INSTITUTE OF ACTUARIAL & QUANTITATIVE STUDIES

Subject: Financial Engineering

Chapter: Unit 4

Category: Practice Questions

1. CT8 April 2008 question 10

In a situation where the zero-coupon bond market is arbitrage-free and complete, consider the following Vasicek model for the short-rate process:

$$dr(t) = a\left(b - r(t)\right)dt + \sigma dW_t$$

where $\left(W_t; t \geq 0\right)$ is a standard Brownian motion with respect to the risk-neutral probability measure **Q**.

(i) State the general expression $r(t)$ of the solution of this stochastic differential equation. [2]

(ii) Derive an expression for $\int_t^T r(u)du$, where t and T are given.

Hint: consider the stochastic differential equation of $r(u)$, for $u \geq t$.

[6]

(iii) State the distribution of $\int_t^T r(u)du$. [1]

(iv) Derive the price of a zero-coupon bond at time t with maturity $T \geq t$ related to the distribution of $\int_t^T r(u)du$. [6]

[Total 15]

2. CT8 April 2010 Question 4

Consider the following stochastic differential equation for the instantaneous risk free rate (also referred to as the short-rate):

$$dr(t) = a\left(b - r(t)\right)dt + \sigma dW_t$$

Its solution is given by:

$$r(t) = r_0 \exp(-at) + b\left(1 - \exp(-at)\right) + \sigma \exp(-at)\int_0^t \exp(as)\, dW_s$$

You may also use the fact that for $T > t$:

$$\int_t^T r(u)du = b(T-t) + \left(r(t) - b\right)\frac{1 - \exp\left(-a(T-t)\right)}{a} + \frac{\sigma}{a}\int_t^T \left(1 - \exp\left(-a(T-s)\right)\right)dW_s$$

(i) Derive the price at time t of a zero-coupon bond with maturity T. [10]

(ii) (a) State the main drawback of such a model for the short-rate.

(b) State the name and stochastic differential equation of an alternative model for the short-rate that is not subject to the drawback.

[2]

3. CT8 April 2011 Question 10

Let $B(t,T)$ be the price at time t of a zero-coupon bond paying £1 at time T, r_t be the short-rate of interest, $\mathbb{P}$ be the real world probability measure and $\mathbb{Q}$ the risk neutral probability measure.

(i) Write down two equations for the price of a zero-coupon bond, one of which uses the risk-neutral approach to pricing and the other of which uses the state-price-deflator approach to pricing. [2]

(ii) State the Stochastic Differential Equation (SDE) of the short rate r_t under Q for the Vasicek model and the general type of process this SDE represents. [3]

(iii) Solve the SDE for the short rate r_t from (ii). [5]

(iv) Deduce the form of the distribution of the zero-coupon bond price under this model. [2]

[Total 12]

4. CT8 September 2011 Question 5

(i) List the desirable characteristics of a model for the term structure of interest rates. [4]

(ii) Write down the stochastic differential equation for the short rate r_t under $\mathbb{Q}$ in the Hull-White model. [1]

(iii) Indicate whether or not the Hull-White model shows the characteristics listed in (i). [4]

[Total 9]

5. CT8 April 2012 Question 5

(i) Write down a stochastic differential equation for the short rate $r(t)$ for the Vasicek model. [1]

(ii) State the type of process of which the Vasicek model is a particular example. [1]

(iii) Solve the stochastic differential equation in (i). [5]

(iv) State the distribution of $r(t)$ for t given. [1]

(v) Derive the expected value and the second moment of $r(t)$ for t given. [3]

(vi) Outline the main drawback of the Vasicek model. [1]

[Total 12]

6. CT8 September 2012 Question 5

State eight desirable characteristics of a term-structure model. [8]

7. CT8 September 2012 Question 6

(i) State the Stochastic Differential Equations for the short rate r(t) in the Vasicek model and the Cox-Ingersoll-Ross model. [2]

(ii) Explain the impact of a movement in the short rate on the volatility term in both models. [2]

[Total 4]

8. CT8 April 2015 Question 10

There are two risk-free zero coupon bonds trading in a market, Bond X and Bond Y.

