INSTITUTE OF ACTUARIAL & QUANTITATIVE STUDIES

Class: TY BSc

Subject : Financial Engineering - 2

Chapter: Unit 1 Chapter 1

Chapter Name: Interest Rate Derivatives and Hedging - 1

Topics to be covered

1. Interest Rate Derivatives (IRD)

1. Forward Rate Agreement (FRA)

1. Interest Rate Futures (IRF)
    1. Example – Eurodollar Futures Contract
    2. Differences between Eurodollar Futures Contracts & FRA
    3. IRF – Indian Scenario

1. Treasury Bond Futures
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    3. Conversion Factors
    4. Cheapest-to-Deliver (CTD) Bonds
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2. Interest Rate Swaps
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6. Swap Options (Swaptions)
    1. Classification of Swaptions
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Discussion

Investments which generate cash-flows have significant risks. These risks stem from the uncertainty regarding the payment of the cash-flows and also about their present or current value. What does the risk for their valuation stem from and how can it be tackled?

A video on what those risks are how they can influence valuation.

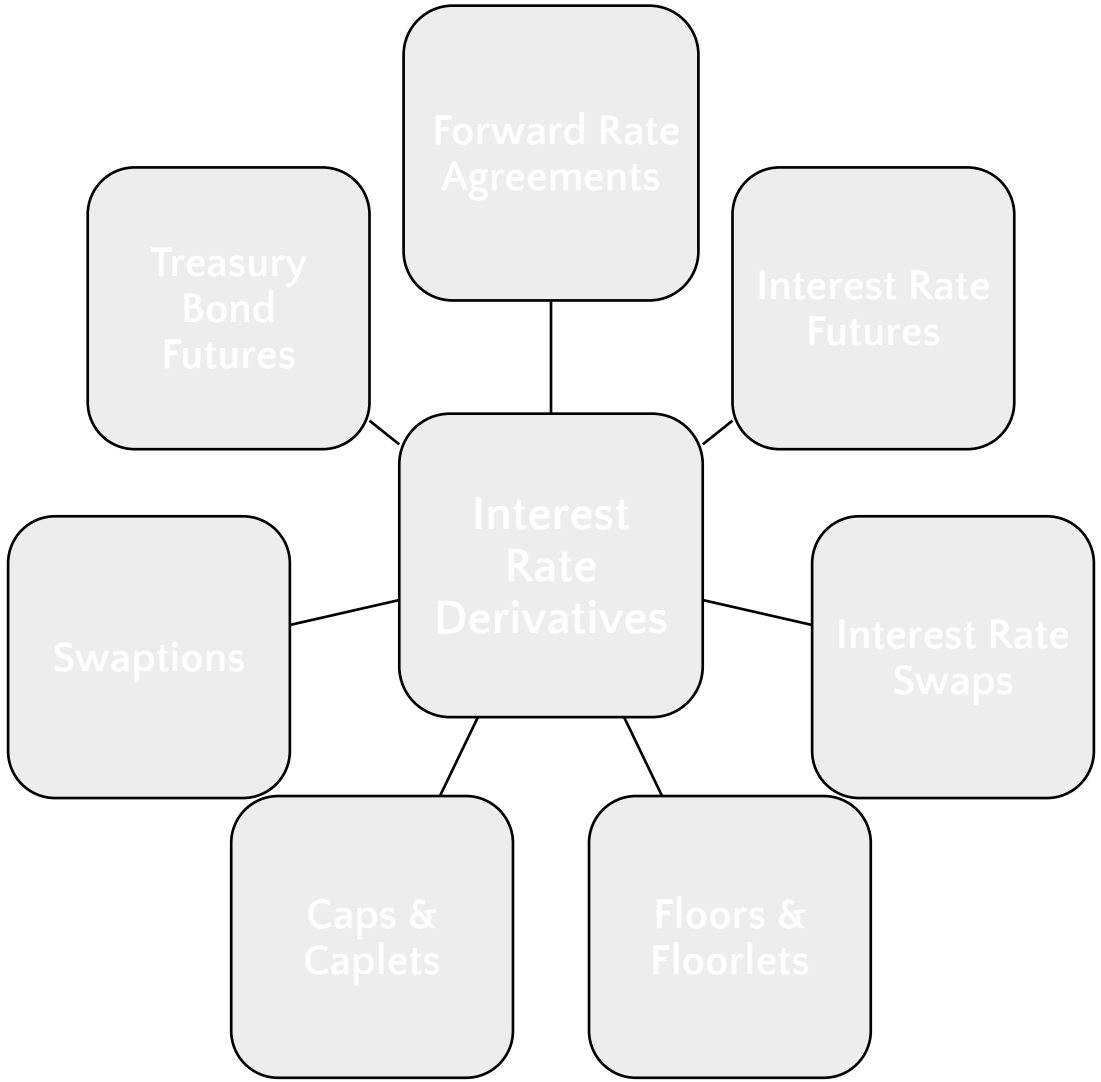
- We've learnt about how stock options are priced and used. Stock options can be used by company's management for safeguarding it's control or it can be used by investors and speculators like hedge fund managers to make profits. Similarly, options exist for interest rate.
- These are used by certain institutions to safeguard their assets and future cashflows while others use it to speculate and invest. We'll learn more about them in this chapter.
- While we were pricing stock/equity options, interest rate was an underlying factor influencing their price. Curiously this underlying factor itself has a lot of derivatives priced to hedge against it's volatility!

