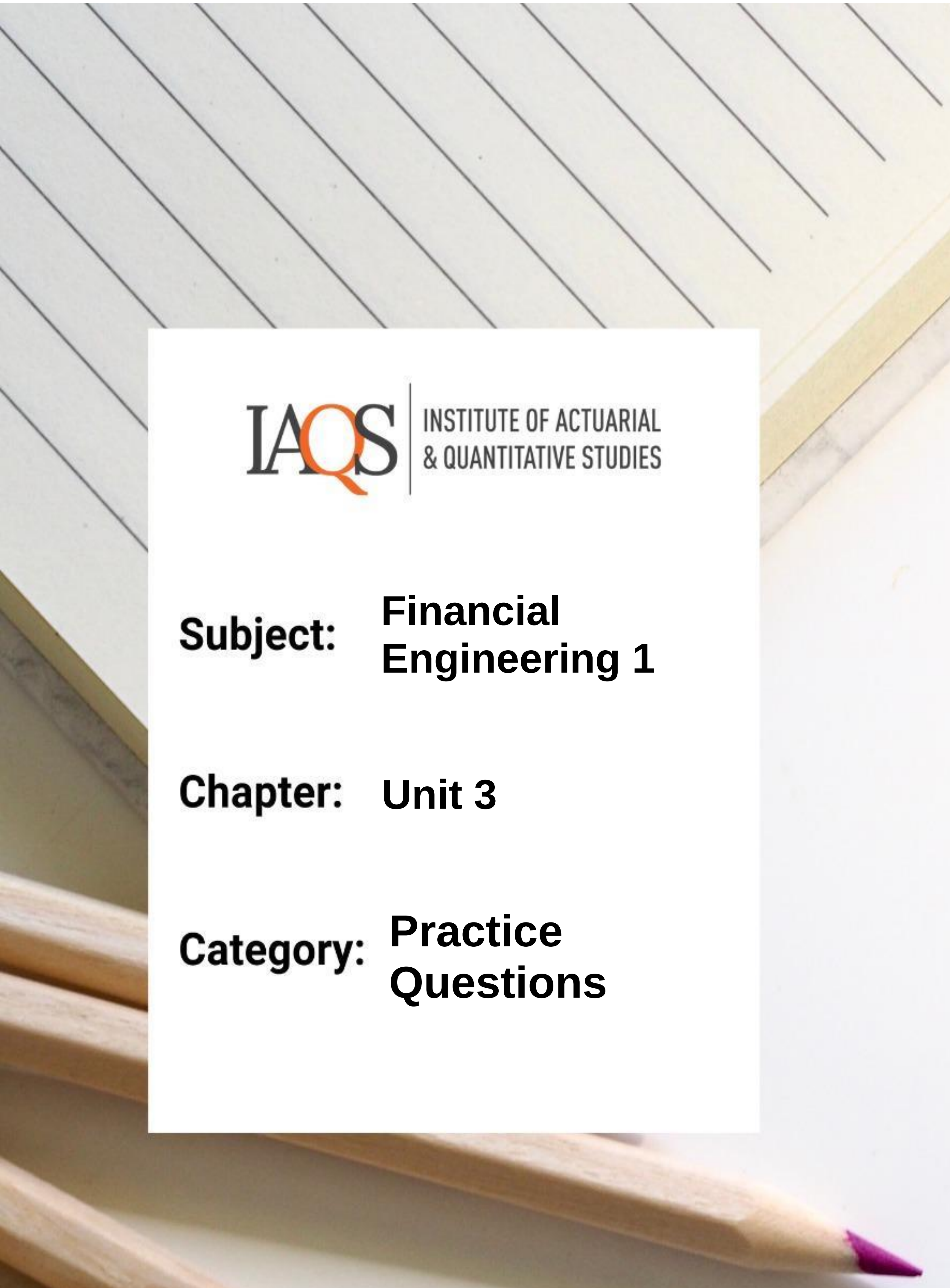

INSTITUTE OF ACTUARIAL & QUANTITATIVE STUDIES

Subject: Financial Engineering 1

Chapter: Unit 3

Category: Practice Questions

1. CM2A April 2022 Q3

Consider a share, S_t, and a derivative on the share with a value at time t of $f(t, S_t)$.

