Lecture

INSTITUTE OF ACTUARIAL & QUANTITATIVE STUDIES

Class: TY BSc

Subject : Financial Engineering - 2

Subject Code: PUSASQF506B

Chapter: Unit 2 Chapter 2

Chapter Name: Interest rate derivatives and hedging - 2

6 Recall back - The Black Model (Swaption)

Special features

- The strike rate is usually chosen to be similar to current swap rates (which, in turn, are similar to current interest rates for the same term).

Pricing/ Valuation

- A European swaption can be valued using Black's formula. The underlying variable is the n-year forward swap rate, whose current value is F_0.

Valuing a European Swaption

Option on a pay-fixed swap: $V_{swaption} = L\,A[F_0\Phi(d_1) - R_X\,\Phi(d_2)]$

Option on a pay-floating swap: $V_{swaption} = L\,A[R_X\,\Phi(-d_2)] - F_0\Phi(-d_1)]$

Where A=$\frac{1}{m}\sum_{i=1}^{mn} P(0, t_i)$

Here, A denotes the current value of an annuity of 1 unit per annum on each interest payment date and $P(0, t_i)$ is the market discount factor for a payment at time t_i, F_0 is the forward swap rate and R_X is the Strike price.

7 Interest Rate Swaption Hedging

Pricing European Swaption

- The cash flow made to the buyer of a payer swaption at time T amounts to

$$\sum_{i=1}^{n} N * e^{-(t_i - T)} * (i_F - i_S) * (t_i - t_{i-1}),$$

Where, F_0 = i_F = forward swap rate and R_X = i_S = Strike price

- if $i_F > i_S$ and 0 otherwise. It's value today therefore equals

$$e^{-iT} * \sum_{i=1}^{n} N * e^{-i(t_i - T)} * (i_F - i_S) * (t_i - t_{i-1}) = N * \sum_{i=1}^{n} e^{-it_i} * (i_F - i_S) * (t_i - t_{i-1})$$

- Now, the price of a European payer swaption is determined using the Black model. The i^{th} term:

$$N(t_i - t_{i-1}) * (i_F - i_S)+$$

- of the cash flow corresponds to the price of a European call option with expiry t_i. According to the Black model the price of this option at time 0 is

$$N * e^{-it_i} * (t_i - t_{i-1}) * [i_F \Phi(d_1) - i_S \Phi(d_2)],$$

- Where. $d_1 = (ln\frac{i_F}{i_S} + \frac{1}{2}\sigma_F^2 T)/\sigma_F\sqrt{T}$, $d_2 = (ln\frac{i_F}{i_S} - \frac{1}{2}\sigma_F^2 T)/(\sigma_F) * \sqrt{T} = d_1 - \sigma_F\sqrt{T}$

7 Interest Rate Swaption Hedging

Pricing European Swaption

- The forward swap rate i_F is computed. The value of the payer swaption P_{PS} itself is obtained by summing up all individual call options:

$$\boldsymbol{P_{PS} = NA[i_F\Phi(d_1) - i_S\Phi(d_2)]},$$

- Whereby, $\boldsymbol{A = \sum_{i=1}^{n} e^{-it_i}(t_i - t_{i-1})}$

- By analogy, in the case of a European receiver swaption, we obtain through,

$$\boldsymbol{P_{RS} = NA[i_S\Phi(-d_2) - i_F\Phi(-d_1)]}$$

Note this formula is same as the one in the previous chapter with slight change in the notations.

7 Interest Rate Swaption Hedging

- Partial derivatives lay the practical foundation for the application of special trading and hedging techniques on options and derivative securities. They quantify the influence (risk) of changes in market factors on the option price. In this respect and in many instances, partial derivatives are as important as the theoretically determined price of the option, as they tell the user in a short and accurate manner which direction to go in the current investment (assets, liabilities): buy, sell or maintain.
- The most important risk parameters the ("Greeks") are presented with respect to swaptions. This is followed by some trading strategies in brief.
- We shall denote the price of a general derivative security with D and the price of its underlying security with B. Concretely, in the case of swaptions (under consideration here) this means: D stands for the price P_{PS} of the Payer swaption and B stands for the corresponding swap price i_F.