1 Interest Rate Derivatives (IRD)

Interest Rate Derivative (IRD) is a financial derivative contract whose value is derived from one or more interest rates, prices of interest rate instruments, or interest rate indices.

- Interest rate derivatives are often used as hedges by institutional investors, banks, companies, and individuals to protect themselves against changes in market interest rates, but they can also be used to increase or refine the holder's risk profile or to speculate on rate moves.
- They may include interest rate futures, options, swaps, swaptions, and FRA's.

1 Interest Rate Derivatives (IRD)



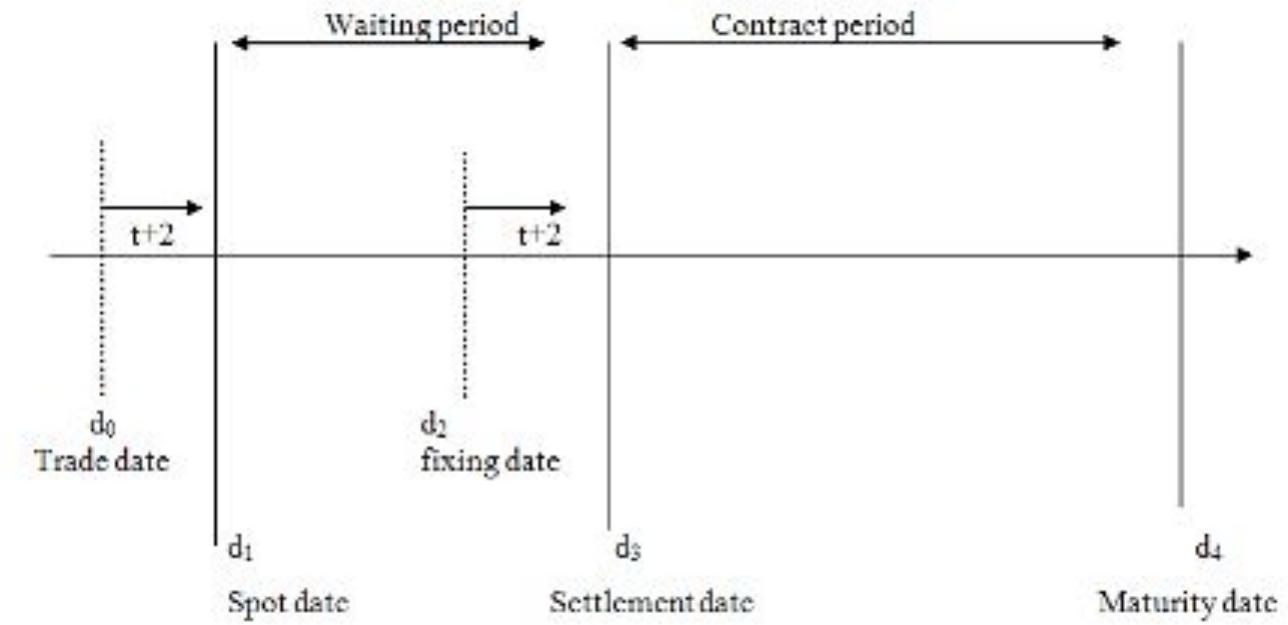
2 Forward Rate Agreement (FRA)

A forward rate agreement (FRA) is an over-the-counter (OTC) contract designed to fix the interest rate that will apply to either borrowing or lending a certain principal amount during a specified future time period.

- When an FRA is first negotiated the specified interest rate usually equals the forward rate. The contract then has zero value.
- Most FRAs are based on London Inter-bank Offered Rate (LIBOR).
- A trader who will borrow a certain principal amount at LIBOR for a future period can enter into an FRA where for the specified time period LIBOR will be received on the principal amount and a predetermined fixed rate will be paid on the principal amount.
- This converts the uncertain floating LIBOR rate to a fixed rate. If LIBOR proves to be greater (less) than the fixed rate the payoff from the FRA is positive (negative).

2 Forward Rate Agreement (FRA)

- A diagram showing the timeline of how a typical FRA is executed :



- The basic purpose of the FRA is to hedge the interest rate risk.

2 Forward Rate Agreement (FRA)

Example

Suppose that a company enters into an FRA that is designed to ensure it will receive a fixed rate of 5% on a principal of $100 million for a 6-month period starting in 3 years. The FRA is an exchange where LIBOR is paid and 5% . is received for the 6-month period. If 6-month LIBOR proves to be 5.5% for the 6-month period, the cash flow to the lender will be :

$$\mathbf{100,000,000} * (\mathbf{0.05} - \mathbf{0.055}) * \mathbf{0.50} = -\$\mathbf{250,000}$$

at the 3.5-year point. This is equivalent to a cash flow equal to the present value of $250,000 at the 3-year point. The cash flow to the party on the opposite side of the transaction will be +$250,000 at the 3.5-year point or the present value of this at the 3-year point.