The short-rate of interest, r_t, follows a Vasicek model:

$$dr_t = \alpha(\mu - r_t)dt + \sigma dW_t$$

where W_t is a standard Brownian motion.

(i) Write down the formula for the price of a risk-free zero coupon bond at time t, with bond maturity at time T, under the Vasicek model. [3]

In this market the parameters for the Vasicek model are $\alpha = 0.5$, $\mu = 4\%$ and $\sigma = 10\%$. The short-rate at time 0, $r(0)$, is 2% p.a. Bond X matures at time 1, and Bond Y matures at time 3. Both bonds are for a nominal value of $100.

(ii) Calculate the fair price of Bond X. [3]

Bond Y has a fair price at time 0 of $90.

(iii) Derive the market-implied risk-free spot rate of interest with maturity 3 years. [2]

(iv) Derive the market-implied risk-free forward rate of interest from time 1 to time 3. [2]

[Total 10]

9. CT8 April 2016 Question 10

In the Vasicek model, the short rate of interest under the risk-neutral probability measure is given by:

$$r_t = \theta + e^{-kt}(r_0 - \theta) + \sigma \int_0^t e^{-k(t-u)} dW_u$$

where k, θ, $\sigma > 0$ and W is a standard Brownian motion.

Consider the related process:

$$R_t = \int_0^t r_s ds$$

where r_t is the short rate defined above.

(i) Show that R_t has a Normal distribution with mean and variance given by:

$$E(R_t) = \theta t + (r_0 - \theta)\frac{1-e^{-kt}}{k} \quad \text{and}$$

$$\text{Var}(R_t) = \frac{\sigma^2}{k^2}\left(t - \frac{2(1-e^{-kt})}{k} + \frac{1-e^{-2kt}}{2k}\right). \qquad [6]$$

Let $P(0,t)$ be the price at time 0 of a zero-coupon bond with redemption date $t > 0$.

(ii) Show that, under the Vasicek model:

$$P(0,t) = e^{-E(R_t) + \frac{\text{Var}(R_t)}{2}}. \qquad [3]$$

(iii) Show, by using the results from parts (i) and (ii), that:

$$P(0,t) = A(t)e^{-B(t)r_0}$$

where $B(t) = \dfrac{1-e^{-kt}}{k}$

and $A(t) = \exp\left[(B(t)-t)\left(\theta - \dfrac{\sigma^2}{2k^2}\right) - \dfrac{\sigma^2}{4k}B(t)^2\right]$. [4]

(iv) State the main drawback of the above model for the term structure of interest rates. [1]

[Total 14]

10. CT8 September 2016 Question 9

Write down the properties of the following two models for interest rates:

(a) the one-factor Vasicek model
(b) the Cox-Ingersoll-Ross model

[You are not required to give any formulae for the models.] [4]

The Vasicek term structure model is described by the following stochastic differential equation:

$$dr_t = a(b - r_t)dt + \sigma dW_t,$$

with initial value r_0 and a, b, $\sigma > 0$.

(ii) Show, by solving the Vasicek stochastic differential equation, that:

$$r_t = r_0 e^{-at} + b\left(1 - e^{-at}\right) + \sigma \int_0^t e^{-a(t-s)} dW_s \ . \qquad [4]$$

(iii) Determine the expectation, the variance and the distribution of the short rate r_t. [3]

[Total 11]

11. CT8 September 2017 Question 9

(i) State the main potential drawback of the Vasicek model. [1]

(ii) Discuss the extent to which this drawback may be a problem. [3]

(iii) Explain how the Cox-Ingersoll-Ross model avoids this drawback. [3]

The Vasicek term structure model is described by the following stochastic differential equation:

$$dr_t = a(b - r_t)dt + \sigma dW_t,$$

and a, b, $\sigma > 0$.

