{\mathrm{cov}[R_i, R_p]}{\sigma_p^2} \times \cancel{\sigma_p}$$

$$= Z_c \times \left[\frac{\text{variance}_i \times \text{weight of value}_i}{\sigma_p^2}\right] \div \cancel{\sigma_p}$$

$$= 1.65 \times \begin{bmatrix} 5\%^2 \times 4 \\ 10\%^2 \times 3 \end{bmatrix} \Big/ \sqrt{0.13}$$

$$= \begin{bmatrix} 0.045705 \\ 0.13728 \end{bmatrix}$$

Change in VaR or IVaR $= \begin{bmatrix} 0.015 & 0 \end{bmatrix} \begin{bmatrix} 0.045705 \\ 0.13728 \end{bmatrix}$

$= \$685.58$