(i) Define, in your own words, what is represented by each of the following Greeks for this derivative:

(a) Delta

(b) Gamma

(c) Vega.

Consider another share, A_t, which pays no dividends. The continuously compounded risk-free rate is r. Let K be the fair price at time 0 of a forward contract on A_t maturing at time T.

(ii) State the formula for K.

Under the risk-neutral measure, Q, the share is expected to grow at the risk-free rate.

(iii) Demonstrate that the expected present value of the forward contract at time t $(0 \le t \le T)$ under the measure Q is $A_t - e^{-r(T-t)}K$.

(iv) Calculate Delta, Gamma and Vega for the forward contract.

(v) Comment on how the Greeks for the forward contract in part (iv) compare to the same Greeks for the underlying share.

(vi) Discuss whether it would be appropriate to use a forward contract to Delta hedge a European call option on the share.

2. CM2A September 2020 Q3

Consider a European call option based on an underlying non-dividend paying share priced at \$34.55 with volatility of 10% per annum. The option has a strike price of \$40 and is 3 years from expiry. The risk-free force of interest is 2.5% per annum.

(i) Calculate the value of the option using the Black–Scholes model.

(ii) Explain why Theta is negative for this option.

(iii) Explain why Delta for this option is positively correlated with the underlying share price.

(iv) Explain, by comparing the cashflows of an option-holder and a shareholder, why the value of this option would be lower if the share was dividend-paying.

Consider now a European put option on the same underlying share, with the same strike price and time to expiry.

(v) Calculate the value of the put option.

3. CM2A September 2020 Q6

A bank sells 1,000 European put options p_t on a non-dividend paying share S_t. The initial share price is S_0 = \$17 and the strike price of the option is \$19 at expiry in 2 years' time. The share has volatility 15% p.a. and the risk-free force of interest is 3% p.a.
The option price is \$1.963 and the Delta of the option is –0.446.

(i) Calculate the initial income for the bank from selling the options.

(ii) Determine the portfolio of shares and cash the bank should hold to Delta hedge its initial position.

One day later the share price has increased to \$19.

(iii) Estimate the new option price, using the value of Delta provided.

(iv) Calculate the new option price, using the Black–Scholes formula.

(v) Comment on the reasons for differences between your answers to parts (iii) and (iv).

(vi) Explain why a low value of Vega for this derivative would be desirable to the bank.

4. CT8 September 2018 Q10

(i) State the main assumptions underpinning the Black-Scholes model.

Consider a put option on a non-dividend paying stock when the stock price is £8, the exercise price is £9, the continuously compounded risk-free rate of interest is 2% per annum, the volatility is 20% per annum. and the time to maturity is three months.

(ii) Calculate the price of the option using the Black-Scholes model.

(iii) Discuss how the price of the contract in part (ii) would change if the rate of interest increases. (There is no need to carry out further calculations.)

5. CT8 April 2018 Q8

The current price of a non-dividend paying stock is £65 and its volatility is 25% per annum. The continuously compounded risk-free interest rate is 2% per annum.
Consider a European call option on this share with strike price £55 and expiry date in six months' time. Assume that the Black-Scholes model applies.

(i) Calculate the price of the call option.

(ii) Define algebraically the delta of the call option.

(iii) Calculate the value of the delta of the call option.

(iv) Calculate the value of the delta of a European put option written on the same underlying, with the same strike and maturity as above.

6. CT8 April 2018 Q10

Consider a call option on a non-dividend paying stock S when the stock price is £15, the exercise price, K, is £12, the continuously compounded risk-free rate of interest is 2% per annum, the volatility is 20% per annum and the time to maturity is three months.

(i) Calculate the price of the option using the Black-Scholes model.

(ii) Determine the (risk neutral) probability of the option expiring in the money.

