INSTITUTE OF ACTUARIAL & QUANTITATIVE STUDIES

Class: TY BSc

Subject : Financial Engineering - 2

Chapter: Unit 1 Chapter 2

Chapter Name: Exotic Options

Topics to be covered

1. Derivatives
1. Exotic Options
1. Differences between Exotic and Vanilla Options
1. Development of Exotic Options
1. Gap Options
1. Forward Start Options
1. Compound Options
1. Chooser Options
1. Barrier Options

Continued…

10. Binary Options
10. Lookback Options
10. Shout Options
10. Asian Options
10. Exchange Options
10. Rainbow Options
10. Basket Options
10. Deferred Premium options

We previously learned

- Investors face significant valuation risks due to the uncertainty around interest rates. These can be hedged with Interest Rate Derivatives.
- Interest Rate Derivative (IRD) is a financial derivative contract whose value is derived from one or more interest rates, prices of interest rate instruments, or interest rate indices.
- There are various IRD like Forward Rate Agreements, Interest Rate Futures, Treasury Bond Futures, Interest Rate Swaps, Swaptions, etc. to hedge against interest rate risk.
- A forward rate agreement (FRA) is an over-the-counter (OTC) contract designed to fix the interest rate that will apply to either borrowing or lending a certain principal amount during a specified future time period.
- Interest Rate Futures (IRF) are standardized IRD contracts to buy or sell a notional security or any other interest-bearing instrument or an index of such instruments or interest rates at a specified future date, at a price determined at the time of the contract.
- Eurodollar Futures Contracts and Federal Funds Futures are the most widely traded examples of IRF are usually sub-year contracts.
- Treasury Bond Futures are financial derivatives that obligate the contract holder to purchase or sell a government or treasury bond on a specified date at a predetermined price.
- US Treasury Bond Futures are very widely traded and hence liquid. A conversion factor defines the price received for the bond by the party with the short position.
- The term cheapest to deliver (CTD) refers to the cheapest security delivered in a futures contract to a long position to satisfy the contract specifications.

Continued

- An interest rate swap is a forward contract in which one stream of future interest payments is exchanged for another based on a specified principal amount.
- Interest rate swaps are of three types : fixed-to-floating, floating-to-fixed and floating-to-floating.
- A swaption, also known as a swap option, refers to an option to enter into an interest rate swap or some other type of swap. In exchange for an options premium, the buyer gains the right but not the obligation to enter into a specified swap agreement with the issuer on a specified future date.
- Swaptions can be classified as payer or receiver swaption based on which type of interest rate is paid. They can further be classified as European, American and Bermudan based on when they can be exercised.
- European swaptions can only be exercised on the expiration day and then the swap can be entered into if the buyer of the swaption so wishes. For a Bermudan swaption, there is a range of pre-determined dates when the swap can be entered into.
- An interest rate cap/caplet is a type of IRD in which the buyer receives payments at the end of each period in which the interest rate exceeds the agreed strike price.
- An interest rate floor/floorlet is a type of IRD in which the buyer receives payments at the end of each period in which the interest rate is below the agreed strike price.
- Caplets and Floorlets are generally shorter term(90 days) instruments compared to caps and floors respectively.

Discussion

Do you know of any non-traditional financial instruments?

1 Derivatives

- In the previous chapter, we learnt about Interest Rate Derivatives(IRD) . They are products which derived value form one or more interest rates. In that we particularly learnt about European and American Swaptions.
- These swaptions were essentially American and European options in which the premium payer could exercise a interest rate swap after crossing the strike price(or rate in this case).
- American and European options are standardized instruments with a very straightforward cashflow structure. But these instruments aren't ideal to use for specialized scenarios where cashflows are unconventional or there is more uncertainty involved than these standardized products are designed to deal with.
- To deal with these requirements, we use non-traditional or exotic options,

2 Exotic Options

Exotic options are options contracts that differ from traditional options in their payment structures, expiration dates, and strike prices.

- Exotic options are hybrids of American and European options and will often fall somewhere in between these other two styles.
- They can be customized to meet the risk tolerance and desired profit of the investor.
- Exotic options have unique underlying conditions that make them a good fit for high-level active portfolio management and situation-specific solutions.

3 Differences between Exotic and Vanilla Options

- An exotic option can vary in terms of how the payoff is determined and when the option can be exercised. These options are generally more complex than plain vanilla call and put options.
- Exotic options usually trade in the over-the-counter (OTC) market. The OTC marketplace is a dealer-broker network, as opposed to a large exchange such as the New York Stock Exchange (NYSE) where vanilla options are traded.
- Exotic options can be used in trading commodities such as lumber, corn, oil, and natural gas as well as equities, bonds, and foreign exchange as against vanilla options which are generally based on a narrower range of assets.

4 Development of Exotic Options

- The emergence of exotic options is driven by the overall evolution of risk management itself. Key factors include:

i. Uncertainty/Volatility in asset markets.
ii. Increased focus on financial risk management.
iii. Demand for highly customized risk-reward profiles.
iv. Development of option pricing and hedging technology.
v. In some cases, exotic options are more cost-effective compared to vanilla options, particularly American options.

5 Gap Options

A **gap option** is an option where there's a **trigger price** that's different from the strike price.

- A gap option has a strike price, K_1 , and a trigger price, K_2 . The trigger price determines whether or not the gap option will have a nonzero payoff. The strike price determines the amount of the nonzero payoff.
- The strike price may be greater than or less than the trigger price. If the strike price is equal to the trigger price, then the gap option is an ordinary option.
- A gap call option has a nonzero payoff (which may be positive or negative) if the final stock price exceeds the trigger price.

Gap Call Option

A gap call option has a payoff of:

$$\text{Gap call option payoff} = \begin{cases} S_T - K_1 & if\ S_T > K_2 \\ 0 & if\ S_T \leq K_2 \end{cases}$$

Gap Put Option

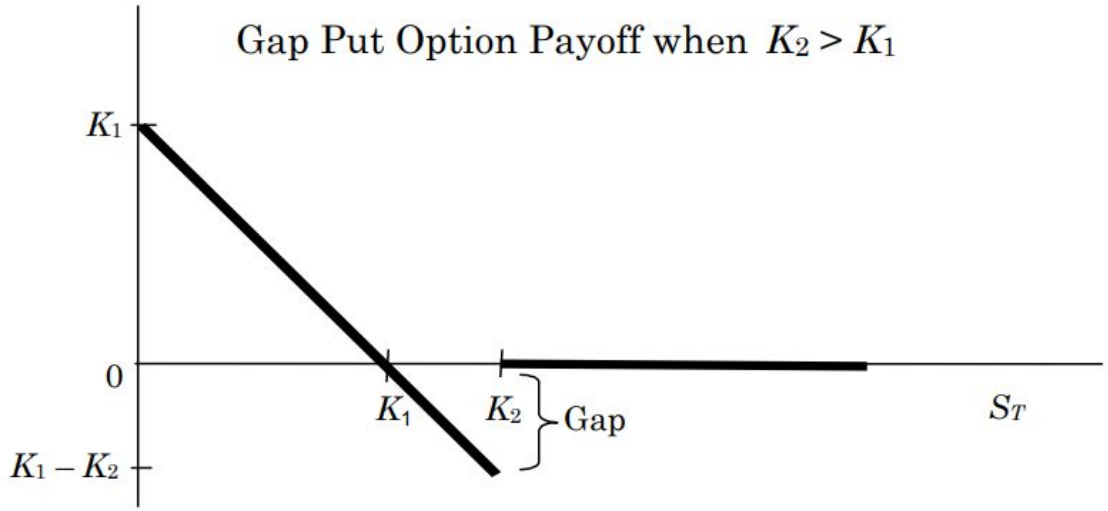
A gap put option has a payoff of:

$$\text{Gap put option payoff} = \begin{cases} K_1 - S_T & if\ S_T < K_2 \\ 0 & if\ S_T \geq K_2 \end{cases}$$

Where K_1 = Strike Price and K_2 = Trigger Price

5 Gap Options with negative payoff

- A rather curious case is possible for a gap put option – a negative payoff.
- If the trigger price is greater than the strike price for a gap put option, then negative payoffs are possible:



- Because negative payoffs are possible, gap options can have negative premiums.
- A gap option must be exercised even if it results in a negative payoff, so perhaps it shouldn't really be called an "option."

