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Subject: Financial Engineering 1

Chapter: Unit 1 & 2

Category: Assignment Questions

1.

i) Explain arbitrage opportunity. (2)

ii) Define law of one price. (2)

iii) Let pt be the price of 3-month European put option on a share with current share price Rs 125 and strike price of Rs 120. The call options on the same share are priced at Rs 30 and the current risk- free force of interest is r= 5% p.a.

a) If dividends are payable continuously at a rate of q= 15% p.a., calculate pt. (2)

b) Explain the strategy for arbitrage profit if, instead, the price of the put option is Rs 23. (4)

2. The process X has the stochastic differential equation
$dX_t = \alpha\,\mu\,(T-t)\,dt + \sigma\sqrt{(T-t)}\,dZ_t$,
where $\alpha > 0$ & μ are fixed parameters and Z is a standard Brownian motion under Q.

The function f is given by
$f(x, t) = e^{(m(T-t)-x)}$
where m is a differentiable function

Find $\frac{\partial m}{\partial t}$ if f(x, t) is a martingale. (6)

3. The stochastic process X follows the SDE given by

$$\frac{dXt}{Xt} = 0.25\ dt + \sigma\ dWt$$

where W is a standard Brownian motion.

Consider the new process Y defined by Yt = f(t, Xt) where

$$f(t, x) = e^{-t}x^2$$

i) Write an expression for dYt (4)

ii) Under what condition will the process be a martingale? (1)

4.

Consider an American call option on a stock with 10 months to maturity and risk free rate of interest is 9.5% per annum, exercise price is Rs. 350. A dividend of Rs.10 is expected after three months from now and again after six months from now.

Show that it can never be optimal to exercise the option on either of the two dividend dates. (3)

5.

A stock price follows Geometric Brownian Motion with an expected return of 16% and a volatility of 35%. The current price is Rs. 254.

What is the probability that a European call option on the stock with an exercise price of Rs. 258 and a maturity date in six months will be exercised?

What is the probability that a European put option on the stock with the same exercise price and maturity will be exercised? (4)

6. The market price of a certain share is modelled as Geometric Brownian motion.

The price S_t at time $t \geq 0$ satisfies:
$\log(S_t/S_0) = \mu t + \sigma B_t$
where B_t is a standard Brownian motion and μ and σ are constants

i. Show that dS_t can be written as: $dS_t/S_t = x\, dB_t + y\, dt$
where x and y are constants to be specified. (4)

ii. Derive $E[S_t]$ and $Var[S_t]$ (4)

iii. Derive $cov[S_{t1}, S_{t2}]$ where $0 < t1 < t2$ (6)

7. A commodity of price C is assumed to follow the process:

$dC= \mu C dt+\sigma C dW_t$

where μ and σ are positive constants and W_t is a standard Brownian motion. The continuously compounded risk free interest rate r is a constant.

You wish to value a special type of option.

You construct a recombining binomial tree algorithm using a proportionate "up step" u and "down step" d for each small time interval Δt, and the stock price at time 0 is S_0.

i) Specify fully the first step of the binomial process, giving formulae for the up and down probabilities and step sizes u and d. (3)

The initial commodity price is 80, σ is 15% per annum and r=0. You may assume that u can be approximated by $\exp(\sigma\sqrt{\Delta t})$ for small Δt

ii) For the tree specified in (i)

a) Draw three steps of the tree with quarter- year time steps and calculate the commodity price at each node. (3)

b) Using this tree, calculate the price of a 9- month European call option with an at-the-money strike. (2)

c) By considering each possible path in the tree, evaluate the price of a 9- month European lookback call option, where the lookback period includes time 0. Note that lookback call pays the difference between the minimum value and the final value of any asset price. (5)

8.

i) Derive the price of an option in one step binominal model using risk neural valuations. (5)

ii) What does the price of an option derived above say about the probabilities of the underlying stock going up or down? (2)

iii) Show that the underlying stock price grows on average at risk free rate (3)

iv) Interpret (i) and (iii) in terms of risk neutral world. (2)

9. Consider the following table of prices of European call and put options on a non-dividend-paying stock Iota Ltd.

Strike (K)	Price of call option	Price of put option	Time to expiry
70	13.334	0.120	3 months
75	8.869	0.568	3 months
a	6.899	1.055	3 months
80	b	1.789	3 months
85	2.594	c	3 months
90	2.569	7.909	6 months

i) Calculate the price of a 3-month forward on Iota Ltd. (4)

ii) Also, calculate the forward rate for delivery between 3 months from now and 6 months from now. (2)

iii) Fill in the missing values a, b, c in the above table. (3)

10.