Under this model, the short rate r_t follows a Normal distribution with mean

$$E(r_t) = r_0 e^{-at} + b\,(1 - e^{-at})$$

and variance $\mathrm{Var}\,(r_t) = \dfrac{\sigma^2}{2a}(1 - e^{-2at})$.

(iv) Assess, using the information provided above, whether the model generates interest rates that are mean reverting and, if so, the value to which they revert. [2]

(v) Assess, using the information provided above, the relevance of the parameter a to any mean reversion. [2]

[Total 11]

Questions on Stochastics Interest rate models

1. a. In any year, the interest rate per annum effective on money invested with a given bank has mean value j and standard deviation s and is independent of the interest rates in all previous years.
Let S_n be the accumulated amount after n years of a single investment of 1 at time t = 0.

(i) Show that E[S_n] = $(1 + j)^n$

(ii) Show that Var[S_n] = $(1 + 2j + j^2 + s^2)^n$ - $(1 + j)^{2n}$

b. The interest rate per annum effective in (a), in any year, is equally likely to be i_1 or i_2 ($i_1 > i_2$). No other values are possible.

(i) Derive expressions for j and s^2 in terms of i_1 and i_2.

(ii) The accumulated value at time t = 25 years of 1 million invested with a bank at time t = 0 has expected value 5.5 million and standard deviation 0.5 million. Calculate the values of i_1 and i_2.

Solution

Let i_t be the (random) rate of interest in year t. Let S_n be the accumulation of a single investment of 1 unit after n years:

$$E(S_n) = E\left[(1+i_1)(1+i_2)\dots(1+i_n)\right]$$

$$E(S_n) = E[1+i_1]E[1+i_2]\dots E[1+i_n] \text{ as } \{i_t\} \text{ are independent}$$

$$E[i_t] = j$$

$$\therefore \quad E(S_n) = (1+j)^n$$

$$E\left(S_n^2\right) = E\left[\left[(1+i_1)(1+i_2)\dots(1+i_n)\right]^2\right]$$

$$= E(1+i_1)^2 E(1+i_2)^2 \dots E(1+i_n)^2 \text{ (using independence)}$$

$$= E\left(1+2i_1+i_1^2\right)E\left(1+2i_2+i_2^2\right)\dots E\left(1+2i_n+i_n^2\right)$$

$$= \left(1+2j+s^2+j^2\right)^n$$

as $E\left[i_t^2\right] = V[i_t] + E[i_t]^2 = s^2 + j^2$

$$\therefore \text{Var}[S_n] = \left(1+2j+s^2+j^2\right)^n - (1+j)^{2n}$$

(a) $E\left[\text{Interest}\right] = j = \frac{1}{2}\left(i_1 + i_2\right)$

$$Var\left[\text{Interest}\right] = s^2 = E\left[\text{Interest}^2\right] - \left[E\left(\text{Interest}\right)\right]^2$$

$$= \frac{1}{2}\left(i_1^2 + i_2^2\right) - \left[\frac{1}{2}\left(i_1 + i_2\right)\right]^2$$

$$= \frac{1}{4}\left(i_1^2 + i_2^2\right) - \frac{1}{2} i_1 . i_2$$

$$= \left[\frac{1}{2}\left(i_1 - i_2\right)\right]^2$$

(b) $E[S_{25}]=(1+j)^{25}=5.5$

$\Rightarrow j=0.0705686$

$\text{Var}[S_{25}]=\left(1+2j+j^2+s^2\right)^{25}-(1+j)^{50}=(0.5)^2$

$\Rightarrow \left(1+2*0.0705686+0.0705686^2+s^2\right)^{25}-(1.0705686)^{50} \qquad =0.25$

$\Rightarrow s^2=0.000377389$

Hence, $s^2=0.000377389=\frac{1}{4}(i_1-i_2)^2$

$\Rightarrow \quad i_1-i_2=0.0388530 \qquad$ (taking positive root since $i_1>i_2$)