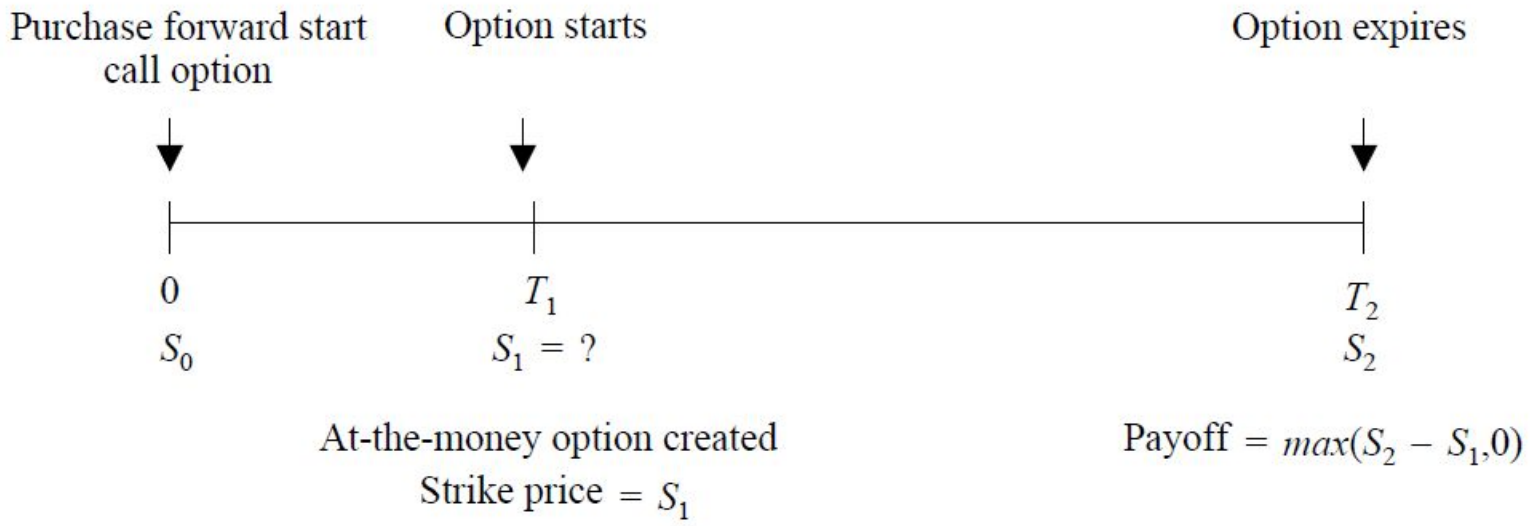
6 Forward Start Options

A **forward start option** is an option that **starts** at some time **in the future.**

- At initiation, a forward start option contract spells out all the defined characteristics relevant to the option, except for its strike price. The expiration date, underlying asset, size, and activation date are set at the time the contract is drawn up.
- In terms of pricing the contract, the future price of the underlying asset is also unknown. The contract typically stipulates some parameters for where the strike price will be in relation to the underlying asset's price.
- Forward start options typically attempt to keep future strike prices at the money or near the money. In this way, the holder will have the right, but not the obligation, to buy (call) or sell (put) the underlying asset in the future at or near the then-current market price.
- If, at the expiration date, the underlying trades below the strike price of the option (for a call), then it expires worthless. If the underlying is above the strike (for a call), then the holder exercises it and owns the underlying at the strike price.
- Once a forward start option becomes active (strike price is set), then the option is valued like a vanilla option.
- A group of consecutive forward start options is called a ratchet option or cliquet option, where the first forward start option is active immediately.

6 Forward Start Options

Example



- The option is **paid** for (in full) at **time** 0.
- The option shown above is an **at-the-money** (ATM) option, with the strike price determined at time T_1, when the **underlying price** S_1 can be established.

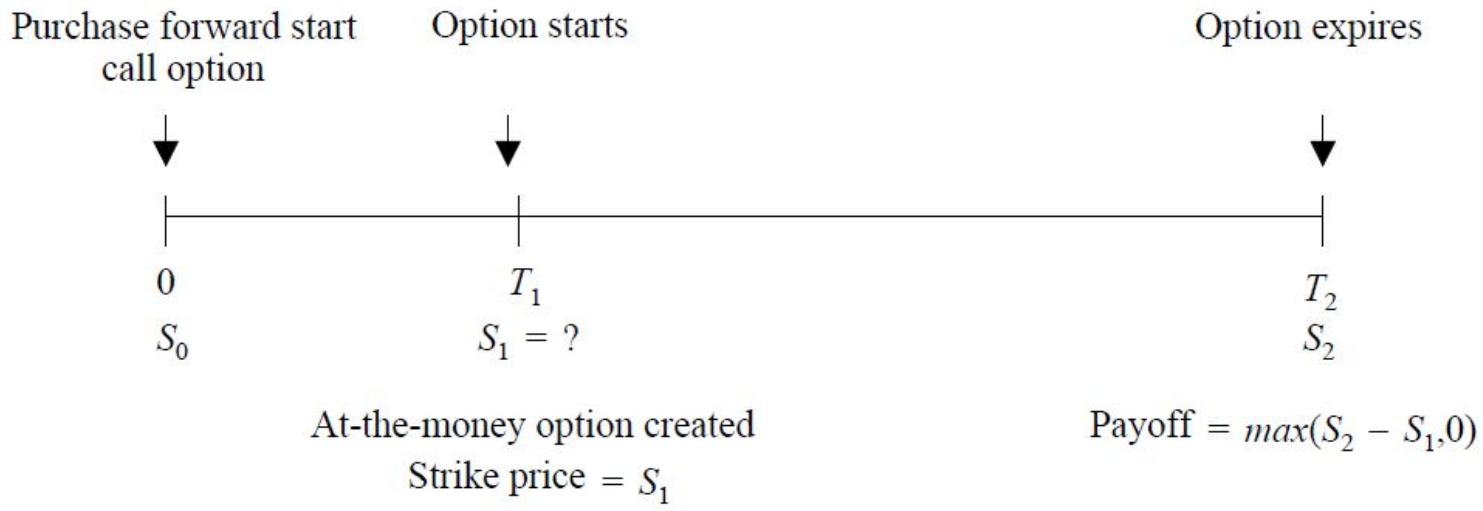
7 Compound Options

A **compound option** is an **option** on an **option.**

- There are four main types of compound options: a **call on a call**, a **put on a call**, a **call on a put**, and a **put on a put**.
- Compound options have two strike prices and two exercise dates.
- The underlying is the second option, while the initial option is called the overlying.
- When the holder exercises a compound call option, called the overlying option, they must then pay the seller of the underlying option a premium based on the strike price of the compound option. This premium is called the back fee.

7 Compound Options

Example



Consider a call on a call option. On the first exercise date, T_1, the holder of the compound option is entitled to pay the first strike price, S_1, and receive a call option. The call option gives the holder the right to buy the underlying asset for the second strike price, S_2, on the second exercise date, T_2. The compound option will be exercised on the first exercise date only if the value of the option on that date is greater than the first strike price.

