Lecture

INSTITUTE OF ACTUARIAL & QUANTITATIVE STUDIES

Class: TY BSc

Subject : Financial Engineering - 2

Subject Code: PUSASQF506B

Chapter: Unit 2 Chapter 2

Chapter Name: Interest rate derivatives and hedging - 2

1 Introduction

- For the derivatives, we need to calculate Greeks for Risk Management and hedging purposes.
- However, because of the term structure of interest rates, there is no longer a single underlying price or volatility parameter to differentiate with respect to.
- So the definitions of the Greeks are no longer clear cut

2 Delta

- Delta could be defined as the price change corresponding to a One basis point (=0.01%) parallel shift in the yield curve (i.e. a uniform increase across all term).

- However, practitioners prefer to define it as a set of partial derivatives with respect to the quoted price of each of the financial instruments from which the yield curve is derived, which they consider to be more fundamental as measures of exposure.

2 Delta

- Delta risk is the risk associated with a shift in the zero curve. Because there are many ways in which the zero curve can shift, many deltas can be calculated. Some alternatives are:

i. Calculate the impact of a 1-basis-point parallel shift in the zero curve. This is sometimes termed a DV01.
ii. Calculate the impact of small changes in the quotes for each of the instruments used to construct the zero curve.
iii. Divide the zero curve (or the forward curve) into a number of sections (or buckets). Calculate the impact of shifting the rates in one bucket by 1 basis point, keeping the rest of the initial term structure unchanged.
iv. Carry out a principal components analysis as outlined in the upcoming example. Calculate a delta with respect to the changes in each of the first few factors. The first delta then measures the impact of a small, approximately parallel, shift in the zero curve; the second delta measures the impact of a small twist in the zero curve; and so on.

2 Delta

- In practice, traders tend to prefer the second approach. They argue that the only way the zero curve can change is if the quote for one of the instruments used to compute the zero curve changes. They therefore feel that it makes sense to focus on the exposures arising from changes in the prices of these instruments.

2 Delta

ALM Delta Hedging

- The asset–liability management (ALM) committees of banks now monitor their exposure to interest rates very carefully. Matching the durations of assets and liabilities is sometimes a first step, but this does not protect a bank against nonparallel shifts in the yield curve.
- A popular approach is known as GAP management. This involves dividing the zero-coupon yield curve into segments, known as buckets. The first bucket might be 0 to 1 month, the second 1 to 3 months, and so on. The ALM committee then investigates the effect on the value of the bank's portfolio of the zero rates corresponding to one bucket changing while those corresponding to all other buckets stay the same.
- If there is a mismatch, corrective action is usually taken. This can involve changing deposit and lending rates in the way described in Section 4.12. Alternatively, tools such as swaps, FRAs, bond futures, Eurodollar futures, and other interest rate derivatives can be used.

3 Gamma

- Gamma could be defined as a second partial derivative corresponding to a parallel shift in the yield curve.
- Alternatively, if the yield curve is derived from n instruments, with prices $x_1, x_2, \dots, x_n,$ we could calculate n possible gammas of the form $\frac{\partial^2 f}{\partial x_i^2}$.
- If we want to explore the interactions between all the underlying instruments, there are a total of 1/2n(n-1) possible second derivatives of the form $\frac{\partial^2 f}{\partial x_i \partial x_j}$ $(i \neq j)$.

3 Gamma

- When several delta measures are calculated, there are many possible gamma measures. Suppose that 10 instruments are used to compute the zero curve and that deltas are calculated by considering the impact of changes in the quotes for each of these.
- Gamma is a second partial derivative of the form $\partial^2\Pi/\partial x_i \partial x_j$, where Π is the portfolio value.
- There are 10 choices for x_i and 10 choices for x_j and a total of 55 different gamma measures. This may be "information overload".
- One approach is ignore cross-gammas and focus on the 10 partial derivatives where i = j.
- Another is to calculate a single gamma measure as the second partial derivative of the value of the portfolio with respect to a parallel shift in the zero curve.
- A further possibility is to calculate gammas with respect to the first two factors in a principal components analysis.

4 Vega

- Vega could be defined as a rate of change of the theoretical portfolio value when the volatility parameter in the Black formulae used to value interest rate options is increased by small amount.