Sheldon has decided to use an n-step recombining binomial tree for pricing a derivative on a non-dividend-paying stock Epsilon Ltd. Each time period in the tree is one month. Interest rates can be assumed zero.

i) Show that the risk-neutral up-step probability q that Sheldon should use is less than 0.5

ii) Use this tree to price a derivative, which expires one year from now, with the following pay-off at expiry (T):

$$Payoff = \sqrt{\frac{S_T}{S_0}}$$

(S0 is the current stock price and St is the price of the stock at time t)

11.

i) Explain what is meant by a recombining binomial tree. State the advantage and disadvantage of using a recombining tree [vis-à-vis a non-recombining tree] to model share price movements. (4)

ii) A trader in derivatives is using a two-step binomial tree to determine the value of a 6- month European put option on a non-dividend-paying share. The put option has a strike price of Rs. 950. The trader assumes that during the first 3 months, the current share price of Rs. 1000 will either increase by 10% or decrease by 5%. The continuously compounded risk free rate during the first 3 months is 1.75%. During the following 3 months, the trader assumes that the share price will either increase by 20% or decrease by 10%. The risk free rate during this period is expected to be 2.5%

a) Calculate the value of the put option. (6)

b) The trader believes that a more accurate value of the put option can be determined by dividing the term of the option into "months". State the disadvantages of applying this modification to the model; and suggest an alternative model based on months that might be efficient numerically. (4)

12.

Let {Z(t)} be a standard Brownian motion. You are given

a. $U(t) = 2Z(t) - 2$
b. $V(t) = [Z(t)]^2 - t$
c. $W(t) = t^2 Z(t) - 2\int_0^t sZ(s)ds$

Derive the SDEs and explain which of the processes defined above has / have zero drift?

13.

Consider an at-the-money American put option on the geometric average of price of a non-dividend paying stock currently priced at Rs. 200. Risk free interest is 10% per annum, volatility is 35% per annum and time to maturity is 2 months.

i) Determine values of u and d assuming stock price follows Geometric Brownian motion with drift μ and volatility σ stating any other assumption used. (5)

ii) Evaluate the value of an American put option using a 2-time step. The geometric average is measured from today till the option matures (4)

14.

The price of a stock is currently Rs. 500. Over each of the next two 3-month periods the stock price is expected to go up by 6% or down by 5%. The risk free interest is 5% per annum with continuous compounding.

i) What is the value of a six-month European call option with a strike price of Rs. 510? (3)

ii) What is the value of a six-month European put option with a strike price of Rs. 510? Verify that the European call and put prices satisfy put-call parity theorem. (3)

iii) What is the value of a six-month American put option with a strike price of Rs. 510? (3)

iv) Suppose in the real world the expected return is 9%. What is the expected payoff of the European call option after 3 months? Explain with reasons whether you would be able to calculate the no-arbitrage value of such an option or not. (2)

15.

Suppose the stock price S follows geometric Brownian motion with expected return μ and volatility σ:

$$dS = \mu S dt + \sigma S dz$$

Where dz is a wiener process

The risk free rate of interest (with continuous compounding) is r.

i) Determine the process followed by the variable S^K (where k is a positive integer). Find the expected return and variance of S^K . (5)

ii) For the same measure corresponding to dz, show that the stock price discounted at risk-free rate is a martingale only for a specific value of μ. (3)

iii) Can you make any similar claim about the process followed by S^K in part (i)? What conditions are needed? (1)