Note: All interest rates in this example are expressed with semi-annual compounding.

3 Interest Rate Futures (IRF)

Interest Rate Futures (IRF) are standardized IRD contracts to buy or sell a notional security or any other interest-bearing instrument or an index of such instruments or interest rates at a specified future date, at a price determined at the time of the contract.

- Interest Rate Futures include Money Market Futures. Currently, IRF contracts are available on GoI securities, Federal Funds Rates, Eurodollar Futures Contracts, etc. The latter is very popular globally because of the wide acceptance of those currencies.
- IRF are widely traded on CME(Chicago Mercantile Exchange) and CBOT(Chicago Board of Trade) and are standardized contracts.

3 Interest Rate Futures (IRF)

- The two most widely traded Interest Rate Futures contracts are the Eurodollar Futures Contract and the Federal Funds Futures.
- The **Eurodollar Futures Contract** is a contract on the 3-month LIBOR interest rate two days before the third Wednesday of the delivery month.
- The Federal Funds Rate is the interest rate that banks charge each other to borrow or lend excess reserves overnight. There are short term futures called **Federal Funds Futures** on these rates.
- Law requires that banks must have a minimum reserve level in proportion to their deposits. This reserve requirement is held at a Federal Reserve Bank. When a bank has excess reserve requirements, it may lend these funds overnight to other banks that have realized a reserve deficit.

3 Interest Rate Futures (IRF)

3.1 Example – Eurodollar Futures Contracts

An investor wants to lock in the interest rate for a three-month period beginning two days before the third Wednesday of September, on a principal of $100 million. We suppose that the September Eurodollar futures quote is 96.50, indicating that the investor can lock in an interest rate of 100 – 96.5 or 3.5% per annum. The investor hedges by buying 100 contracts. Suppose that, two days before the third Wednesday of September, three-month LIBOR turns out to be 2.6%. The final settlement in the contract is then at a price of 97.40. The investor gains:

$$\mathbf{100 * 25 * (9,740 - 9,650) = 225,000}$$

or $225,000 on the Eurodollar futures contracts. The interest earned on the 3-month investment is:

$$\mathbf{100,000,000 * 0.25 * 0.026 = 650,000}$$

or $650,000. The gain on the Eurodollar futures brings this up to $875,000, which is what the interest would be at 3.5% = $\boldsymbol{(100,000,000 * 0.25 * 0.035 = 875,000)}$.

3 Interest Rate Futures (IRF) – Indian Scenario

With the commencement of Interest Rate Futures trading under a new framework, the Indian financial markets would achieve another milestone. Interest rates are linked to a variety of economic conditions. They can change rapidly, influencing investments and debt obligations. In a market environment where long term debt issuance by the government is increasing and the demand for it is growing, there is a strong need for a cost efficient hedging instrument against interest rate risk. Electronic trading platform of NSE ensures transparency of prices, volumes and trade data.

Products

Currently, Interest Rate Futures segment of NSE offers two instruments i.e. Futures on 6 year, 10 year and 13 year Government of India Security (NBF II) and 91-day Government of India Treasury Bill (91DTB).

The NSE Bond Futures II (NBF II) contracts are available for trading based on Government of India (GOI) security of face value 100 with semi-annual coupon and residual maturity between 4 and 8 years, 8 and 11 years and 11 and 15 years on the day of expiry of IRF contract, as decided by stock exchanges in consultation with FIMMDA.
Three Serial monthly contracts followed by three quarterly contracts of the cycle March/June/September/December will be made available along with functionality for spread contract trading on the NSE electronic trading platforms.

4 Treasury Bond Futures

Treasury Bond Futures are financial derivatives that obligate the contract holder to purchase or sell a government or treasury bond on a specified date at a predetermined price.

- Treasury bond futures are available on various treasury bonds issued by different national governments. The US T-bonds and their futures are the most widely traded ones.
- Treasury bonds (T-bonds) are a type of debt issued by the U.S Department of Treasury to finance the U.S. government's spending activities. T-bonds have long durations, issued with maturities of between 20 and 30 years.
- Bond futures trade primarily on the Chicago Board of Trade (CBOT), which is part of the Chicago Mercantile Exchange (CME). Contracts typically expire quarterly: March, June, September, and December.

4 Treasury Bond Futures

4.1 Delivery Options

- In the US Treasury bond futures contracts, the party with the short position has a number of interesting delivery options:
  i. Delivery can be made on any day during the delivery month.
  ii. There are a number of alternative bonds that can be delivered.
  iii. On any day during the delivery month, the notice of intention to deliver at the 2:00 p.m. settlement price can be made later in the day.