$i_1+i_2=2\times 0.07056862 = 0.1411372$

$\Rightarrow \quad 2i_1=0.0388530+0.1411372$

$i_1=0.089995 \quad (8.9995\%\text{p.a.})$

and $i_2=0.051142 \quad (5.1142\%\text{p.a.})$

2. £80,000 is invested in a bank account which pays interest at the end of each year. Interest is always reinvested in the account. The rate of interest is determined at the beginning of each year and remains unchanged until the beginning of the next year. The rate of interest applicable in any one year is independent of the rate applicable in any other year. During the first year, the annual effective rate of interest will be one of 4%, 6% or 8% with equal probability. During the second year, the annual effective rate of interest will be either 7% with probability 0.75 or 5% with probability 0.25. During the third year, the annual effective rate of interest will be either 6% with probability 0.7 or 4% with probability 0.3.

a. Derive the expected accumulated amount in the bank account at the end of three years.

b. Derive the variance of the accumulated amount in the bank account at the end of three years.

c. Calculate the probability that the accumulated amount in the bank account is more than £97,000 at the end of three years.

Solution:

Let $S_3 =$ Accumulated fund after 3 years of investment of 1 at time 0

i_t = Interest rate for year t

Then, fund after 3 years

$$= 80{,}000 \;\; S_3 = 80000(1+i_1)(1-i_2)(1+i_3)$$

$$E(i_1) = \frac{1}{3}(0.04 + 0.06 + 0.08) = 0.06$$

$$E(i_2) = 0.75 \times 0.07 + 0.25 \times 0.05 = 0.065$$

$$E(i_3) = 0.7 \times 0.06 + 0.3 \times 0.04 = 0.054$$

Then:

$$E[80000 S_3] = 80{,}000 \; E[S_3]$$

$$- E[80{,}000(1+i_1)(1+i_2)(1+i_3)]$$

$$= 80{,}000 \; E(1+i_1) . E(1+i_2) . E(1+i_3)$$

since i_t's are independent

$$= 80{,}000 \times 1.06 \times 1.065 \times 1.054 = £95{,}188.85$$

(ii) $\text{Var}[80000S_3] = 80{,}000^2 \times \text{Var}[S_3]$

where $\text{Var}[S_3] = E\left[S_3^2\right] - \left(E[S_3]\right)^2$

$$E\left[S_3^2\right] = E\left[(1+i_1)^2(1+i_2)^2(1+i_3)^2\right]$$

$$= E\left[(1+i_1)^2\right].E\left[(1+i_2)^2\right].E\left[(1+i_3)^2\right]$$

using independence

$$= \left(1+2E[i_1]+E\left[i_1^2\right]\right).\left(1+2E[i_2]+E\left[i_2^2\right]\right)\cdot\left(1+2E[i_3]+E\left[i_3^2\right]\right)$$

Now,

$$E\left(i_1^2\right) = \tfrac{1}{3}\left(0.04^2+0.06^2+0.08^2\right) = 0.0038667$$

$$E\left(i_2^2\right) = 0.75\times0.07^2+0.25\times0.05^2 = 0.0043$$

$$E\left(i_3^2\right) = 0.7\times0.06^2+0.3\times0.04^2 = 0.0030$$

Hence, $E\left[S_3^2\right]$

$= (1+2\times0.06+0.0038667)\times(1+2\times0.065+0.0043)\times(1+2\times0.054+0.003)$

$=1.41631$

Hence $\text{Var}\left[80,000 S_3\right]$

$$= 80,000^2 \ \text{Var}\left[S_3\right]$$

$$= 80,000^2 \left(1.41631 - (1.18986)^2\right)$$

$$= 3,476,355$$

(iii) *Note:* $80,000 \times 1.08 \times 1.07 \times 1.06 = 97,995 > 97,000$

But, if in any year, the highest interest rate for the year is not achieved then the fund after 3 years falls below £97,000.

Hence, answer is probability that highest interest rate is achieved in each year

$$= \frac{1}{3} \times 0.75 \times 0.7 = 0.175$$

3. By adopting a particular investment strategy a company expects that on average the annual yield on its funds will be 8% with a standard deviation of 7%. The yields in different years may be assumed to be independently distributed.

a. Find the expected value and standard deviation of the accumulated amount after 15 years of a single investment of Rs.1000 made now.