7 Interest Rate Swaption Hedging

Delta

- Given a specific yield curve (interest rate structure) plus swap as underlying, the price of a swaption depends on the expiry date T and the strike price S (see, e. g., [11]). Therefore the impact of a shift in the swap price (strike rate i_S kept constant) on the swaption price is dependent on the variables T and i_F .
- Generally, the parameter delta describes the rate of change of the price of the derivative security with respect to the asset (price of the underlying):

$$\boldsymbol{\Delta = \frac{\partial D}{\partial B}}$$

- In the case under consideration, our underlying is the fixed forward swap rate i_F . By analogy with the delta of an equity call ,the following holds for the delta Δ_{PS} of a European payer swaption

$$\boldsymbol{\Delta_{PS} = \frac{\partial P_{PS}}{\partial i_F} = NA\Phi(d_1)},$$

- whereby $\boldsymbol{A = \sum_{i=1}^{n} e^{-it_i}(t_i - t_{i-1})}$
- By analogy, we obtain for the delta of a European receiver swaption the following value

$$\boldsymbol{\Delta_{RS} = NA(\Phi(d_1) - 1)}$$

7 Interest Rate Swaption Hedging

Gamma

- The gamma of a derivative product (e.g., swaption or portfolio thereof) is the second derivative of the price of the derivative with respect to the underlying:

$$\boldsymbol{\Gamma = \frac{\partial^2 D}{\partial B^2} = \frac{\partial \Delta}{\partial B}}$$

- Like in the case of delta, a closed form formula can be derived for gamma through differentiation:

$$\boldsymbol{\Gamma_{PS} = \frac{NA\varphi(d_1)}{\sqrt{T} i_F \sigma_F}}$$

- whereby $\boldsymbol{\varphi(d_1) = \Phi'(d_1)}$ stands for the probability density function of the gaussian distribution.
- According to the definition, gamma is the sensitivity of the delta to the underlying security. Therefore, it measures how much and how often Gamma must be rehedged in order to maintain a delta-neutral portfolio.

7 Interest Rate Swaption Hedging

Theta

- The theta of a portfolio of derivative products is the rate of change of the portfolio price with respect to time to maturity T (time left for option to expire):

$$\Theta = \frac{\partial D}{\partial T}$$

- In the special case of a payer swaption one obtains

$$\Theta = \frac{\partial P_{PS}}{\partial T} = -i_F P_{PS} + \frac{NAi_F \sigma_F \varphi(d_1)}{2\sqrt{T}}$$

7 Interest Rate Swaption Hedging

Vega

- Vega is the sensitivity of the derivative (or portfolio of financial derivative products) price to volatility σ

$$\boldsymbol{V = \frac{\partial D}{\partial \sigma}}$$

- In the special case of swaptions one obtains the following equation through differentiation:

$$\boldsymbol{V_{PS} = V_{RS} = NAi_F\sqrt{T}\varphi(d_1)}$$

- The implicit quantity σ_F is the estimate for the expected forward swap rate. A vega-hedged portfolio is protected against fluctuations of volatility.

7 Interest Rate Swaption Hedging

Delta-Hedging

- The so called delta-hedging is a dynamic hedging strategy. Here, it is sought, price changes of the swap to be compensated with price changes of the swaption. This is achieved by setting up a portfolio by holding (or shorting) the derivative (swaption) and shorting (or holding) a quantity Δ of the underlying (swap); this is referred to as hedge portfolio.
- In this way, within the portfolio, price increases of the swap are compensated by price drops of the swaption and vice-versa. Risks caused by fluctuations of the underlying security are practically eliminated. As can be verified, this portfolio has a delta of zero (let P_{Port} be the price of the portfolio):

$$\boldsymbol{\Delta_{Port} = \frac{\partial P_{Port}}{\partial B} = \Delta * \frac{\partial B}{\partial B} - \frac{\partial D}{\partial B} = \Delta * 1 - \Delta = 0}$$

7 Interest Rate Swaption Hedging

Delta-Hedging

- Therefore, by way of delta-hedging, one can eliminate (at least theoretically and to a great extent practically) the risk. The proportion of the underlying security in the portfolio must be continuously changed since the quantity Δ depends on both the price of the underlying and the remaining period to maturity of the swaption. This process is called dynamic hedging (or rebalancing) of the portfolio. Therefore (theoretically), one continuously has to buy and sell swaps. However, in the case of a discrete model, rebalancing of delta is done at discrete time intervals Δt.