4 Treasury Bond Futures

4.2 Conversion Factors

- The Treasury bond futures contract allows the party with the short position to choose to deliver any bond that has a maturity between 15 and 25 years. When a particular bond is delivered, a parameter known as its **conversion factor defines the price received for the bond by the party with the short position**.
- The applicable quoted price for the bond delivered is the product of the conversion factor and the most recent settlement price for the futures contract.
- Taking accrued interest into account , the cash received for each $100 face value of the bond delivered = *(Most recent settlement price x Conversion factor) + Accrued interest*
- Each contract is for the delivery of $100,000 face value of bonds.
- Suppose that the most recent settlement price is 120-00, the conversion factor for the bond delivered is 1.3800, and the accrued interest on this bond at the time of delivery is $3 per $100 face value.
- The cash received by the party with the short position (and paid by the party with the long position) is then: *(1.3800 x 120.00) + 3.00 = $168.60 per $100 face value*. A party with the short position in one contract would deliver bonds with a face value of $100,000 and receive $168,600.

4 Treasury Bond Futures

4.3 Cheapest-to-Deliver (CTD) bonds

- At any given time during the delivery month, there are many bonds that can be delivered in the Treasury bond futures contract.
- The party with the short position can choose which of the available bonds is ‘‘cheapest’’ to deliver. Because the party with the short position receives:

  (Most recent settlement price x Conversion factor) + Accrued interest

- The cost of purchasing a bond is:

  Quoted bond price + Accrued interest

- The cheapest-to-deliver (CTD) bond is the one for which ***[Quoted bond price – (Most recent settlement price x Conversion factor)]*** is least. Once the party with the short position has decided to deliver, it can determine the cheapest-to-deliver bond by examining each of the deliverable bonds in turn.

4 Treasury Bond Futures

4.4 Example – US Treasury Bonds

An exact theoretical futures price for the US Treasury bond contract is difficult to determine because the short party's options concerned with the timing of delivery and choice of the bond that is delivered cannot easily be valued. However, if we assume that both the cheapest-to-deliver bond and the delivery date are known, the Treasury bond futures contract is a futures contract on a traded security (the bond) that provides the holder with known income. The futures price, F_0, is related to the spot price, S_0, by:

$$\mathbf{F_0 = (S_0 - I) * e^{rT}}$$

where;
I : the present value of the coupons during the life of the futures contract
T is the time until the futures contract matures
r : the risk-free interest rate applicable to a time period of length T.

5 Interest Rate Swaps

5.1 Origin

The birth of the over-the-counter swap market can be traced to a currency swap negotiated between IBM and the World Bank in 1981. The World Bank had borrowings denominated in U.S. dollars while IBM had borrowings denominated in German deutsche marks and Swiss francs. The World Bank (which was restricted in the deutsche mark and Swiss franc borrowing it could do directly) agreed to make interest payments on IBM's borrowings while IBM in return agreed to make interest payments on the World Bank's borrowings. Since that first transaction in 1981, the swap market has seen phenomenal growth.

5 Interest Rate Swaps

An interest rate swap is a forward contract in which one stream of future interest payments is exchanged for another based on a specified principal amount.

- Interest rate swaps usually involve the exchange of a fixed interest rate for a floating rate, or vice versa, to reduce or increase exposure to fluctuations in interest rates or to obtain a marginally lower interest rate than would have been possible without the swap.

- By far the most common over-the-counter derivative is a "plain vanilla" interest rate swap. In this a company agrees to pay cash flows equal to interest at a predetermined fixed rate on a notional principal for a number of years. In return, it receives interest at a floating rate on the same notional principal for the same period of time.

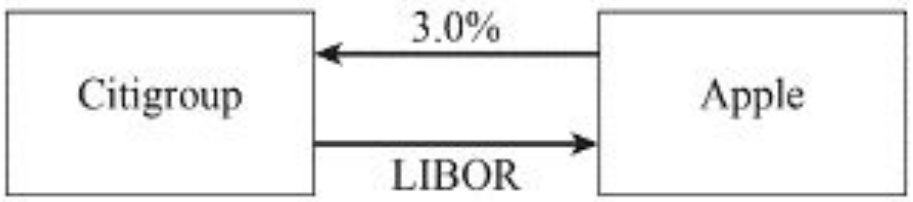
5 Interest Rate Swaps

Illustration

- Consider a hypothetical three-year swap initiated on March 8, 2017, between Apple and Citigroup. We suppose Apple agrees to pay to Citigroup an interest rate of 3% per annum on a notional principal of $100 million, and in return Citigroup agrees to pay Apple the six-month LIBOR rate on the same notional principal. (LIBOR is the rate of interest at which a AA-rated bank can borrow money from other banks.)

- Apple is the fixed- pate payer; Citigroup is the floating-rate payer. We assume the agreement specifies that payments are to be exchanged every six months and that the 3% interest rate is quoted with semi-annual compounding.

- The swap is shown in Figure



5 Interest Rate Swaps

5.2 Types of Interest Rate Swaps

- There are three types of interest-rate swaps:
  - **Fixed-to-Floating** : A company that wants access to a potentially attractive future floating rate will enter into a swap by borrowing at a fixed rate.
  - **Floating-to-Fixed** : A company that does not have access to a fixed-rate loan may borrow at a floating rate and enter into a swap to achieve a fixed rate.
  - **Float-to-Float** : A swap can also involve the exchange of one type of floating rate for another, which is called a **basis swap**. A company can swap from three-month LIBOR to six-month LIBOR, for example, either because the rate is more attractive or it matches other payment flows.

