



Subject: Statistical & Risk Modelling - 01

Chapter: Unit 03

Category: Practice Questions

Note - Graphical Graduation is no longer a part of the IAQS syllabus. Whichever answer has links to graphical graduation, please consider that as out of syllabus. You have to answer three methods as

1. Parametric formula
2. Standard Table
3. Spline functions

1. CT4 September 2010 Question 8

Rocky Bay is a small seaside town in the north of Europe. In a leaflet advertising the town, the tourist office has claimed that ‘in August, Rocky Bay has a Mediterranean climate’. An actuarial student spent August 2009 on holiday in Rocky Bay with his family, and became sceptical of this claim. When he returned home, he thought it might be interesting to examine the claim by applying some of the methods he had learned while studying for the Core Technical subjects. For each of the 31 days in August 2009 he collected data recorded by various meteorological offices on the maximum temperature in Rocky Bay and the mean of the maximum temperatures reported on the same day at a range of places in the Mediterranean region. The data are shown below, where, for each of the days in August, ‘+’ means that Rocky Bay had the higher maximum temperature and ‘-’ means that the Mediterranean average was higher.

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
-	-	-	-	-	-	-	-	-	-	-	-	+	+	+	+	-	-	-	-
21	22	23	24	25	26	27	28	29	30	31									
-	-	-	-	-	-	-	-	-	+	+	+								

- (i) Carry out a statistical test to examine the tourist office’s claim.
- (ii) Suggest reasons why the test might not be an appropriate way to examine the tourist office’s claim.

2. CT4 April 2012 Question 3

A graduation of a set of crude mortality rates is tested for goodness-of-fit using a chi-squared test.

Discuss the factors to be considered in determining the number of degrees of freedom to use for the test statistic.

3. CT4 April 2013 Question 9

A life office compared the mortality of its policyholders in the age range 30 to 60 years inclusive with a set of mortality rates prepared by the continuous mortality investigation (CMI). The mortality of the life office policyholders was higher than the CMI rates at ages 30-35, 38-41, 41-50 and 54-59 years inclusive, and lower than the CMI rates at all other ages in the age range.

- (i) Perform two tests of the null hypothesis that the underlying mortality of the life office policyholders is represented by the CMI rates.
- (ii) Comment on your results from part (i).
- (iii) Explain the problem which duplicate policies cause in the context of the CMI mortality investigation.

4. CT4 September 2015 Question 11

- (i) Describe why an insurance company might want to compare the results of a mortality investigation with previous experience.

A large life insurance company has undertaken an investigation of the mortality of its policyholders. Currently it assumes that mortality at age x , m_x , is equal to a standard table. The company wishes to use the results from the investigation to see whether the standard table is still appropriate. Below are shown some data from the investigation.

Age x	Number of policies in force	Actual death claims	Expected death claims from standard table
70	1,000	13	23.74
71	1,200	28	31.80
72	1,100	31	32.50
73	1,100	34	36.20
74	1,000	39	36.63
75	1,000	41	40.73
76	950	41	42.99
77	900	40	45.20
78	850	46	47.34
79	800	48	49.35

(ii) Perform an overall test of the hypothesis that the underlying mortality of the company's policyholders is, over this range of ages, represented by the standard table.

(iii) Evaluate the suitability of the standard table for use in the company's financial modelling by performing two additional tests for different possible inconsistencies between the actual death rates and those represented by the standard table.

5. CT4 April 2015 Question 6

A health insurance company has collected data on sickness rates during the calendar year 2013 among a sample of its policyholders aged 40–64 years inclusive. It compares these to the rates among its policyholders of the same age in 2012. It finds that at ages 40–50 years inclusive, and at ages 56–61 years inclusive, the sickness rates in 2013 are higher than those in 2012. At other ages, the sickness rates in 2013 were lower than those in 2012.

(i) Carry out two tests of the null hypothesis that the underlying sickness rates in 2013 are the same as those in 2012.