8 Chooser Options

A **chooser option** (or **"as-you-like-it"** option) is an option where the holder has the right to **choose** whether it is a **call** or a **put** option at some point during it's life.

- Due to its greater flexibility, a chooser option will be more expensive than a comparable vanilla option.
- Chooser options are typically European style, and have one strike price and one expiration date regardless of whether the option is exercised as a call or put.
- Chooser options tend to be more expensive than European vanilla options, and high implied volatility will increase the premium paid for the chooser option.

8 Chooser Options

Put-Call Parity and Pricing

- Suppose that the value of a chooser option at time, T_1 is $\max(c, p)$ where c is the value of the call underlying the option and p is the value of the put underlying the option.
- If the options underlying the chooser option are both European and have the same strike price, put–call parity can be used to provide a valuation formula. Suppose that S_1 is the asset price at time T_1, K is the strike price, T_2 is the maturity of the options, q is the constant dividend yield and r is the risk-free interest rate. Put–call parity implies that:

$$\begin{aligned}\max(c, p) &= \max(c, c + Ke^{-r(T_2-T_1)} - S_1 e^{-q(T_2-T_1)}) \\ &= c + e^{-q(T_2-T_1)}\max(0, Ke^{-(r-q)(T_2-T_1)} - S_1\end{aligned}$$

- This shows that the chooser option is a package consisting of:
  i. A call option with strike price K and maturity T_2
  ii. $e^{-q(T_2-T_1)}$ put options with strike price $Ke^{-(r-q)(T_2-T_1)}$ and maturity T_1
- In this manner, a chooser European option can easily be priced

9 Barrier Options

A **barrier option** is an option where the payoff depends on whether the price of the underlying asset reaches a certain level (**barrier**) during a certain period of time.

- Barrier options are also considered a type of path-dependent option because their value fluctuates as the underlying's value changes during the option's contract term.
- Barrier options offer cheaper premiums as compared to standard options and are also used to hedge positions.

9 Barrier Options

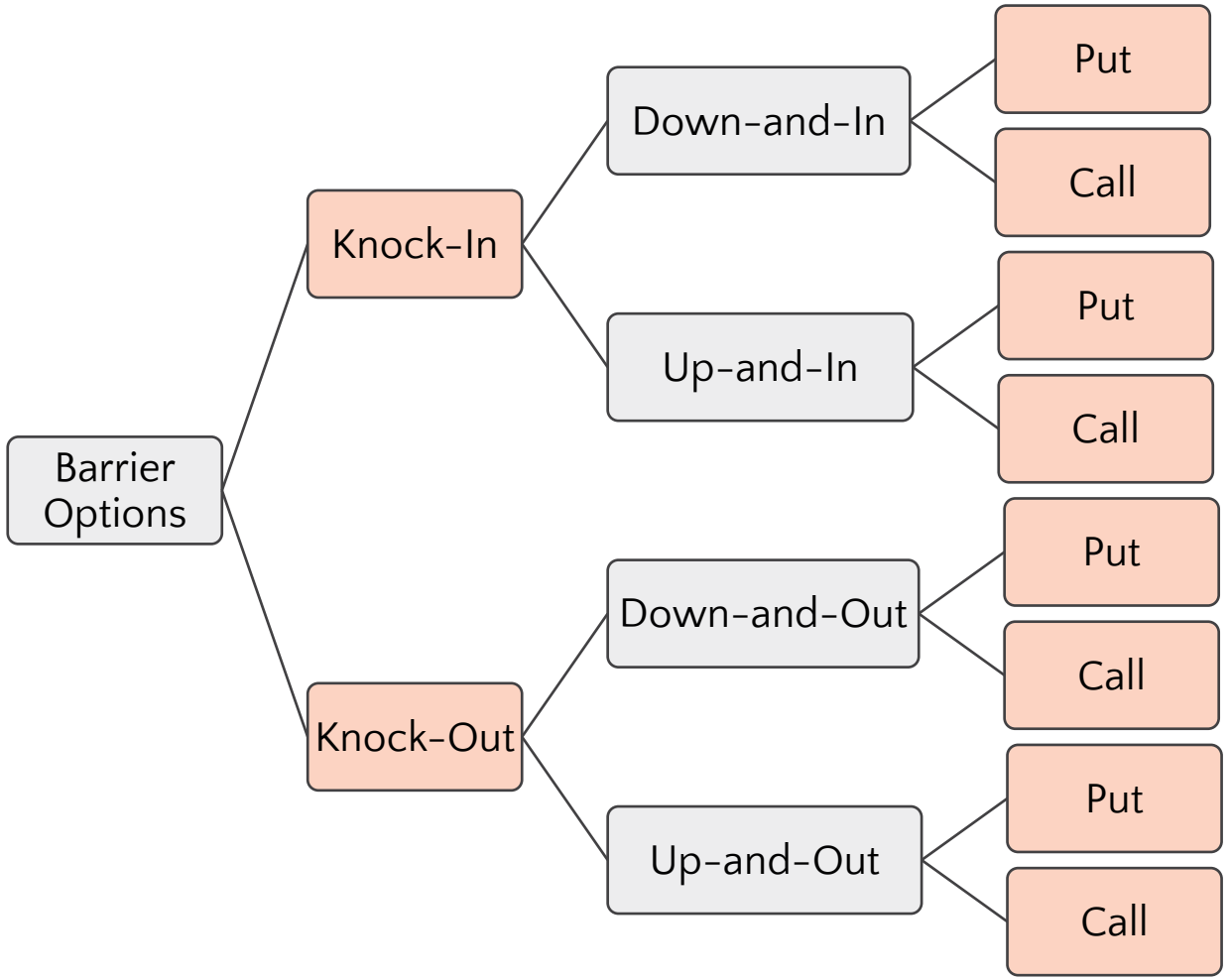
Classification

- A Knock-In option is a type of barrier option where the right to exercise comes into existence (knocks in) when the barrier is hit. They can be:
  i. Down-and-In : Option is knocked-in when the underlying price goes below the barrier.
  ii. Up-and-In : Option is knocked-in when the underlying price goes above the barrier.

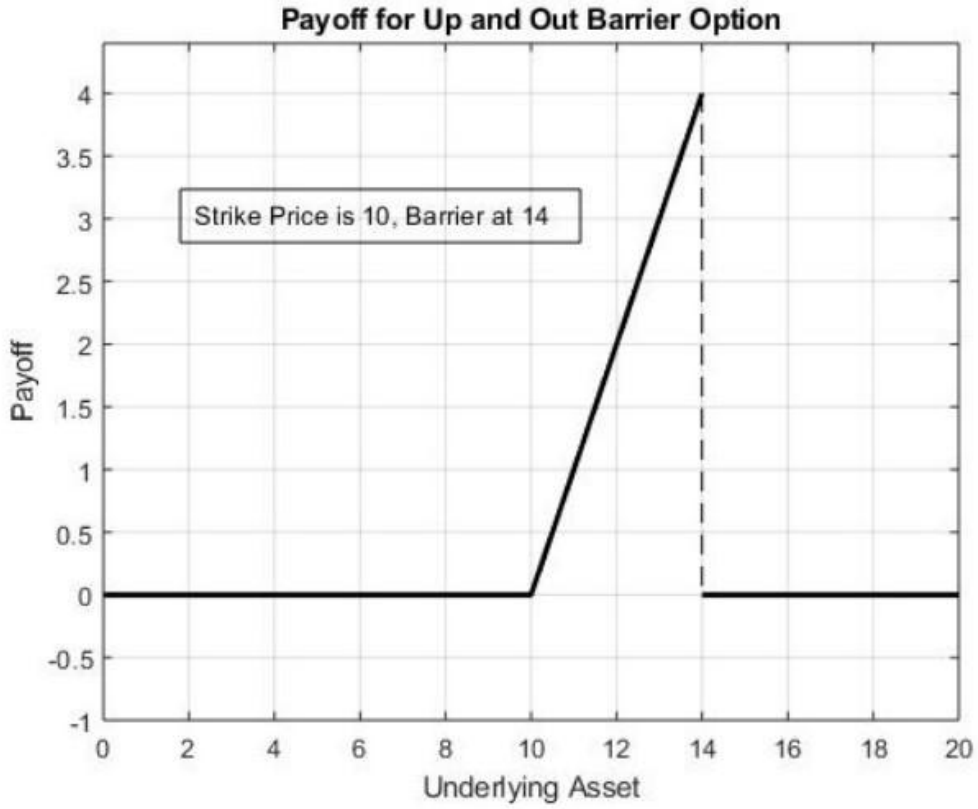
- A Knock-Out option is a type of barrier option where the right to exercise ceases to exist (knocks out) when the barrier is hit. They can be:
  i. Down-and-Out : Option is knocked-out when the underlying price goes below the barrier.
  ii. Up-and-Out : Option is knocked-out when the underlying price goes above the barrier.