5 Interest Rate Swaps

5.3 Question

Suppose that some time ago a financial institution entered into a swap where it agreed to make semiannual payments at a rate of 3% per annum and receive LIBOR on a notional principal of $100 million. The swap now has a remaining life of 1.25 years. Payments will therefore be made 0.25, 0.75, and 1.25 years from today. The risk-free rates with continuous compounding for maturities of 3 months, 9 months, and 15 months are 2.8%, 3.2%, and 3.4%. We suppose that the forward LIBOR rates for the 3- to 9-month and the 9- to 15-month periods are 3.4% and 3.7%, respectively, with continuous compounding. The 3- to 9-month forward rate becomes $2 \times (e^{0.034 \times 0.5} - 1)$ or 3.429% with semiannual compounding. Similarly, the 9- to 15-month forward rate becomes 3.734% with semiannual compounding. The LIBOR rate applicable to the exchange in 0.25 years was determined 0.25 years ago. Suppose it is 2.9% with semi-annual compounding.
Calculate the PV of the swap.

5 Interest Rate Swaps

5.3 Solution

The calculation of swap cash flows on the assumption that LIBOR rates will equal forward rates and the discounting of the cash flows are shown in the following table (all cash flows are in millions of dollars) :

Time (years)	Fixed cash flow	Floating cash flow	Net cash flow	Discount factor	Present value of net cash flow
0.25	-1.5000	+1.4500	-0.0500	0.9930	-0.0497
0.75	-1.5000	+1.7145	+0.2145	0.9763	+0.2094
1.25	-1.5000	+1.8672	+0.3672	0.9584	+0.3519
Total					0.5117

Consider, for example, the 0.75 year row. The fixed cash flow is (-0.5 x 0.03 x 100), or $1.5000 million. The floating cash flow, assuming forward rates are realized, is (0.5 x 0.03429 x 100), or $1.7145 million. The net cash flow is therefore $0.2145 million. The discount factor is $e^{-0.032*0.75} = 0.9763$, so that the present value is (0.2145 x 0:9763) = 0.2094.

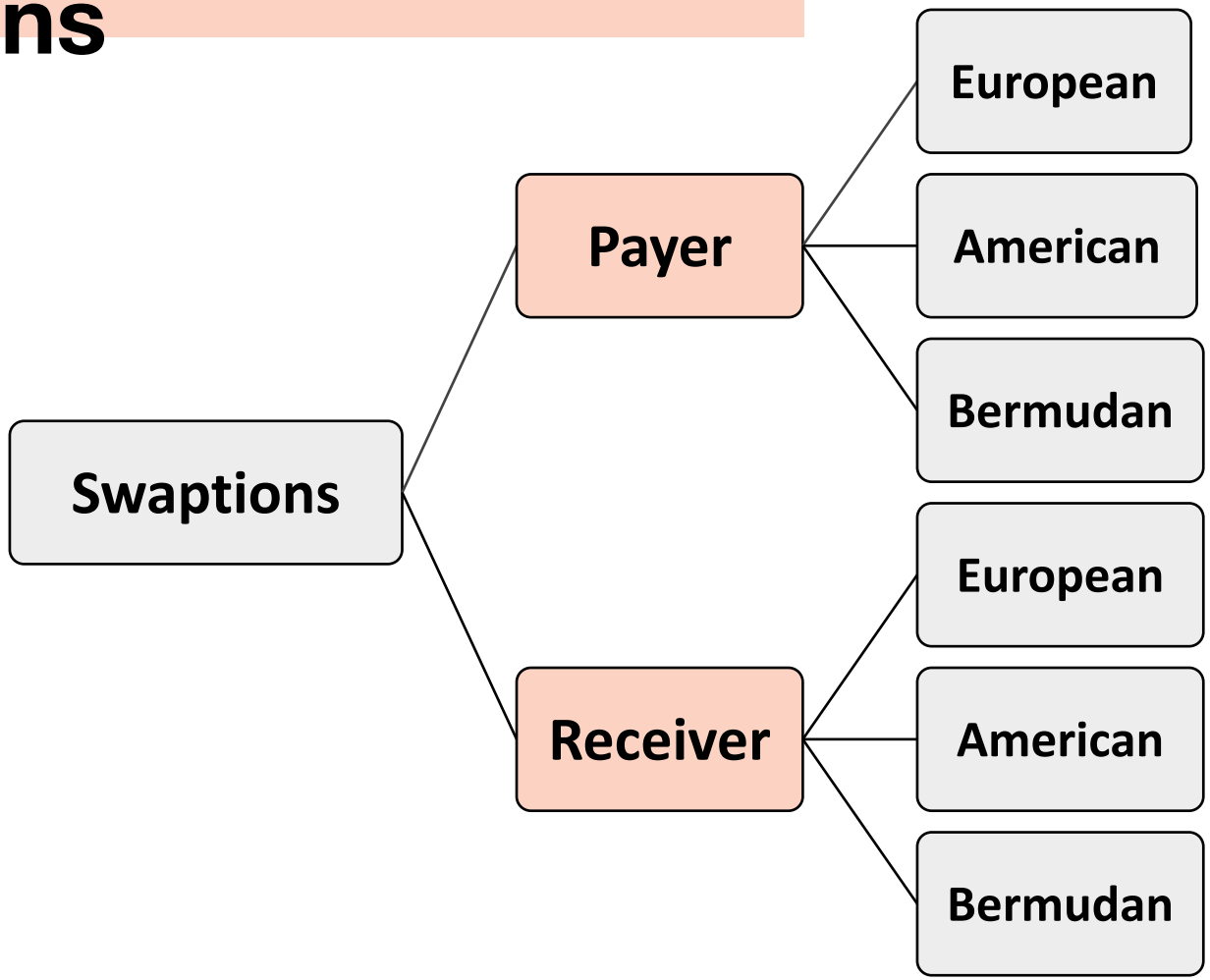
The value of the swap is obtained by summing the present values. It is $0.5117 million.