7 Interest Rate Swaption Hedging

Delta-Gamma-Hedging

- A little value for gamma indicates that by definition, the rate of change of delta is little. This means rebalancing of the hedge-portfolio may be carried out in larger intervals of time. Conversely, larger gamma values are an indication that delta is very sensitive with respect to shifts in the underlying, resulting in the increase in risk inherent in a shift in portfolio value.
- Because of the cost of frequent hedging, it is natural to try to minimize the need to rebalance the portfolio too frequently. The corresponding hedging procedure is called a gamma-neutral strategy . To achieve this objective, we have to buy and sell more swaptions, not just the swap. By simple differentiation, you can check that a position in the underlying asset has zero gamma:

$$\frac{\partial^2 B}{\partial B^2} = 0, particularly, \frac{\partial^2 i_F}{\partial i_F^2} = 0$$

7 Interest Rate Swaption Hedging

Delta-Gamma-Hedging

- Thus, we cannot change the gamma of our position by adding the underlying. However, we can add another swaption in quantity, which will make the portfolio gamma-neutral. By holding two different swaptions we can make the portfolio both delta- and gamma-neutral. Note that a delta-neutral portfolio Δ_{Port} = 0 has gamma equal to Γ and a traded swaption has gamma equal to Γ_o. If the number of traded swaptions added to the portfolio is w0, the gamma of the portfolio is

$$\widetilde{\boldsymbol{\Gamma_{Port}}} = \boldsymbol{\Gamma} + \boldsymbol{w_0 \Gamma_0}$$

- Hence, the portfolio becomes gamma-neutral, if our position in the traded swaption is equal to $w_o = -\Gamma/\Gamma_0$. Of course, as we add the traded swaption, the delta of the portfolio changes. So the position in the underlying (swap) then has to be changed to maintain delta-neutrality. Due to

$$\widetilde{\boldsymbol{\Delta}}_{\boldsymbol{Port}} = \boldsymbol{0} + \boldsymbol{w_0 \Delta_0} = \boldsymbol{0} - \left(\frac{\boldsymbol{\Gamma}}{\boldsymbol{\Gamma_0}}\right)\boldsymbol{\Delta_0}$$

- the quantity $\widetilde{\Delta}_{Port}$ of the underlying (swap) has to be added for hedging.

7 Interest Rate Swaption Hedging

Practical Approach

- In practice, portfolio rebalancing to achieve delta-, gamma-, vega-neutrality etc. is not a continuous process. If it were, transaction costs would render it extremely expensive. Instead, the individual risks are analyzed to find out if they are worth taking or not. The aforementioned risk parameters play the role of quantifying various aspects of portfolio risk. If the risk is acceptable, no action is taken. Otherwise rebalancing is carried out as outlined above.

8 SABR Model

- The stochastic alpha beta rho model is a stochastic volatility model for forward prices commonly used in the modelling of interest rate derivatives. The alpha, beta and rho in the name are parameters to be calibrated. Alpha describes the magnitude of the volatility in the price of the underlying asset; beta describes the sensitivity of forward price movements to the spot price; and rho describes the correlation between movements in the forward price and movements in the volatility of the price of the underlying asset. The model first appeared in a 2002 paper by Patrick Hagan, Deep Kumar, Andrew Lesniewski and Diana Woodward.
- SABR Model has the distinctive advantage of being useful for IRD modelling even when the interest rates are negative.
- They are used for hedging widely in the industry.