- Note: They can further be classified as Put and Call Options

9 Barrier Options



9 Barrier Options



Up and Out Barrier Call Option - Payoff at expiry

10 Binary Options

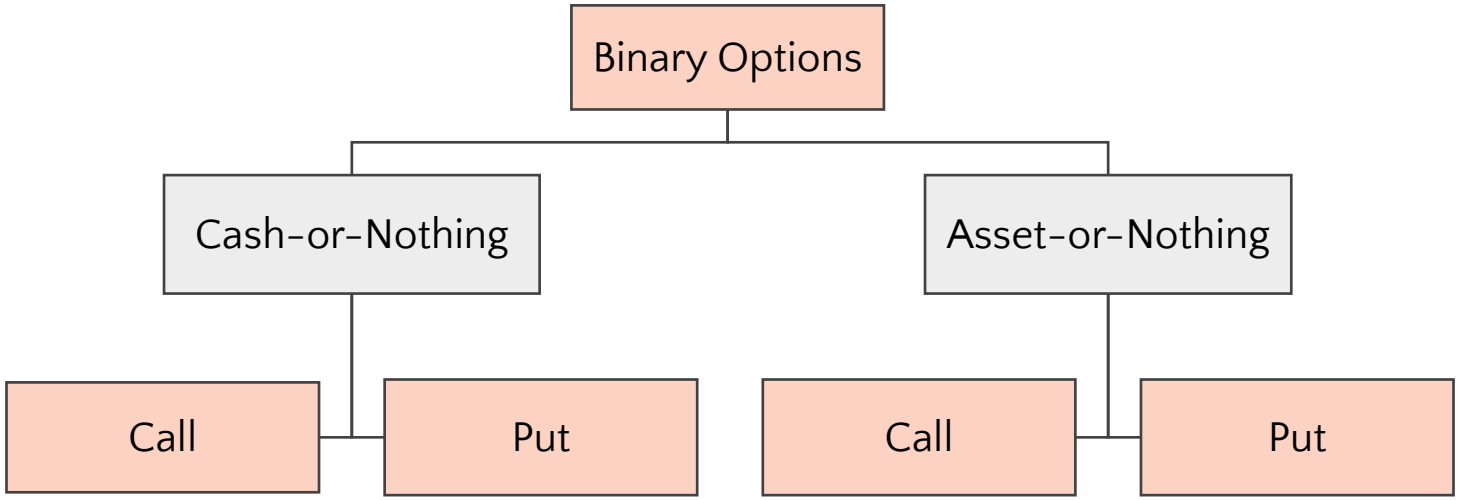
A **binary** (or **digital**) **option** is an option where the payoff is **“all-or-nothing”** (and so they have a discontinuous payoff)

- Binary options depend on the outcome of a "yes or no" proposition.
- Traders receive a payout if the binary option expires in the money and incur a loss if it expires out of the money.
- Binary options set a fixed payout and loss amount.
- A binary option automatically exercises, meaning the gain or loss on the trade is automatically credited or debited to the trader's account when the option expires. That means the buyer of a binary option will either receive a payout or lose their entire investment in the trade—there is nothing in between.
- Binary options have discontinuous payoffs. This can create problems if the underlying asset is thinly traded so that relatively small buy or sell trades move the price.
- Most binary options trading occurs outside the United States.

10 Binary Options

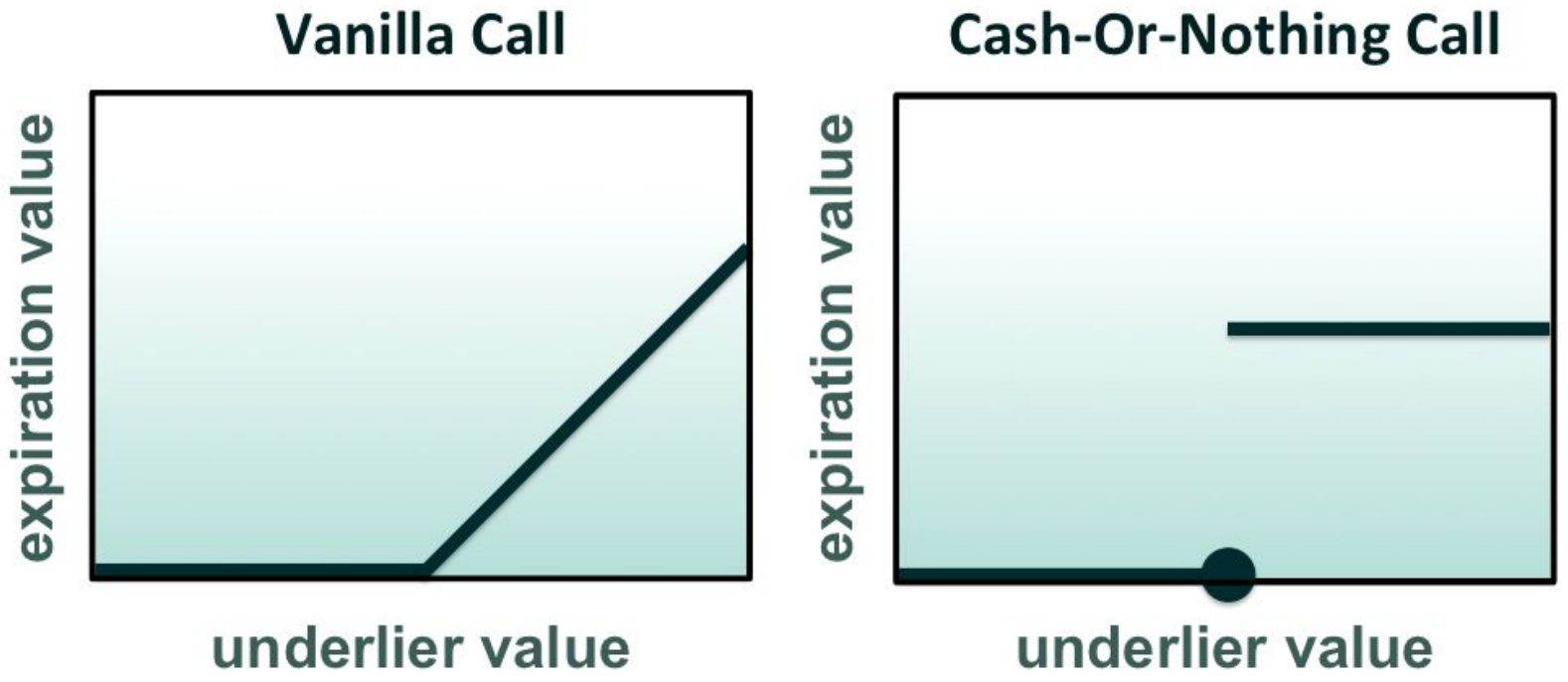
Classification

- A **Cash-or-Nothing call/put** option pays a **fixed amount** (Q) if the price of the underlying asset at expiry time (T) is **above/below** the strike price (K), and **nothing** otherwise.
- An **Asset-or-Nothing call/put** option pays an amount equal to the price of the underlying asset at expiry time (T) if this is **above/below** the strike price (K), and **nothing** otherwise.