6 Swap Options (Swaptions)

A swaption, also known as a swap option, refers to an option to enter into an interest rate swap or some other type of swap. In exchange for an options premium, the buyer gains the right but not the obligation to enter into a specified swap agreement with the issuer on a specified future date.

- With a swaption, the company is able to benefit from favorable interest rate movements while acquiring protection from unfavorable interest rate movements.
- The difference between a swaption and a forward swap is analogous to the difference between an option on a foreign currency and a forward contract on the currency.

6.1 Classification of Swaptions



6.1 Classification of Swaptions

- Swaptions can be classified broadly on two basis. The method of execution and the type of option/right associated with it.
- Based on the type of option, swaptions can be of the following types:
  i. **Payer Swaption**: The purchaser has the right but not the obligation to enter into a swap contract where they become the fixed-rate payer and the floating-rate receiver.
  ii. **Receiver Swaption**: The purchaser has the right but not the obligation to enter into a swap contract where they become the floating-rate payer and the fixed-rate receiver.

- Based on the method in which they are executed, swaptions can be of the following types:
  i. **European Swaption**: The purchaser is only allowed to exercise the option and enter into the swap on the expiration date of the swaption.
  ii. **American Swaption**: The purchaser can exercise the option and enter into the swap on any day between the origination of the swap and the expiration date.
  iii. **Bermudan Swaption**: The purchaser is allowed to exercise the option and enter into the specified swap on a predetermined set of specific dates.

6.2 European Swaptions

An European Swaption, refers to an option in which the buyer gains the right but not the obligation to enter into a specified swap agreement with the issuer on the expiration date, in return for a premium.

- Black's model and its extensions provide a popular approach for valuing European-style interest rate options. The essence of Black's model is that the value of the variable underlying the option (interest rate in this case) is assumed to be lognormal at the maturity of the option.
- The swap rate for a particular maturity at a particular time is the (mid-market) fixed rate that would be exchanged for LIBOR in a newly issued swap with that maturity.

6.3 Bermudan Swaptions

A Bermudan Swaption, refers to an option in which the buyer gains the right but not the obligation to enter into a specified swap agreement with the issuer on a predetermined set of specific dates , in return for a premium.

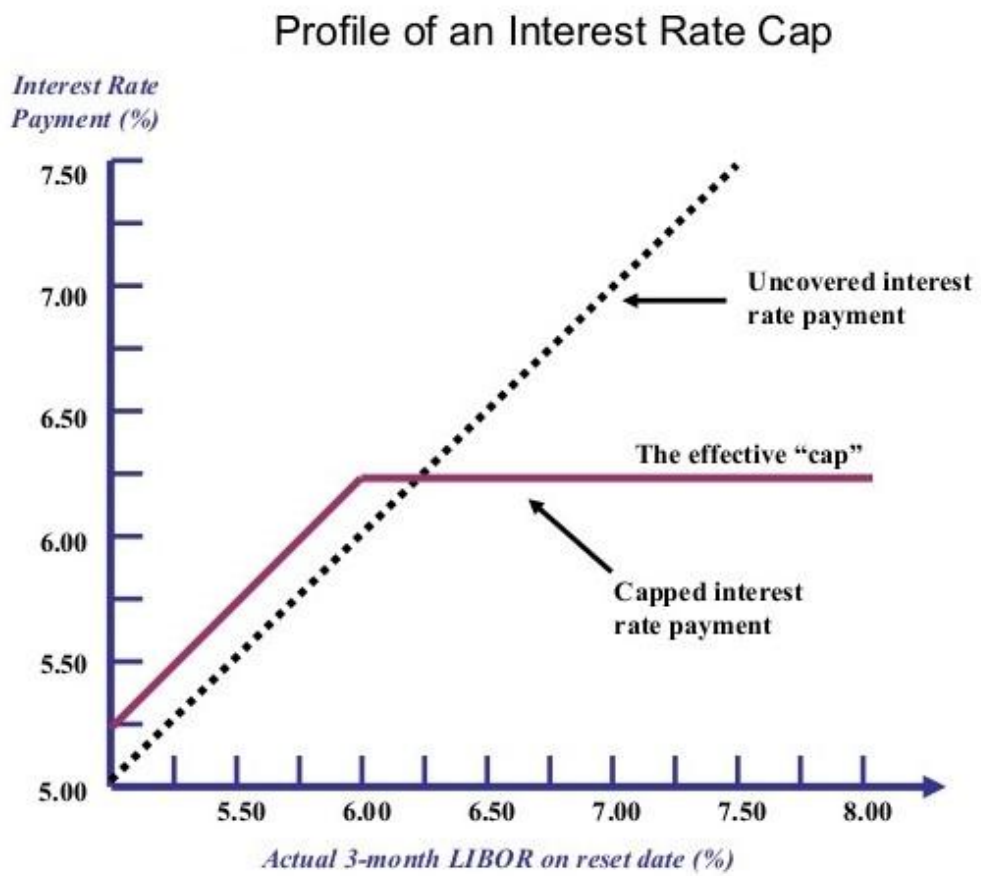
- A Bermudan swaption is more complex to value. We use Black-Derman-Toy or Hull-White models to value Bermudan swaption.
- Example: A Bermudan Swaption has a maturity of 5 years. The holder and issuer agree that the holder can exercise it any one of the first four quarterly dates. This swaption is not as rigid as European swaption and not as flexible as the American swaption.