Assume further that each year $1+i_t$ has a lognormal distribution where it is the annual yield on company's funds in year t.

b. Calculate the parameters of the lognormal distribution.

c. Calculate the probability that a single investment of Re.1/- made now will be less than 60% of its expected value.

Solution:

(a) Given $E(i_t) = .08$ and $V(i_t) = .07^2$
i.e., $j = .08$ and $s = .07$ using usual notations.

Required to calculate $1000.E[S_{15}]$ and $1000.\ V[S_{15}]^{1/2}$
$E[S_{15}] = (1+j)^{15} = 1.08^{15} = 3.172169$
$1000.E[S_{15}] = 3172.17$
Expected value of accumulation of Rs.1000 after 15 years = $\underline{\mathbf{3172.17}}$

$V[S_{15}] = (1+j^2 + 2j + s^2)^{15} - (1+j)^{30}$
$= (1+.08^2 + 2*.08 + .07^2)^{15} - (1.08)^{30} = 0.653083$
$S.D[S_{15}] = V[S_{15}]^{1/2} = 0.808135$

Standard Deviation of accumulation of Rs.1000 after 15 years = $\underline{\mathbf{808.135}}$

(b) Expected accumulation of a single investment of Re.1 = $E[S_{15}]$
$=(1+j)^{15} = 1.08^{15} = 3.172169$

If $1+i_t \sim \log N(\mu, \sigma^2)$ then
$E[1+i_t] = \exp(\mu + \sigma^2/2)$ and $V[1+i_t] = \exp(2\mu + \sigma^2)[\exp(\sigma^2) - 1]$
i.e., $E[i_t] = \exp(\mu+\sigma^2/2) - 1$ and $V[i_t] = \exp(2\mu + \sigma^2)[\exp(\sigma^2) - 1]$

We know that $E(i_t) = .08$ and $V(i_t) = .07^2$
$\therefore$ We get the two equations :-

$\exp(\mu + \sigma^2/2) - 1 = .08$
$\exp(2\mu + \sigma^2)[\exp(\sigma^2) - 1] = .07^2$

=> $1.08^2[\exp(\sigma^2) - 1] = .07^2$
$\exp(\sigma^2) = 1.004201$ => $\sigma^2 = .004192$ => $\underline{\sigma = 0.064747}$
and, $\underline{\mu = 0.074865}$

(c) **Using multiplicative of lognormal random variables,**
$S_{15} \sim \log N(15\,\mu, 15\,\sigma^2)$ or $(\log S_{15} - 15\,\mu)/\sqrt{15}\,\sigma \sim N(0,1)$

Prob that accumulation of Re.1 will
be less than 60% of Exp value(3.172169)= $P(S_{15} \leq 1.903301)$
$= P((\log S_{15} - 15\,\mu)/\sqrt{15}\,\sigma \leq (\ln(1.903301) - 15\,\mu)/\sqrt{15}\,\sigma)$
$= P(Z \leq -1.9117) = 0.028$

4. An insurance company holds a large amount of capital and wishes to distribute some of it to the policyholders by way of two possible options.

Option A
£100 for each policyholder will be put into a fund from which the expected annual effective rate of return from the investments will be 5.5% and the standard deviation of annual returns 7%. The annual effective rates of return will be independent and $(1+ i_t)$ is log normally distributed, where i_t is the rate of return in year t. The policyholder will receive the accumulated investment at the end of ten years.

Option B
£100 will be invested for each policyholder for five years at a rate of return of 6% per annum effective. After 5 years, the accumulated sum will be invested for a further 5 years at the prevailing five-year spot rate. This spot rate will be 1% per annum effective with probability 0.2, 3% per annum effective with probability 0.3, 6% per annum effective with probability 0.2, and 8% per annum effective with probability 0.3. The policyholder will receive the accumulated investment at the end of ten years.