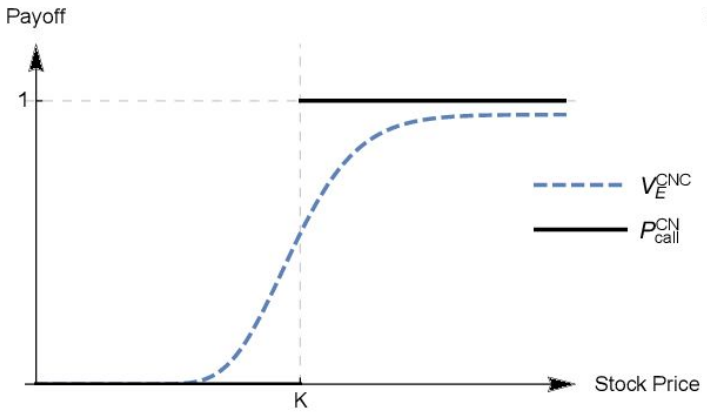


10 Binary Options

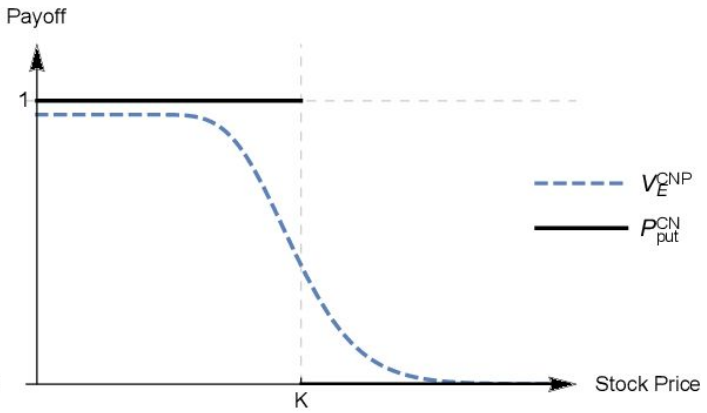
Comparing a Vanilla call option payoff with a Cash-or-Nothing call option payoff:



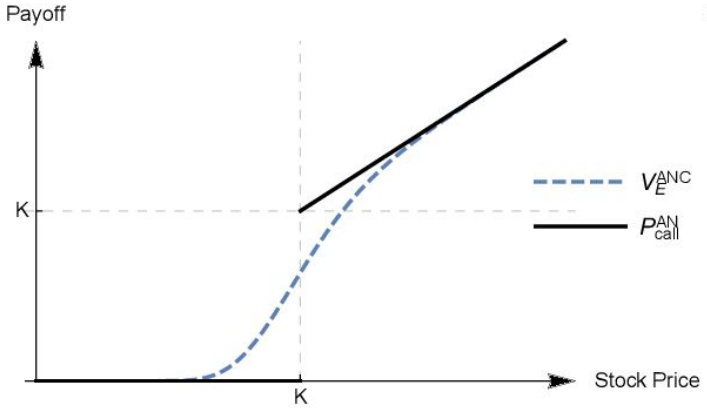
10 Binary Options



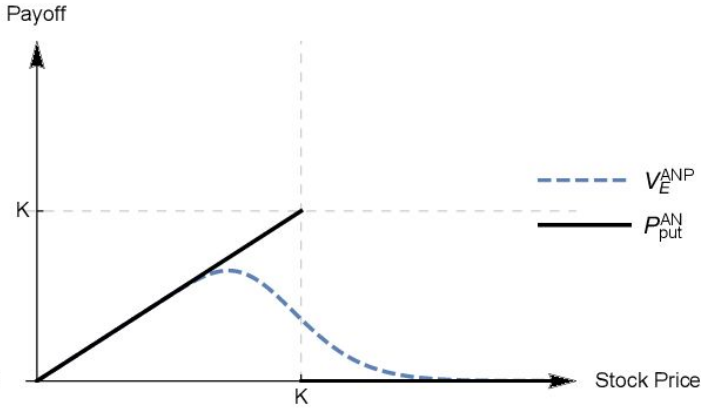
Cash-or-Nothing Call (CNC)



Cash-or-Nothing Put (CNP)



Asset-or-Nothing Call (ANC)



Asset-or-Nothing Put (ANP)

11 Lookback Options

A **lookback** (or **hindsight**) **option** allows the holder the advantage of **knowing history** when determining when to exercise their option.

- Lookback options are exotic options that allow a buyer to minimize regret.
- The payoffs from lookback options depend on the maximum or minimum asset price reached during the life of the option.
- As a type of exotic option, the lookback allows the user to "look back," or review, the prices of an underlying asset over the lifespan of the option after it has been purchased. The holder may then exercise the option based on the most beneficial price of the underlying asset.
- Due to this very convenient customization, lookback options are traded OTC and are cash-settled.
- The ability to maximize profits for the option-holder also means that these options are generally very expensive and potential profits are often nullified by the costs.

11 Lookback Options

Classification

- **Fixed-Strike**: The strike price is set or **fixed at purchase**, similar to most other types of option trades. Unlike other options, however, at the time of exercise, the most beneficial price of the underlying asset over the life of the contract is used instead of the current market price.

- **Floating-Strike**: The strike price is set automatically **at maturity** to the most favorable underlying price reached during the contract's life. The option will then settle against the market price calculating the profit or loss against the floating strike.

- These can be of the call and put variety and the strikes are fixed accordingly.