7 Caps and Caplets

An **interest rate cap/caplet** is a type of IRD in which the buyer receives payments at the end of each period in which the interest rate exceeds the agreed strike price.

- Caps/Caplets are interest rate options designed to "cap" the risk of rising rates.
- These options use an interest rate, rather than a price, as the basis for a strike.
- Caplets are shorter term (90 days) in duration compared to caps which may be a year or longer.
- Caplets are usually based on an interbank interest rate, such as LIBOR. That's because they are typically used for hedging the risk of LIBOR rising.

7 Caps and Caplets

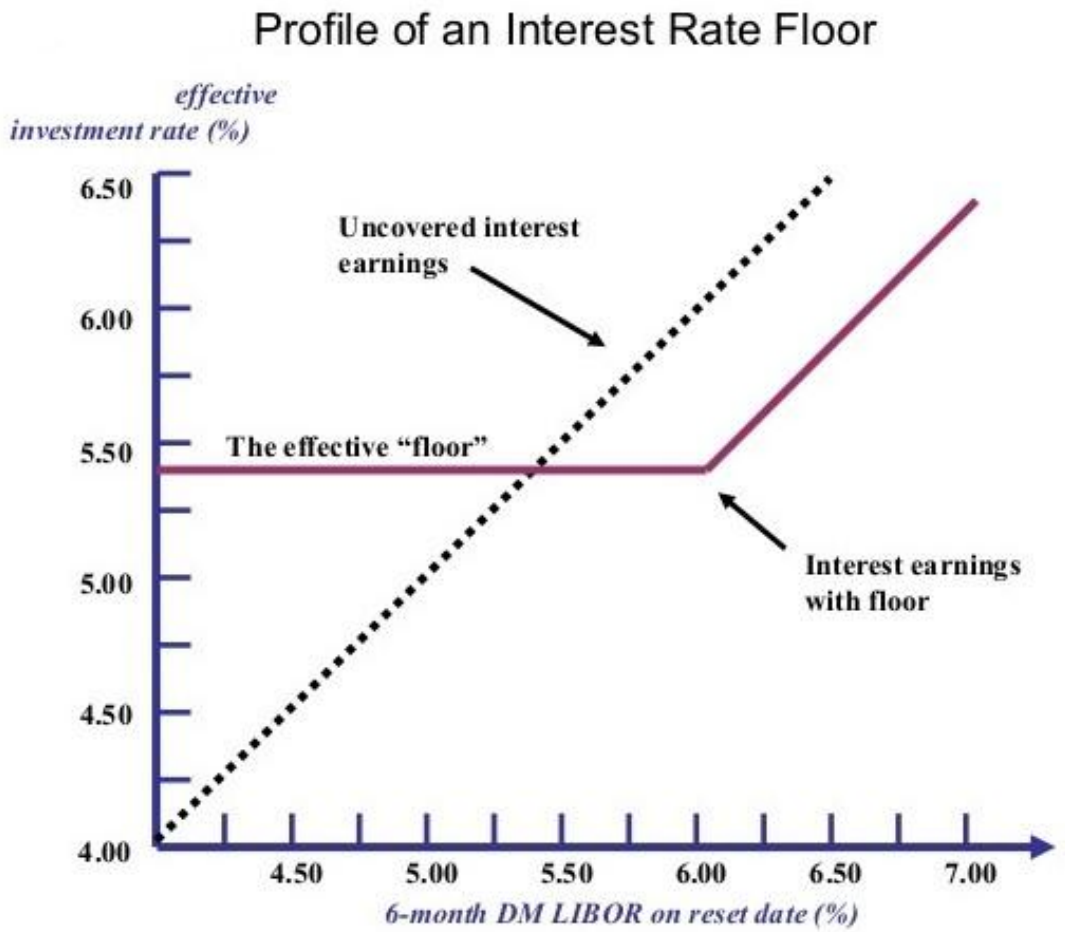


8 Floors and Floorlets

An **interest rate floor/floorlet** is a type of IRD in which the buyer receives payments at the end of each period in which the interest rate is below the agreed strike price.