(ii) Comment on the implications of the results of your test for the company's sickness insurance business.

6. CT4 April 2016 Question 4

The manager of a life insurance company wishes to revise the premiums for term assurance policies. He has asked a trainee to compare the latest mortality estimates from the Continuous Mortality Investigation (CMI) for ages 40–64 years inclusive with the estimates the company has been using in its premium calculations, using a 95% significance level. The trainee says: 'I've done the Signs Test and we just pass – one more positive sign and we would have failed!'.

(i) Calculate the number of ages for which the company's mortality estimate was higher than that produced by the CMI.

Ten minutes later the trainee says: 'I tried the Grouping of Signs test and we just fail. We needed one more positive run!'.

(ii) Determine the number of runs of positive signs in the company's data.

7. CT4 October 2016 Question 5

(i) Describe why the raw data gathered from a mortality investigation need to be graduated.

(ii) Explain which method of graduation would be most suitable for each of the following mortality investigations:

- (a) the female population of a large European country
- (b) a study of the mortality of rhinoceroses in the safari parks of South Africa
- (c) the pension scheme of a large company

8.CT4 April 2017 Question 10

The government statistical service in a country with a population of 10 million has estimated mortality rates among males in that country aged 20 to 99 years inclusive. It wishes to create a new standard mortality table.

(i) Describe why the crude mortality rates should be graduated during the production of this standard mortality table.

(ii) Describe a suitable method of graduation for these mortality rates.

(iii) Explain the limitations of the method described in your answer to part (ii) in this situation.

The government performs the graduation and compares the crude and graduated rates. Below are some of the results of the comparison:

<i>Value of individual standardised deviation at age x, z_x</i>	<i>Number of ages</i>
$z_x < -3$	0
$-2 > z_x \geq -3$	7
$-1 > z_x \geq -2$	16
$0 > z_x \geq -1$	26
$1 > z_x \geq 0$	16
$2 > z_x \geq 1$	10
$3 > z_x \geq 2$	2
$z_x \geq 3$	3

(iv) Assess the quality of the graduated rates for use as a new standard mortality table by applying TWO statistical tests to the above information. The two tests should each examine a different aspect of the graduation.

(v) Comment on the implications of your results in part (iv) for the government using the new standard mortality table for economic and financial planning purposes.

9. CT4 April 2018 Question 3

(i) List the advantages of graduation:

- by parametric formula.
- with reference to a standard table.
- using a graphical method.

- (ii) Outline the steps involved in graduating mortality rates with reference to a standard table.

10. CT4 September 2018 Question 2

Describe how you would determine the number of degrees of freedom to use in a chi-square test when graduating a set of crude mortality rates.

11. CS2A September 2020 Question 5

An investigation was undertaken into the mortality of policyholders for a large life insurance company. The crude mortality rates were graduated using a formula of the form:

$$\mu_x = \exp(ax + bx^2).$$

An extract of the results is set out below. All data have been collated between 1 January 2018 and 1 January 2019 on an 'age last birthday' basis:

<i>Age</i>	<i>Exposed-to-risk (years)</i>	<i>Observed deaths</i>	<i>Graduated rates</i>
50	23,308	70	0.00368
51	19,316	58	0.00379
52	16,914	54	0.00391
53	21,082	90	0.00402
54	14,820	70	0.00415
55	24,084	96	0.00428
56	28,076	114	0.00441
57	22,958	86	0.00455
58	24,960	102	0.00469
59	21,134	86	0.00485
60	18,374	94	0.00500

- (i) Perform the signs test to explore the hypothesis that the graduated mortality rates are the true rates underlying the observed data from the life insurance company.
- (ii) State the main limitation of the signs test in assessing the suitability of the graduated rates.

(iii) Perform a chi-square goodness-of-fit test to explore further the hypothesis that the graduated mortality rates are the true rates underlying the observed data from the life insurance company.

(iv) Comment on your answers to parts (i) and (iii).