11 Lookback Options

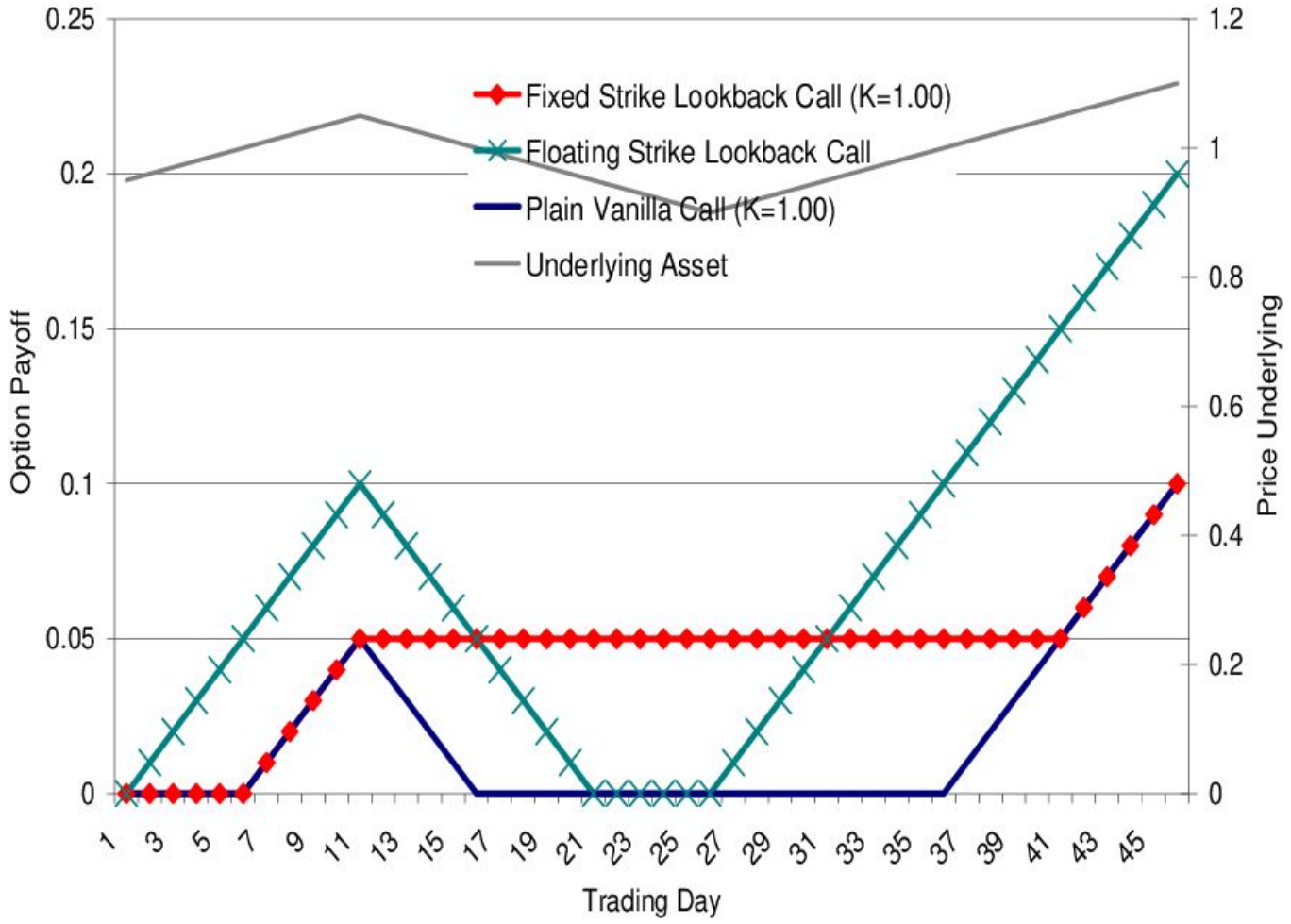
Example

- Assume a stock trades at $50 at both the start and end of the three-month option contract, so there is no net change, gain or loss. The path of the stock will be the same for both the fixed and floating strike call versions. At one point during the life of the option, the highest price is $70, and the lowest price is $40.

- For a fixed strike lookback call option, the strike price is $50. The best price during the lifespan is $70. At strike, the stock is $50. The profit for the call holder is $70 - 50 = $20.

- For a floating strike lookback call option, the lowest price during the lifespan is $40, which becomes the strike price. At maturity, the stock is $50. The holder's profit is $50 - 40 = $10.

11 Lookback Options



12 Shout Options

A **shout option** is an European option where the holder can "shout" at various points of the duration of the option. At expiry, the holder receives the greater of the payoff from the **European option** and the **intrinsic values** at shouts.

- A shout option allows the holder to lock in intrinsic value at pre-determined intervals while maintaining the right to continue participating in gains without a loss of locked-in monies.
- The shout guarantees a minimum of profit, even if the intrinsic value decreases after the shout. If the option increases in value after the shout, the option buyer can still participate in that.
- After each shout, the profit & the underlying price moves higher for calls options.
- The profit moves higher while the price of the underlying will have to decrease further for another "shout" for put options.

12 Shout Options

Example

- Continuing on the example used for the lookback option:
- Assume a stock trades at $50 at both the start of the three-month shout call contract. Only one shout at a pre-determined time is allowed. At that time, the price is $70, so the shout is exercised.
- If the final asset price is less than $70, the holder receives a payoff of $20. If it is greater than $70, the holder receives the excess of the asset price over $50.

- Note: Because of the flexibility and chance for greater profits associated with, lookback options, they are generally more expensive than shout options of similar types.

13 Asian Options

An **asian** (or **average**) **option** is an option where the **payoff** depends on the **average price** of the underlying during at least some part of the life of the option.

- The average price is a geometric or arithmetic average of the price of the underlying asset at discreet intervals, which are also specified in the options contract.
- In general (but not always), Asian options are less expensive than their standard counterparts, as the volatility of the average price is less than the volatility of the spot price.

- Typical uses include:
  - When a business is concerned about the average exchange rate over time.
  - When a single price at a point in time might be subject to manipulation.
  - When the market for the underlying asset is highly volatile.
  - When pricing becomes inefficient due to thinly traded markets (low liquidity markets).

13 Asian Options

Classification

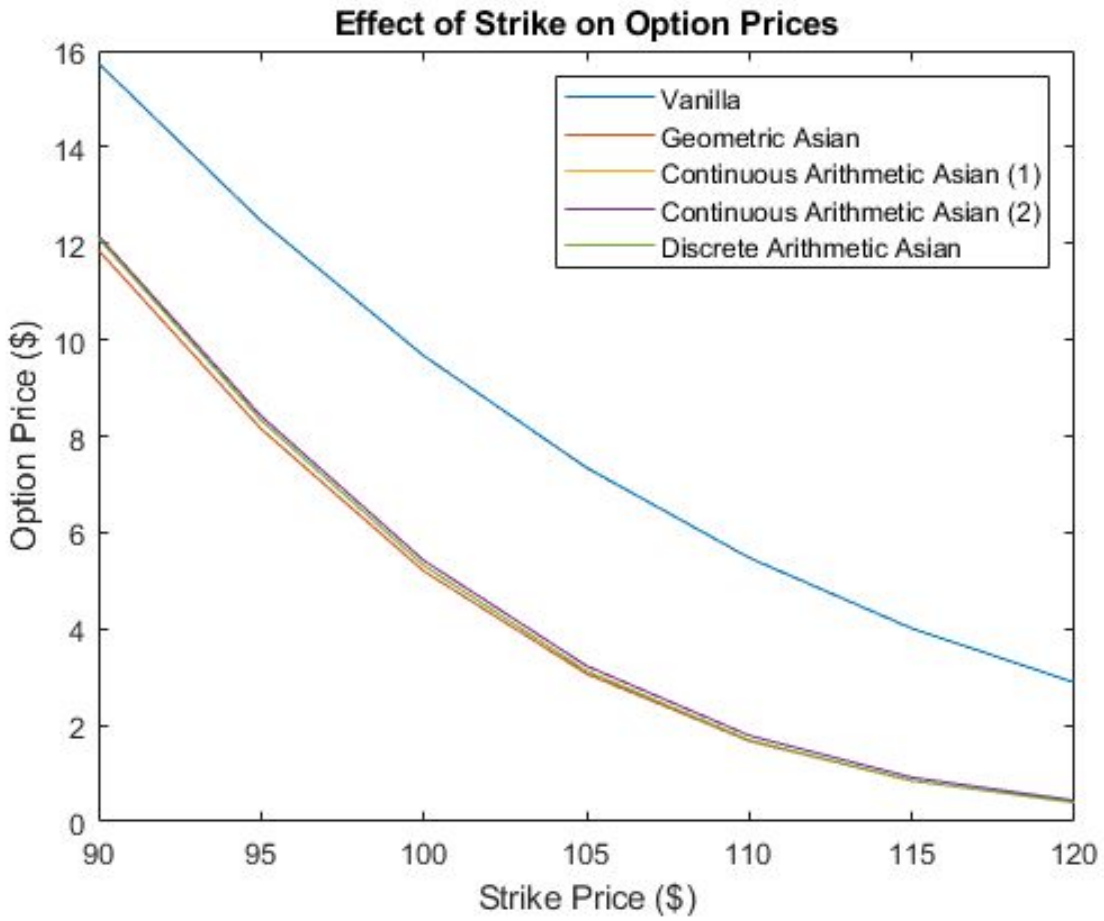
- **Average-Price**: The strike price is set or **fixed at purchase**, similar to most other types of option trades but the payoff depends on the average price of underlying. The payoff functions are:

$$Payoff_{call} = \max\left(S_{average} - K, 0\right)$$
$$Payoff_{put} = \max\left(K - S_{average}, 0\right)$$