A special option called a "digital cash-or-nothing" option has a payoff in three months' time of:

$$\begin{array}{ll} 1 & \text{if } S_T > K \\ 0 & \text{otherwise} \end{array}$$

(iii) Calculate the price of the digital option.

(iv) Describe the limitations of the Black-Scholes model.

7. CT8 September 2017 Q6

(i) Write down an expression for the price of a derivative in a Black-Scholes market in terms of an expectation under the risk-neutral measure, defining any additional notation that you use.
Consider an option on a non-dividend-paying stock when the stock price is £50, the exercise price is £49, the continuously compounded risk-free rate of interest is 5% per annum, the volatility is 25% per annum, and the time to maturity is six months.

(ii) Calculate the price of the option using the Black-Scholes formula, if the option is a European call.

(iii) Determine the price of the option if it is an American call.

(iv) Calculate the price of the option if it is a European put.

(v) Determine how the prices of the contracts in parts (ii) to (iv) would change in the case of a dividend-paying underlying stock. [Note that you do not have to perform any further calculations.]

8. CT8 April 2017 Q4

(i) Define the following Greeks algebraically:
(a) delta
(b) vega
(c) theta
(d) gamma

Consider a call option with price c_t at time t (in years) written on an underlying nondividend-paying asset with price S_t at time t and volatility σ.

Using Taylor's expansion, it can be shown that the change in value of the option is approximately given by:
$dc_t = \text{delta} \times dS_t + 0.5 \times \text{gamma} \times (dS_t)^2 + \text{ theta } \times dt + \text{ vega } \times d\sigma$

At time t = 0, the underlying asset price is €23 and the volatility is 20% per annum. The option is priced at €6.17 and has the following properties:
• delta = 0.822
• vega = 0.104
• theta = –0.855
• gamma = 0.033

At time t = 1, the security price has fallen to €20 and its volatility is now 15% per annum.

(ii) Estimate the value of the call option at time t = 1.

The delta of a call option is always positive, whilst the delta of a put option is always negative.

(iii) Justify this result.

The vega of both call and put options is always positive.

(iv) Justify this result.

9. CT8 April 2017 Q7

The current price of a non-dividend-paying share is £7 and its volatility is thought to be 40% per annum. The continuously compounded risk-free interest rate is 5% per annum. A European call option on this share has a strike price of £6.50 and term to maturity of one year.

(i) Calculate the price of this call option, assuming that the Black-Scholes model applies.

The market price for the option is actually £2.

(ii) Show that the volatility of the share implied by the true market price of the option is 60% per annum, to the nearest 1% per annum.

10. CT8 September 2016 Q8

Consider a non-dividend-paying stock, with price S_t, and a European call option on that stock, whose value can be modelled using the Black-Scholes model.

(i) Write down the formula for the delta of this option under this model.

Suppose that the stock price at time 0 is $S_0 = \$40$ and the continuously compounded risk-free rate is 2% per annum. The call option has strike price $\$45.91$, term to maturity 5 years and a delta of $\Delta = 0.6179$.
(ii) Determine the implied volatility of the stock to the nearest 1%.

A second stock with price R_t is currently priced at $R_0 = \$30$ and has volatility $\sigma_R = \sqrt{15\%}$ per annum.

An exotic option pays an amount c at time T if $S_1/S_0 < k_S$ and $R_1/R_0 < k_R$.

(iii) Give a formula for the value of the option at time 0 if the two stocks are independent, defining any additional notation used.

(iv) Explain how the structure of the option could be simplified if the assets were perfectly correlated.

Assume now that the stock prices are independent. The option has term $T = 1$ year, payoff $c = \$50$ and strike prices $k_S = 0.8$ and $k_R = 0.6$.

(v) Determine the value of the option at time 0.

11. CT8 April 2016 Q7

Consider a non-dividend-paying share with price S_t at time t (in years) in a market with continuously compounded risk-free rate of interest r.

(i) Show that the fair price at t = 0 of a forward contract on the share maturing at time T is $K = S_0 e^{rT}$.