- However, the parameters value will normally be different for different options and different payment dates.

- Alternatively, we can apply principal component analysis to determine the predominant volatility factors driving the prices (which will effectively be weighted averages of the individual volatilities).

- We can then calculate vegas as Partial derivatives with respect to these factors

4 Vega

- The vega of a portfolio of interest rate derivatives measures its exposure to volatility changes.
- One approach is to calculate the impact on the portfolio of making the same small change to the Black volatilities of all caps and European swap options. However, this assumes that one factor drives all volatilities and may be too simplistic.
- A better idea is to carry out a principal components analysis on the volatilities of caps and swap options and calculate vega measures corresponding to the first 2 or 3 factors.

5 Principal Component Analysis (PCA)

- One approach to handling the risk arising from groups of highly correlated market variables, like interest rates, is principal components analysis (PCA). This is a standard statistical tool with many applications in risk management. It takes historical data on movements in the market variables and attempts to define a set of components or factors that explain the movements.
- The approach is best illustrated with an example. The market variables we will consider are swap rates with maturities 1 year, 2 years 3 years, 4 years, 5 years, 7 years, 10 years, and 30 years.

5 Principal Component Analysis (PCA)

The yield curve is a line that plots the various interest rates of bonds with equal credit quality and different maturities.

According to the expectations theory of interest rates, the yield curve is made up of two aspects:
1. *An average* of market expectations concerning future short-term interest rates.
2. *The term premium* — the extra compensation an investor receives for holding a longer-term bond

- We can model these aspects of the yield curve using principal components decomposition. Data has two main properties: noise and signal.

- Principal components analysis aims to extract the signal and reduce the dimensionality of a dataset; by finding the **least amount of variables that explain the largest proportion of the data.** It does this by transforming the data from a correlation/covariance matrix onto a subspace with fewer dimensions, where all explanatory variables are orthogonal (perpendicular) to each other, i.e there is no multicollinearity.

5 Principal Component Analysis (PCA)

- When finding the principal components of the yield curve, the main theory held by econometricians is that:
  PC1 = constant ≈ long term interest rate ≈ R*
  PC2 = slope ≈ term premium
  PC3 = curvature
  There is a function in scikit-learn to perform PCA.

- The next step is to standardize the data into z-scores, assuming a mean of 0 and a variance of 1.

- We then form a covariance matrix from the standardized data using numpy.

- We can then create the eigenvalues and eigenvectors from the matrix. This performs eigendecomposition on our standardized data. We can use eigenvalues to find the proportion of the total variance that each principal component explains

- To form a time series for the principal components, we simply need to calculate the dot product between the eigenvectors and the standardized data.

5 Principal Component Analysis (PCA)

- The next two tables show results produced for these market variables using 2,780 daily observations between 2000 and 2011. The first column in The following table shows the maturities of the rates that were considered. The remaining eight columns in the table show the eight factors (or principal components) describing the rate moves.

Factor loadings for swap data.

	PC1	*PC2*	*PC3*	*PC4*	*PC5*	*PC6*	*PC7*	*PC8*
1y	0.216	−0.501	0.627	−0.487	0.122	0.237	0.011	−0.034
2y	0.331	−0.429	0.129	0.354	−0.212	−0.674	−0.100	0.236
3y	0.372	−0.267	−0.157	0.414	−0.096	0.311	0.413	−0.564
4y	0.392	−0.110	−0.256	0.174	−0.019	0.551	−0.416	0.512
5y	0.404	0.019	−0.355	−0.269	0.595	−0.278	−0.316	−0.327
7y	0.394	0.194	−0.195	−0.336	0.007	−0.100	0.685	0.422
10y	0.376	0.371	0.068	−0.305	−0.684	−0.039	−0.278	−0.279
30y	0.305	0.554	0.575	0.398	0.331	0.022	0.007	0.032

The factor loadings have the property that the sum of their squares for each factor is 1.0. Also, note that a factor is not changed if the signs of all its factor loadings are reversed.