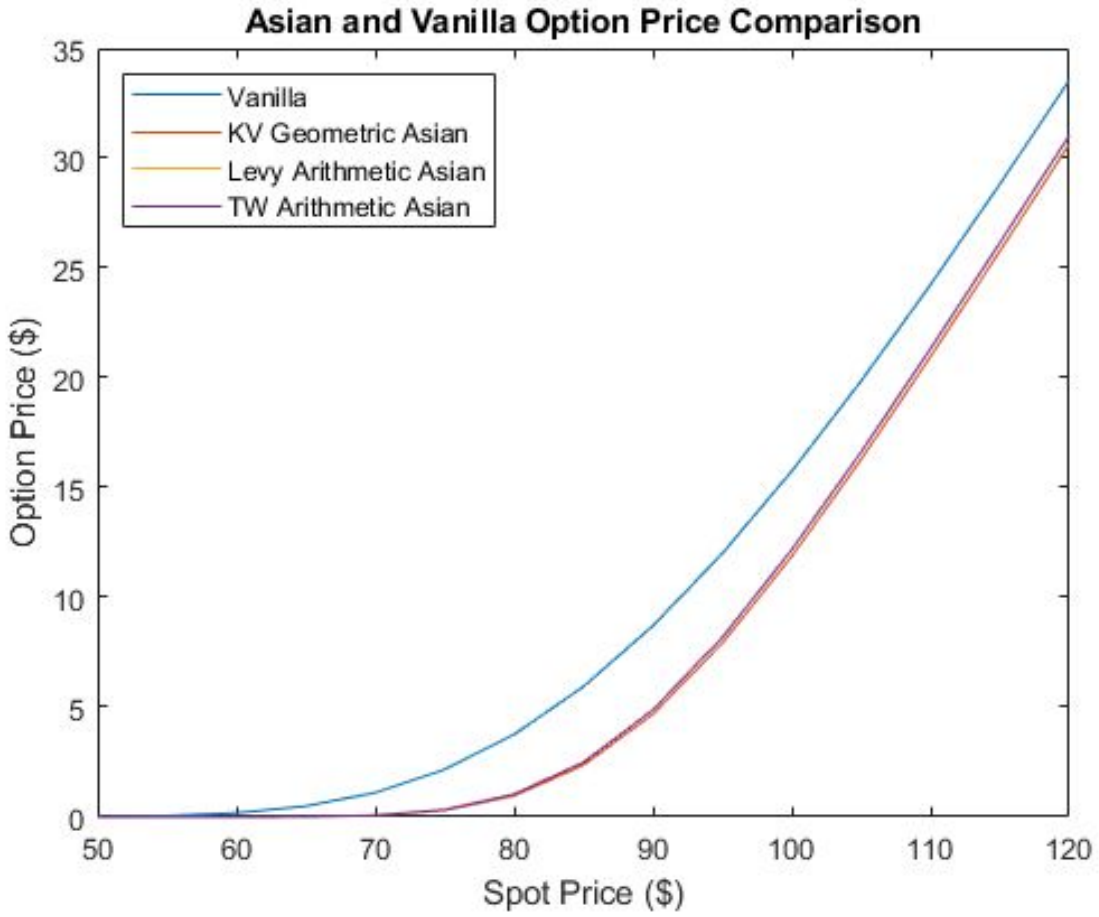
- **Average-Strike**: The strike price is an average of prices around the strike price in this option. It is very similar to an exchange-option which we'll learn further on. The payoff functions are:

$$Payoff_{call} = \max\left(S_T - S_{average}, 0\right)$$
$$Payoff_{put} = \max\left(S_{average} - S_T, 0\right)$$

13 Asian Options



13 Asian Options



14 Exchange Options

An **exchange option** is an option to exchange one asset for another.

- Options to exchange one asset for another arise in various contexts. Some examples are:
  - A stock tender offer is an option to exchange shares in one stock for shares in another stock.
  - An option to buy yen with Australian dollars is, from the point of view of a U.S. investor, an option to exchange one foreign currency asset for another foreign currency asset.

- Consider a European option to give up an asset worth U_T at time T and receive in return an asset worth V_T .
  The payoff from the option is : $\max(V_T - U_T, 0)$

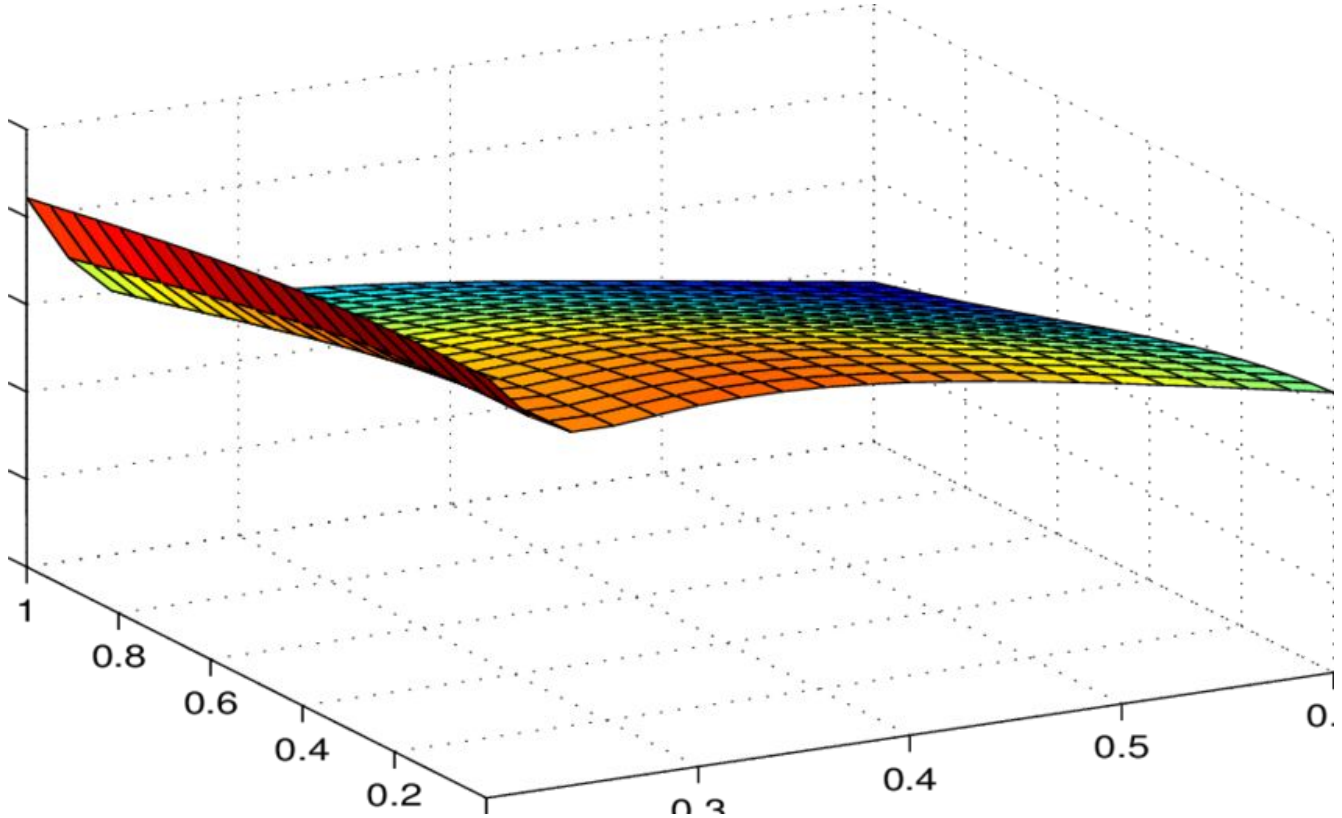
15 Rainbow Options

A **rainbow option** is an option where the payoff is dependent on **two or more** underlying variables.