A share is currently worth S_0 = €20. The continuously compounded risk-free rate of interest is 1%

per annum.

(ii) Calculate the fair price at t = 0 of a forward contract written on the share with delivery at t = 2.

(iii) Give an expression for the value to the investor of the forward contract in part (ii) at time $t \leq 2$, in terms of S_t, t and r.

An investor enters into the above forward contract at time t = 0. At time t = 1 the risk-free rate of interest has increased to 4% per annum. The share price has not changed.

(iv) Calculate the value to the investor of the forward contract at t = 1.

(v) Determine each of the following Greeks for the contract value at time t = 1:
• delta
• theta
• vega

12. CT8 September 2015 Q7

A non-dividend paying share currently trades at S_0 = \$10. An investor is considering buying a European call option on the share with a strike price of \$12 and expiry in five years. The continuously compounded risk-free rate of interest is 4% p.a.

(i) Determine lower and upper bounds for the price of the call option at time 0.

The call option is currently priced at \$1.50. The assumptions of the Black-Scholes model apply.

(ii) Calculate the implied volatility of the share.

(iii) Determine the corresponding hedging portfolio in shares and cash for 100 call options.

13. CT8 April 2015 Q3

A share is currently priced at 640p. A writer of 100,000 units of a one year European put option with an exercise price of 630p has delta-hedged the option with a portfolio which holds cash and is short 24,830 shares. The continuously compounded risk-free rate of interest is 3% p.a. and no dividends are payable during the life of the option.
The assumptions of the Black-Scholes model apply.

(i) (a) Write down an expression for the delta of the option.

(b) Calculate its value in this case.

(ii) Prove that the volatility of the share implied by the delta is 7.1% p.a. (assuming it is less than 100%).

(iii) (a) Calculate the price of the option.

(b) Determine the value of the cash holding in the hedging portfolio.

14. CT8 April 2015 Q7

(i) Define delta, gamma and vega for an individual derivative.

A bank is considering selling a European call option on a share and wants to hedge some of its risk.

The share is non-dividend paying and has the following properties:
Strike price = \$50
Option price = \$17.91
Underlying share price = \$60
Volatility = 25% p.a.
Time to expiry = 3 years

The continuously compounded risk-free rate of interest is 3% p.a. and the vega for this option is \$29.00.

(ii) Calculate delta for this option.

(iii) Identify a delta-hedged replicating portfolio using the share and the risk-free asset.

Assume that the volatility has instantaneously increased to 27% p.a., with everything else except the option price remaining the same.

(iv) Estimate the new option price.

15. CT8 September 2014 Q7

Let B be a standard Brownian motion.
(i) Derive the probability density function of $\max_{0\le s\le t}(B_s + \mu s)$, where μ is a constant, using the formula in section 7.2 of the Actuarial Formulae and Tables.

In a Black-Scholes market, let S be the stock price.

(ii) Give the expression for the fair price at time t of a derivative written on S paying an amount D_T at time T, defining any terms you use.

Suppose that S has an initial price of $S_0 = £1.20$ and a volatility $\sigma = 30\%$ p.a. and that the continuously compounded risk-free rate is $r = 3\%$ p.a.

(iii) Calculate the fair-price at time zero of the derivative paying £10 at time $T = 2$ if and only if $\max_{0\le s\le T}(B_s + \mu s) > £1.44$.

16. CT8 September 2014 Q8

In a Black-Scholes model, the delta of a call option is $\Delta = \Phi(d_1)$.
(i) Define delta.

Suppose that the stock price at time zero is $S_0 = \$100$, the continuously compounded risk-free rate is 3% and that a European call option written on S with strike price $\$109.42$ and maturity $t = 1$ year has a delta of $\Delta = 0.42074$.

(ii) Find the implied volatility of the stock to the nearest 1%.

An exotic option written on S with strike prices K_1 and K_2 and exercise times τ and T is defined as follows:

- The option may be exercised at time τ in which case the holder receives $\$100$ if and only if the price of the underlying, S_τ is at least K_1.
- If the option is not exercised at time τ, then the holder will receive an amount c if and only if the price at expiry T, S_T, satisfies $S_T/S_\tau \geq K_2$.