12. CS2A April 2021 Question 7

An Actuarial Analyst is investigating the forces of mortality for males aged 65 and over. The Analyst has studied a group of 100 male lives, all of whom were exactly 65 years old at the beginning of the study, over a 10-year period and has estimated the following forces of mortality based on the lives observed in the study:

<i>Duration from age 65 (years)</i>	<i>Force of mortality (p.a.)</i>
0–1	0.040
1–8	0.005
8+	0.080

(i) Determine, to six decimal places, the probability that a 65-year-old male is alive at age 75, using the estimated forces of mortality in the table above.

A colleague has suggested that the Analyst use the following formula to model the forces of mortality at age x : $\mu_x = 0.0020291 + 0.0001000 \times 1.0793496^x$

(ii) Verify that the probability that a 65-year-old male is alive at age 75 using the suggested model matches the result calculated in part (i) to six decimal places.

The Analyst decides, on the basis of the matching probabilities in parts (i) and (ii), to use the suggested formula to graduate the estimated forces of mortality in the table above.

(iii) Comment on the Analyst's decision.

13. CS2A September 2021 Question 4

An Actuary has graduated the mortality experience of a population aged 55 to 65 years using the following formula:

$$\mu_x = \begin{cases} ax + b \exp(cx) & \text{if } x < 65 \\ d & \text{if } x = 65 \end{cases}$$

where a , b , c and d are constants and x is age in years.

The mortality experience data and the graduated rates calculated by the Actuary are shown in the table below. All data have been collated between 1 January 2020 and 31 December 2020 inclusive on an 'age nearest birthday' basis:

<i>Age x</i>	<i>Exposed-to-risk (years)</i>	<i>Observed deaths</i>	<i>Graduated rates</i>
55	2,737	53	0.02094
56	2,610	57	0.02357
57	2,649	86	0.02636
58	2,611	77	0.02930
59	2,449	74	0.03238
60	2,213	96	0.03555
61	2,025	79	0.03880
62	1,969	68	0.04208
63	1,900	78	0.04537
64	1,803	83	0.04860
65	1,736	105	<i>y</i>

Let y be the graduated rate at age 65 and let the null hypothesis be that the graduated rates are the true rates underlying the observed data.

Determine the range of values that y needs to take so that there is insufficient evidence, at the 97.5% confidence level, to reject the null hypothesis under a chi-square goodness-of-fit test.

14. CS2A September 2022 Question 5

(i) An insurance company has graduated the experience of one block of its life business using the following quadratic-Gompertz formula:

$$M_x = \exp(a_0 + a_1x + a_2x^2)$$

where a_0 , a_1 and a_2 are constants and x is age in years.

The data used for this graduation exercise, together with the graduated rates, are shown in the table below. All data have been collated between 1 January 2018 and 31 December 2018 inclusive on an 'age nearest birthday' basis.

<i>Age</i>	<i>Exposed to risk (years)</i>	<i>Death counts</i>	<i>Graduated rates</i>
60	11,362	80	0.0069
61	11,086	85	0.0077
62	10,816	100	0.0085
63	10,530	105	0.0095
64	10,301	108	0.0105
65	10,233	117	0.0115
66	9,970	110	0.0126
67	9,708	131	0.0138
68	9,667	145	0.015
69	9,807	140	0.0163

(i) Perform a Chi-square test to assess the overall goodness of fit of this graduation, stating the null and alternative hypotheses.

The company would like to extend the graduation to age 70. The death count and exposed to risk at age 70 are 150 deaths and 10,000 years, respectively.

Let $\hat{\mu}_{70}$ be the graduated rate at age 70, and let the null and alternative hypotheses be the same as in part (i), except that the age range is now from 60 to 70.

(ii) Determine, using the data in the table above, the condition that $\hat{\mu}_{70}$ would need to fulfil in order for there to be insufficient evidence, at the 5% significance level, to reject the null hypothesis under the cumulative deviations test.