5 Principal Component Analysis (PCA)

- The first factor, shown in the column labeled PC1, corresponds to a roughly parallel shift in the yield curve. When we have one unit of that factor, the 1-year rate increases by 0.216 basis points, the 2-year rate increases by 0.331 basis points, and so on.
- The second factor is shown in the column labeled PC2. It corresponds to a “twist” or change of slope of the yield curve. Rates between 1 year and 4 years move in one direction; rates between 5 years and 30 years move in the other direction. The third factor corresponds to a “bowing” of the yield curve. Relatively short rates (1 year and 2 year) and relatively long rates (10 year and 30 year) move in one direction; the intermediate rates move in the other direction. The interest rate move for a particular factor is known as factor loading. In our example, the first factor’s loading for the 1-year rate is 0.216.
- Because there are eight rates and eight factors, the interest rate changes observed on any given day can always be expressed as a linear sum of the factors by solving a set of eight simultaneous equations. The quantity of a particular factor in the interest rate changes on a particular day is known as the factor score for that day.

5 Principal Component Analysis (PCA)

- The importance of a factor is measured by the standard deviation of its factor score. The standard deviations of the factor scores in our example are shown in the table below and the factors are listed in order of their importance. The numbers in the table are measured in basis points. A quantity of the first factor equal to 1 standard deviation, therefore, corresponds to the 1-year rate moving by 0.216 x 17.55 = 3.78 basis points,
- the 2-year rate moving by 0.331 x 17.55 = 5.81 basis points, and so on.

Standard deviation of factor scores (basis points).

PC1	*PC2*	*PC3*	*PC4*	*PC5*	*PC6*	*PC7*	*PC8*
17.55	4.77	2.08	1.29	0.91	0.73	0.56	0.53

5 Principal Component Analysis (PCA)

- The factors have the property that the factor scores are uncorrelated across the data. For instance, in our example, the first factor score (amount of parallel shift) is uncorrelated with the second factor score (amount of twist) across the 2,780 days. The variances of the factor scores have the property that they add up to the total variance of the data. From the last table, the total variance of the original data (that is, sum of the variance of the observations on the 1-year rate, the variance of the observations on the 2-year rate, and so on) is

$$17.55^2 + 4.77^2 + 2.08^2 + \cdots + 0.53^2 = 338.8$$

- From this it can be seen that the first factor accounts for $\frac{17.55^2}{338.8} = 90.9\%$ of the variance in the original data; the first two factors account for $\frac{17.55^2+4.77^2}{338.8} = 97.7\%$ of the variance in the data; the third factor accounts for a further 1.3% of the variance. This shows that most of the risk in interest rate moves is accounted for by the first two or three factors. It suggests that we can relate the risks in a portfolio of interest rate dependent instruments to movements in these factors instead of considering all eight interest rates.

Example

- To illustrate how a principal components analysis can be used to hedge, consider a portfolio with the exposures to interest rate moves shown in table below. A 1-basis-point change in the 3-year rate causes the portfolio value to increase by $10 million, a 1-basispoint change in the 4-year rate causes it to increase by $4 million, and so on.

Change in portfolio value for a 1-basis-point rate move ($ millions).

3-year rate	*4-year rate*	*5-year rate*	*7-year rate*	*10-year rate*
+10	+4	−8	−7	+2

Example

- Suppose the first two factors are used to model rate moves. (As mentioned above, this captures 97.7%
- of the variance in rate moves.) Using the data in 1st PCA Table, the exposure to the first factor (measured in millions of dollars per factor score basis point) is

  10 x 0.372 + 4 x 0.392 - 8 x 0.404 - 7 x 0.394 + 2 x 0.376 = +0:05
- and the exposure to the second factor is

  10 x (-0.267) + 4 x (0.110) - 8 x 0.019 - 7 x 0.194 + 2 x 0.371= -3:88
- Suppose that f_1 and f_2 are the factor scores (measured in basis points). The change in the portfolio value is, to a good approximation, given by

$$\Delta P = +0.05 f_1 - 3.88 f_2$$
- The factor scores are uncorrelated and have the standard deviations given in 2nd PCA table. The standard deviation of ΔP is therefore

$$\sqrt{0.05^2 \text{x} 17.55^2 + 3.88^2 \text{x} 4.77^2} = 18.48$$

Example

- The delta was calculated for the pair of factor 1 & 2.
- The first delta then measures the impact of a small, approximately parallel, shift in the zero curve; the second delta measures the impact of a small twist in the zero curve; and so on.