(iii) Explain why, if $c \leq \$100$, the option will always be exercised at time τ when S_τ is at least K_1.

(iv) Give a formula for the value of the option just after the first exercise time τ (i.e. just after the first exercise option has expired).

(v) Explain why this value does not depend on the stock price at time τ.

Suppose that $K_1 = \$10, K_2 = e^{-0.09}, \tau = 1$ year, $T = 2$ years and $c = \$200$.

(vi) Determine the fair price of the exotic option just after time one and hence at time one and at time zero.

17. CT8 April 2014 Q8

(i) State and prove the put-call parity for a stock paying no dividends.

In a Black-Scholes market, a European call option on the dividend-free stock, with strike price \$120 and expiry T = 1 year is priced at \$10.09. The continuously compounded risk-free rate is 2% p.a. and the stock is currently priced at \$110.

(ii) Estimate the implied volatility of the stock to the nearest 1%.

A European put option on the same stock has strike price \$121 and the same maturity. An investor holds a portfolio which is long one call and short one put.
(iii) Sketch a graph of the payoff at maturity of the portfolio against the stock price

(iv) (a) Determine an upper and a lower bound on the value of the portfolio at maturity.

(b) Deduce bounds for the current put price.

(v) Determine the fair price of the put.

18. CT8 September 2013 Q9

A one-year European call option on a non-dividend paying stock in Company ABC has a strike of \$150.

The continuously compounded risk-free rate is 2% p.a. The current stock price is \$117.98. Assume that the market follows the assumptions of a Black-Scholes model.

An institutional investor holds a delta-hedged portfolio with 100,000 call options, no cash and short 18,673 shares of Company ABC.

(i) Calculate the delta of the call option.

(ii) Calculate the implied volatility for the underlying.

(iii) Calculate the price of a one-year put on the same stock with a strike of \$150.

The investor retains their holding of call options and trades in the put and the stock to achieve a delta and gamma-hedged portfolio.

(iv) Calculate the investor's new holdings of the put and the stock.

19. CT8 April 2013 Q9

In a Black-Scholes market, we consider a special option with strike K and expiry in 2 years on an underlying (non-dividend bearing) stock with price process S_t.

Its payoff at maturity is $\$100\text{Max}\,(S_2/S_1 - 1; 0)$ if and only if the stock price has not exceeded \$2 by time 1.

The volatility of the stock is 25% p.a. and the continuously compounded risk-free rate is 3% p.a. The initial stock price is \$1.

(i) Calculate $Q(\text{Max}_{t<1}\ S_t < 2)$, where Q is the EMM, using the formula in the actuarial tables and the representation of a geometric Brownian Motion.

(ii) (a) Write down an expression for the price of this option at time 1. You should consider separately the two cases $(\text{Max}_{t<1}\ S_t) < 2$ and $(\text{Max}_{t<1}\ S_t) \geq 2$

(b) Show that the value of this option at time 1 is $11.348 in the case $(\text{Max}_{t<1}\ S_t) < 2$

Hint: S_2/S_1 is independent of the values of S_t up to time 1 under the EMM.

(c) Determine, using the result in (i), the fair price at time 0 for the option.

20. CT8 September 2012 Q7

A three-month European call option on a non-dividend paying stock in Universal Widget Inc with a strike price of $1.30 has current price of $0.8557.

The continuously compounded risk-free rate is 0.5% p.a. The current stock price is $1.20. Assume all the Black-Scholes assumptions hold.

(i) Calculate the implied volatility for the underlying stock to within 1% p.a.

It is known that in three months Universal Widget Inc will embark on a major restructuring. It is anticipated that this will double the volatility of the stock price thereafter.

(ii) Write down a formula in terms of the underlying Brownian motion, Z, for the stock price in three months' and in six months' time.

(iii) Derive the corresponding price of a six month European put on the Universal Widget Inc stock with strike price $1.20.