- Floors/Floorlets are interest rate options designed to prevent the risk of falling rates.
- Floorlets are shorter term (90 days) in duration compared to floors which may be a year or longer.
- In the case of an interest rate floor, the buyer of an interest rate floor contract seeks compensation when the floating rate falls below the contract's floor. This buyer is buying protection from lost interest income paid by the borrower when the floating rate falls.
- Interest rate floors are often used in the adjustable-rate mortgage(ARM) market. Often, this minimum is designed to cover any costs associated with processing and servicing the loan.

8 Floors and Floorlets



9.1 Differences between Eurodollar Futures Contracts & FRA

- The Eurodollar futures contract is similar to a forward rate agreement (FRA) in that it locks in an interest rate for a future period. For short maturities (up to a year or so), the Eurodollar futures interest rate can be assumed to be the same as the corresponding forward interest rate. For longer-dated contracts, differences between the contracts become important.
- Compare a Eurodollar futures contract on an interest rate for the period between times T_1 and T_2 with an FRA for the same period. The Eurodollar futures contract is settled daily. The final settlement is at time T_1 and reflects the realized interest rate for the period between times T_1 and T_2. By contrast the FRA is not settled daily and the final settlement reflecting the realized interest rate between times T_1 and T_2 is made at time T_2.

9.1 Differences between Eurodollar Futures Contracts & FRA

- There are therefore two differences between a Eurodollar futures contract and an FRA. These are:
  i. The difference between a Eurodollar futures contract and a similar contract where there is no daily settlement. The latter is a hypothetical forward contract where a payoff equal to the difference between the forward interest rate and the realized interest rate is paid at time T_1.
  ii. The difference between the hypothetical forward contract where there is settlement at time T_1 and a true forward contract where there is settlement at time T_2 equal to the difference between the forward interest rate and the realized interest rate.

9.2 Differences between IRF & Treasury Bond Futures

- Broadly speaking the two instruments differ by their underlying used to price these contracts. T-Bond futures have Treasury Bonds as underlying while interest rate futures have average daily Fed Funds effective rate / London inter-bank offered rate (LIBOR) as underlying.
- Treasury Bond Futures are available on the 2-year, 5-year, 10-year, and 30-year tenors treasury bonds. Interest rate futures are popular as Eurodollar Futures Contract and based on 1-month and 3-month LIBOR rate (12 months at most).
- The significant thing about T-Bond Futures is that these are deliverable contracts i.e. seller of the futures contract has to deliver bonds that satisfy the certain conditions at the expiry of the contract. A delivery feature of the contract makes the trading in T-Bond Futures exciting. While the Interest rate is not market traded instrument, so Interest-Rate Futures are cash settle instruments.

Quick Recap

- Investors face significant valuation risks due to the uncertainty around interest rates. These can be hedged with Interest Rate Derivatives.
- Interest Rate Derivative (IRD) is a financial derivative contract whose value is derived from one or more interest rates, prices of interest rate instruments, or interest rate indices.
- There are various IRD like Forward Rate Agreements, Interest Rate Futures, Treasury Bond Futures, Interest Rate Swaps, Swaptions, etc. to hedge against interest rate risk.
- A forward rate agreement (FRA) is an over-the-counter (OTC) contract designed to fix the interest rate that will apply to either borrowing or lending a certain principal amount during a specified future time period.
- Interest Rate Futures (IRF) are standardized IRD contracts to buy or sell a notional security or any other interest-bearing instrument or an index of such instruments or interest rates at a specified future date, at a price determined at the time of the contract.
- Eurodollar Futures Contracts and Federal Funds Futures are the most widely traded examples of IRF are usually sub-year contracts.
- Treasury Bond Futures are financial derivatives that obligate the contract holder to purchase or sell a government or treasury bond on a specified date at a predetermined price.
- US Treasury Bond Futures are very widely traded and hence liquid. A conversion factor defines the price received for the bond by the party with the short position.
- The term cheapest to deliver (CTD) refers to the cheapest security delivered in a futures contract to a long position to satisfy the contract specifications.

Continued

- An interest rate swap is a forward contract in which one stream of future interest payments is exchanged for another based on a specified principal amount.
- Interest rate swaps are of three types : fixed-to-floating, floating-to-fixed and floating-to-floating.
- A swaption, also known as a swap option, refers to an option to enter into an interest rate swap or some other type of swap. In exchange for an options premium, the buyer gains the right but not the obligation to enter into a specified swap agreement with the issuer on a specified future date.
- Swaptions can be classified as payer or receiver swaption based on which type of interest rate is paid. They can further be classified as European, American and Bermudan based on when they can be exercised.
- European swaptions can only be exercised on the expiration day and then the swap can be entered into if the buyer of the swaption so wishes. For a Bermudan swaption, there is a range of pre-determined dates when the swap can be entered into.
- An interest rate cap/caplet is a type of IRD in which the buyer receives payments at the end of each period in which the interest rate exceeds the agreed strike price.
- An interest rate floor/floorlet is a type of IRD in which the buyer receives payments at the end of each period in which the interest rate is below the agreed strike price.
- Caplets and Floorlets are generally shorter term(90 days) instruments compared to caps and floors respectively.