Deriving any necessary formulae:
a. Calculate the expected value and the standard deviation of the sum the policyholders will receive at the end of the ten years for each options A and B.

b. Determine the probability that the sum the policyholders wil receive at the end of ten years will be less than £115 for each options A and B.

c. Comment on the relative risk of the two options from the policyholder's perspective.

Solution:

(i) Option A:

$$(1+i_t) \sim Lognormal(\mu, \sigma^2)$$

$$\ln(1+i_t) \sim N(\mu, \sigma^2)$$

$$\ln(1+i_t)^{10} = \ln(1+i_t) + \ln(1+i_t) + \ldots + \ln(1+i_t) \sim N(10\mu, 10\sigma^2)$$

since i_t's are independent

$$(1+i_t)^{10} \sim Lognormal(10\mu, 10\sigma^2)$$

$$E(1+i_t) = \exp\left(\mu + \frac{\sigma^2}{2}\right) = 1.055$$

$$Var(1+i_t) = \exp(2\mu+\sigma^2)\left[\exp(\sigma^2)-1\right] = 0.07^2$$

$$\frac{0.07^2}{1.055^2} = \left[\exp(\sigma^2)-1\right] \therefore \sigma^2 = 0.0043928$$

$$\exp\left(\mu + \frac{0.0043928}{2}\right) = 1.055 \Rightarrow \mu = \ln 1.055 - \frac{0.0043928}{2} = 0.051344$$

$$10\mu = 0.51344, 10\sigma^2 = 0.043928$$

Let S_{10} be the accumulation of one unit after 10 years:

$$E(S_{10}) = \exp\left(0.51344 + \frac{0.043928}{2}\right) = 1.70814$$

Accumulated sum is $100E(S_{10}) = £170.81$

Option B:

The accumulated sum at the end of five years is:

$$100\times 1.06^5 = 100\times 1.33823 = £133.823$$

The expected value of the accumulated sum at the end of ten years is:

$$133.823\left(0.2\times 1.01^5 + 0.3\times 1.03^5 + 0.2\times 1.06^5 + 0.3\times 1.08^5\right)$$

$$= 133.823(0.2\times 1.05101 + 0.3\times 1.15927 + 0.2\times 1.33823 + 0.3\times 1.46933)$$

$$= £169.48$$

Option A:

$$Var(S_{10}) = \exp(2\times 0.51344 + 0.043928)\left[\exp(0.043928) - 1\right]$$

$$= 2.91776\times 0.04491 = 0.13103$$

Therefore standard deviation of £100 is $100\sqrt{0.13103} = £36.20$

Option B:

Here we need to find the expected value of the square of the accumulation as follows:

$$133.823^2\left(0.2\times 1.05101^2 + 0.3\times 1.15927^2 + 0.2\times 1.33823^2 + 0.3\times 1.46933^2\right)$$

$$= 29{,}189.86$$

The variance of the accumulation is therefore:

$$29{,}189.86 - 169.48^2 = £^2 467.54$$

and the standard deviation is £21.62

(ii) For option A we require $P[S_{10} < 1.15]$

$P[\ln S_{10} < \ln 1.15]$ where $\ln S_{10} \sim N(0.51344, 0.043928)$

$$\Rightarrow P\left[N(0,1) < \frac{\ln 1.15 - 0.51344}{\sqrt{0.043928}}\right]$$
$$\Rightarrow P[N(0,1) < -1.7829] = 0.0373 \approx 4\%$$

For option B we first examine the lowest payout possible.

There is a probability of 0.2 that the amount will be $100 \times 1.06^5 \times 1.01^5$ or less which equals $133.823 \times 1.05101 = £140.65$. Therefore the probability of a payment of less than £115 is zero.

(iii) Option A is riskier both from the perspective of having a higher standard deviation of return and also a higher probability of a very low value.