- Rainbow options are structured as calls and/or puts on the best or worst performer as it relates to the underlying assets involved in a group of assets.
- Some pay off based on the best or worst performance among the covered underlying assets. These are sometimes called "best of" or "worst of" rainbow options.
- A common type of rainbow option is an option that allows the party with the short position to **choose** between several **different assets** to deliver.
- **Bond futures** are an example of rainbow option as the short party can deliver *any* "long" government bond (generally the **CTD** (cheapest-to-deliver) are delivered).

15 Rainbow Options

- The spectrum of price for a call on the minimum of three assets:



16 Basket Options

A **basket option** is an option where the payoff is based on the value of a **basket** (ie portfolio) of assets.

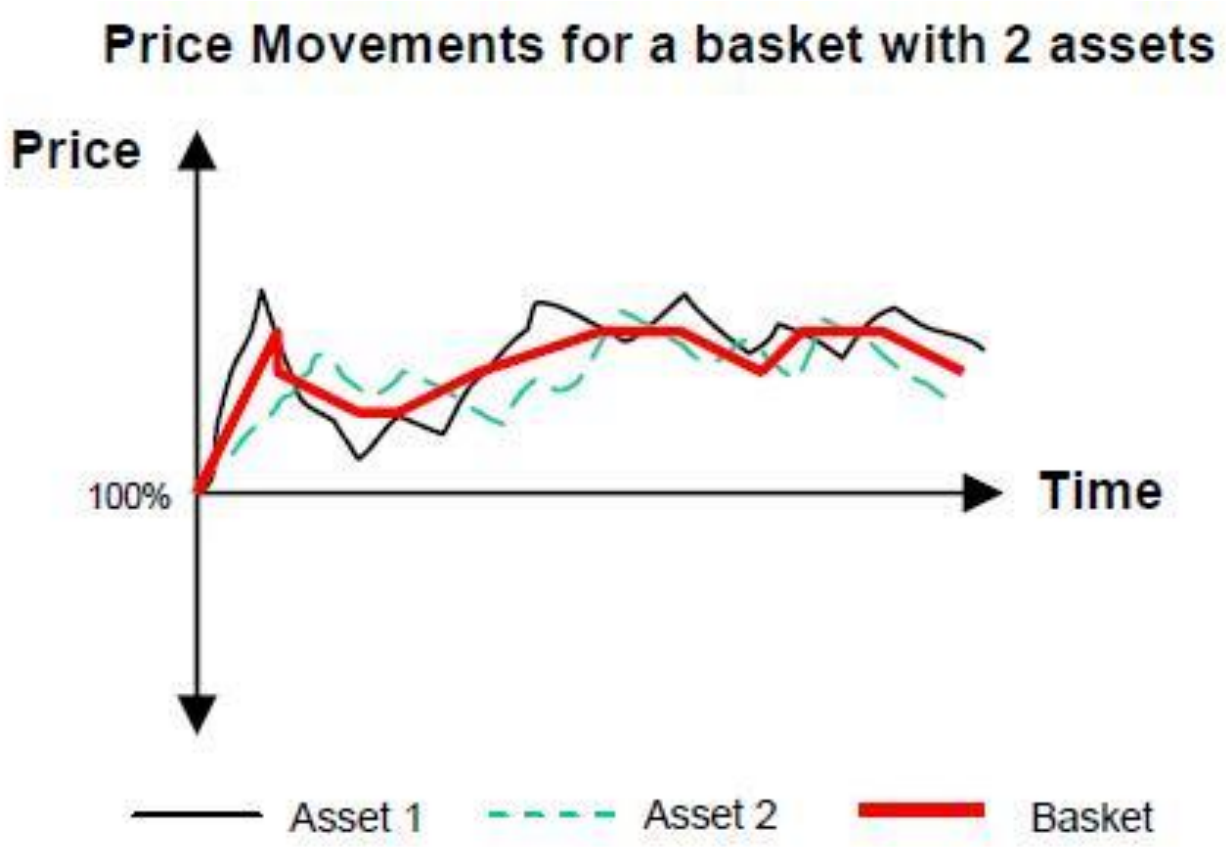
- Basket option has all the characteristics of a standard option, but with strike prices based on the weighted values of its components.
- The underlying asset can be a group, or basket, of commodities, securities, or currencies.
- Because it involves just one transaction, a basket option often costs less than multiple single options as it saves on commissions and fees.
- The most important feature of a basket option is its ability to efficiently hedge risk on multiple assets at the same time.
- Since each basket is unique, these options involve two counterparties and trade over-the-counter. This type of trading limits liquidity, and there is not a guaranteed way to close the options trade ahead of expiration.

16 Basket Options

Valuation

- A European basket option can be valued with Monte Carlo simulation, by assuming that the assets follow correlated geometric Brownian motion processes.
- A much faster approach is to calculate the first two moments of the basket at the maturity of the option in a risk-neutral world, and then assume that value of the basket is lognormally distributed at that time. The option can then be valued using Black's model.

16 Basket Options



17 Deferred Premium Options

A **Boston option**, also known as a **deferred premium option**, is a type of American option. It is identical to a standard call or put option, except that the premium isn't paid until the expiration of the option. The seller of a deferred premium option is simply financing the cost of the option to the buyer of the option.

These forms of options tend to be an exotic option because of how they are structured. What makes this different than standard options is that a Boston option is usually created through a personalized contract instead of an open obligation from standardized market regulations.

Quick Recap

- Exotic options are options contracts that differ from traditional options in their payment structures, expiration dates, and strike prices.
- Exotic options usually trade in the over-the-counter (OTC) market.
- Exotic options have unique underlying conditions that make them a good fit for high-level active portfolio management and situation-specific solutions.
- A **gap option** is an option where there's a **trigger price** that's different from the strike price.

- A **forward start option** is an option that **starts** at some time **in the future.**

- A **compound option** is an **option** on an **option.**

- A **chooser option** (or **"as-you-like-it"** option) is an option where the holder has the right to **choose** whether it is a **call** or a **put** option at some point during it's life.

- A **barrier option** is an option where the payoff depends on whether the price of the underlying asset reaches a certain level (**barrier**) during a certain period of time.

- A **binary** (or **digital**) **option** is an option where the payoff is **"all-or-nothing"** (and so they have a discontinuous payoff)

Continued

- A **lookback** (or **hindsight**) **option** allows the holder the advantage of **knowing history** when determining when to exercise their option.
- A **shout option** is an European option where the holder can "shout" at various points of the duration of the option. At expiry, the holder receives the greater of the payoff from the **European option** and the **intrinsic values** at shouts.
- An **asian** (or **average**) **option** is an option where the **payoff** depends on the **average price** of the underlying during at least some part of the life of the option.
- An **exchange option** is an option to exchange one asset for another.
- A **rainbow option** is an option where the payoff is dependent on **two or more** underlying variables.
- A **basket option** is an option where the payoff is based on the value of a **basket** (ie portfolio) of assets.