INSTITUTE OF ACTUARIAL & QUANTITATIVE STUDIES

Subject: Financial Engineering 2

Chapter:

Category: Assignment Questions

1. Consider the following stochastic differential equation for the instantaneous risk free rate (also referred to as the short-rate):

$$dr(t) = a\left(b - r(t)\right)dt + \sigma dW_t$$

Its solution is given by:

$$r(t) = r_0 \exp(-at) + b\left(1 - \exp(-at)\right) + \sigma \exp(-at)\int_0^t \exp(as)\,dW_s$$

You may also use the fact that for $T > t$:

$$\int_t^T r(u)du = b(T-t) + (r(t) - b)\frac{1-\exp\left(-a(T-t)\right)}{a} + \frac{\sigma}{a}\int_t^T \left(1 - \exp\left(-a(T-s)\right)\right)dW_s$$

i) Derive the price at time t of a zero-coupon bond with maturity T.

ii) a) State the main drawback of such a model for the short-rate.
b) State the name and stochastic differential equation of an alternative model for the short-rate that is not subject to the drawback.

2. State eight desirable characteristics of a term-structure model.

3. Consider the following model for the short-rate r:

$$dr_t = \mu r_t dt + \sigma dZ_t$$

where μ and σ are fixed parameters and Z is a standard Brownian motion.

i) Comment on the suitability of this model for the short-rate.

An alternative model for the short-rate is the Vasicek model:

$$dr_t = a\left(\mu - r_t\right)dt + \sigma dZ_t$$

ii) Derive an expression for $\int_t^T r(u)du$

iii) State the distribution of $\int_t^T r(u)du$

4. Write down the properties of the following two models for interest rates:

a) The one-factor Vasicek model

b) The Cox-Ingersoll-Ross model

[You are not required to give any formulae for the models.]

The Vasicek term structure model is described by the following stochastic differential equation:

$$dr_t = a(b - r_t)dt + \sigma dW_t$$

With initial value r_0 and $a,b,\sigma > 0$.

i) Show, by solving the Vasicek stochastic differential equation, that:

$$r_t = r_0 e^{-at} + b(1 - e^{-at}) + \sigma \int_0^t e^{-a(t-s)} dWs$$

ii) Determine the expectation, the variance and the distribution of the short rate r_t

5. Consider a market with the following bonds in issue.

Principal value	*Expire (years) T*	*Coupon (annual*)*	*Price*	*Zero rate R(0, T)*	*Forward rate F(0, S, T)*
100	0.25	0	97.5	(a)	
100	0.5	0	94.9	(b)	F(0, 0.25, 0.5) = 10.81%
100	1	0	90.0	10.54%	F(0, 0.5, 1) = (d)
100	1.5	8%	(c)	10.68%	F(0, 1, 1.5) = (e)
(* half the stated coupon is paid every 6 months)					

i) Calculate the values of (a), (b), (c), (d), (e) in the table above.

ii) Write down the stochastic differential equations of two standard models for the short rate of interest.

6. A Eurodollar futures price changes from 96.76 to 96.82. What is the gain or loss to an investor who is assuming that a company has fixed debt of £40m with a term of 10 years, the value of the equity in the company is £20m and the Merton model for credit risk holds true. The risk free rate of return is 5% p.a. and there are no other dividends or interest payments.

i) Explain how to calculate the (risk neutral) probability of default. You do not have to calculate the probability, but should state how each value would be calculated.

In a particular two state model for credit rating with deterministic transition intensity, the risk free rate is a constant, r, the recovery rate is δ and the zero coupon bond price is given by:

$$B(t,T) = e^{-r(T-t)}\left[1-(1-\delta)\left(1-e^{-\frac{(T^2-t^2)}{4}}\right)\right].$$

ii) a) State the general formula for the zero coupon bond prices in a two state model for credit ratings.
b) Deduce the risk-neutral default intensity for the particular two state model above.

7. Define default risk and describe default correlation.

8. Define and Explain:

i) Default Risk
ii) Recovery Risk
iii) Exposure Risk

9. What are credit events? Which types of credit events can occur?