5. A company Arbitrage Ltd offers a financial product with an investment guarantee at maturity @ 9.5% p.a. for 10 years. It invests the money at a floating rate it , with (1+ it) following a log-normal distribution with the following parameters $\mu = 0.1$, $\sigma = 0.05$ for $t < 5$ and $\mu = 0.1$, $\sigma = 0.015$ for $t \geq 5$ where t is the time in years since initial investment.

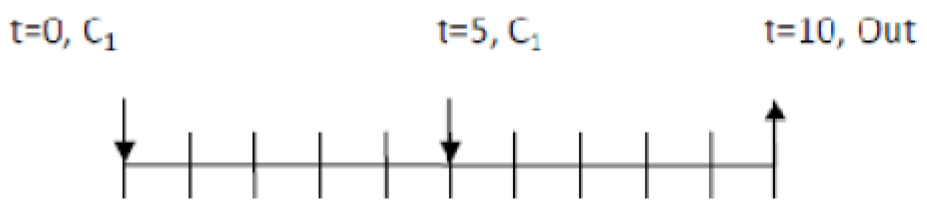
The product was offered to a company Profit Booker Ltd which agrees to invest €9,000 initially and €6,000 at the end of the 5th year. The maturity amount will be payable at the end of the 10th year.

a. Calculate, with a 99.5% confidence level, the minimum accumulated amount that company Arbitrage Ltd. will have at the end of 10 years, if it invests the money received from Profit Booker Ltd. at this floating rate.

b. Calculate the amount that Arbitrage Ltd need to invest now in a risk-free bond in order to mitigate the possible amount of loss, if any, at the 99.5% confidence level assuming the interest rate for the risk free bond to be 6% p.a

Solution:

(a)



Let C_1 = 9,000 amount invested at start

C_2 = 6,000 amount invested time t = 5

$\ln(1+i) \sim N(0.1, 0.05)$ for $t < 5$

$\ln(1+i) \sim N(0.1, 0.015)$ for $t \geq 5$.

The accumulation factor S_{n1} for first 5 years follows the distribution

$\ln S_{n1} \sim N(5*0.1, 5*0.05^2)$

Calculation for minimum value of accumulation factor S_{n1} at 99.5% confidence interval at time t= 5 :

We require to find x such that

$$\Pr(\ln(S_{n1}) > \ln x) = 0.995$$

$$\Rightarrow \Pr\left(Z > \frac{\ln x - 0.5}{\sqrt{5} * 0.05}\right) = 0.995$$

Using tables

$$\frac{\ln x - 0.5}{\sqrt{5} * 0.05} = -2.58$$

So, x $= e^{0.5-2.58*\sqrt{5}*0.05}$

$= 1.23559$

So minimum accumulated amount at 99.5% confidence interval at time t=5 just after new investment of C_2

9000 * 1.23559 + 6000 = 17,120.31

The accumulation factor S_{n2} for time t=5 to t=10 follows the distribution

$\ln S_{n2} \sim N(5*0.1, 5*0.015^2)$

Calculation for minimum value of accumulation factor S_{n2} at 99.5% confidence interval for time t=5 to t=10 :

We require to find y such that

$$\Pr(\ln(S_{n2}) > \ln y) = 0.995$$

$$\Rightarrow \Pr\left(Z > \frac{\ln y - 0.5}{\sqrt{5} * 0.015}\right) = 0.995$$

Using tables

$$\frac{\ln y - 0.5}{\sqrt{5} * 0.015} = -2.58$$

So, $y = e^{0.5 - 2.58 * \sqrt{5} * 0.015}$

$= 1.51205$

So minimum accumulated amount at 99.5% confidence interval at time t=10 is

$= 17{,}120.31 * 1.51205$

$= 25{,}886.76$

(b) Guaranteed maturity outgo

$= 9000 * 1.095^{10} + 6000 * 1.095^{5}$

$= 31{,}749.48$

Possible loss amount at 99.5% confidence level

$= 31{,}749.48 - 25{,}886.76$

$= 5862.72$

The amount required at time t = 0 in order to mitigate the loss at 99.5% confidence interval is

$= 5862.72 * 1.06^{-10}$

$= 3273.71$