INSTITUTE OF ACTUARIAL & QUANTITATIVE STUDIES

Subject: Financial Engineering 2

Chapter: Unit 1 & 2

Category: Assignment Solutions

1. The forward rate is

$$\frac{0.06 \times 0.75 - 0.05 \times 0.50}{0.25} = 0.08$$

or 8%. The FRA rate is 7%. A profit can therefore be made by borrowing for six months at 5%, entering into an FRA to borrow for the period between 6 and 9 months for 7% and lending for nine months at 6%.

2. :

In total the gain or loss under a futures contract is equal to the gain or loss under the corresponding forward contract. However the timing of the cash flows is different. When the time value of money is taken into account a futures contract may prove to be more valuable or less valuable than a forward contract. Of course the company does not know in advance which will work out better. The long forward contract provides a perfect hedge. The long futures contract provides a slightly imperfect hedge.

a) In this case the forward contract would lead to a slightly better outcome. The company will make a loss on its hedge. If the hedge is with a forward contract the whole of the loss will be realized at the end. If it is with a futures contract the loss will be realized day by day throughout the contract. On a present value basis the former is preferable.

b) In this case the futures contract would lead to a slightly better outcome. The company will make a gain on the hedge. If the hedge is with a forward contract the gain will be realized at the end. If it is with a futures contract the gain will be realized day by day throughout the life of the contract. On a present value basis the latter is preferable.

c) In this case the futures contract would lead to a slightly better outcome. This is because it would involve positive cash flows early and negative cash flows later.

d) In this case the forward contract would lead to a slightly better outcome. This is because, in the case of the futures contract, the early cash flows would be negative and the later cash flow would be positive.

3. :

The cheapest-to-deliver bond is the one for which

$$\text{Quoted Price} - \text{Futures Price} \times \text{Conversion Factor}$$

is least. Calculating this factor for each of the 4 bonds we get

$$\text{Bond 1: } 125.15625 - 101.375 \times 1.2131 = 2.178$$
$$\text{Bond 2: } 142.46875 - 101.375 \times 1.3792 = 2.652$$
$$\text{Bond 3: } 115.96875 - 101.375 \times 1.1149 = 2.946$$
$$\text{Bond 4: } 144.06250 - 101.375 \times 1.4026 = 1.874$$

Bond 4 is therefore the cheapest to deliver.

4. :

The Eurodollar futures price has increased by 6 basis points. The investor makes a gain per contract of $25 \times 6 = \$150$ or \$300 in total.

5. :

The number of short futures contracts required is

$$\frac{100{,}000{,}000 \times 4.0}{122{,}000 \times 9.0} = 364.3$$

Rounding to the nearest whole number 364 contracts should be shorted.

a) This increases the number of contracts that should be shorted to

$$\frac{100{,}000{,}000 \times 4.0}{122{,}000 \times 7.0} = 468.4$$

or 468 when we round to the nearest whole number.

b) In this case the gain on the short futures position is likely to be less than the loss on the loss on the bond portfolio. This is because the gain on the short futures position depends on the size of the movement in long-term rates and the loss on the bond portfolio depends on the size of the movement in medium-term rates. Duration-based hedging assumes that the movements in the two rates are the same.

6. :

X has a comparative advantage in yen markets but wants to borrow dollars. Y has a comparative advantage in dollar markets but wants to borrow yen. This provides the basis for the swap. There is a 1.5% per annum differential between the yen rates and a 0.4% per annum differential between the dollar rates. The total gain to all parties from the swap is therefore $1.5-0.4=1.1\%$ per annum. The bank requires 0.5% per annum, leaving 0.3% per annum for each of X and Y. The swap should lead to X borrowing dollars at $9.6-0.3=9.3\%$ per annum and to Y borrowing yen at $6.5-0.3=6.2\%$ per annum. The appropriate arrangement is therefore as shown All foreign exchange risk is borne by the bank.

7. :

A swap rate for a particular maturity is the average of the bid and offer fixed rates that a market maker is prepared to exchange for LIBOR in a standard plain vanilla swap with that maturity. Par yields on the swap is that fixed rate which gives a NPV of zero basis the current term structure of interest rates.

8. :

(i) *S*

a. The forward swap rate Y (in %) satisfies:

$$100e^{-2\times 0.044}=Y\left[e^{-3\times 0.046}+e^{-4\times 0.048}+e^{-5\times 0.05}\right]+100e^{-5\times 0.05}$$

Solving this, $Y=5.533\%$.

b. Because the 5.75% rate within the forward agreement is higher than the forward swap rate, the forward agreement has <u>negative</u> value to the financial institution.

ii)

The financial institution needs to buy a receiver swaption with:

- Maturity 2 years
- Strike 5.75% pa
- Notional €1m
- Swap length 3 years
- Annual tenor

The swaption can be priced using Black's formula:

$$\text{Price} = XA\,[KN(-d_2) - YN(-d_1)]$$

Where

X = notional, K = strike, Y = forward swap (fixed rate),

A = value of payment of 1 for each period of the swaption life

$$d_1 = \frac{\ln Y/K + \frac{1}{2}\sigma^2 T}{\sigma\sqrt{T}}, \quad d_2 = d_1 - \sigma\sqrt{T}$$

In this case:

$$A = \left[e^{-3\times0.046} + e^{-4\times0.048} + e^{-5\times0.05}\right] = 2.47521$$

$$d_1 = \frac{\ln 0.0553/0.0575 + \frac{1}{2}(0.12)^2 2}{0.12\sqrt{2}} = -0.14183$$

so $N(-d_1) = 0.55639$

and $d_2 = d_1 - 0.12\sqrt{2} = -0.31154$

so $N(-d_2) = 0.62230$

Hence the price of the swaption is:

$1{,}000{,}000 \times 2.47521 \times [5.75\% \times 0.62230 - 5.533\% \times 0.55639] = €12{,}369$

iii)

) Rather than buying a receiver swaption, the financial institution could:

- Pay a premium to enter into a forward-starting swap commencing at end year 2, running for 3 years paying floating rates but receiving a fixed rate of 5.75%
- Buy a payer swaption with:
  - Maturity 2 years
  - Strike 5.75% pa
  - Notional €1m
  - Swap length 3 years
  - Annual tenor

9. :

The swap can be regarded as a long position in a floating-rate bond combined with a short position in a fixed-rate bond. The correct discount rate is 12% per annum with quarterly compounding or 11.82% per annum with continuous compounding.

Immediately after the next payment the floating-rate bond will be worth \$100 million. The next floating payment (\$ million) is

$$0.118 \times 100 \times 0.25 = 2.95$$

The value of the floating-rate bond is therefore

$$102.95e^{-0.1182\times 2/12} = 100.941$$

The value of the fixed-rate bond is

$$2.5e^{-0.1182\times 2/12} + 2.5e^{-0.1182\times 5/12} + 2.5e^{-0.1182\times 8/12}$$
$$+2.5e^{-0.1182\times 11/12} + 102.5e^{-0.1182\times 14/12} = 98.678$$

The value of the swap is therefore

$$100.941 - 98.678 = \$2.263\text{million}$$

As an alternative approach we can value the swap as a series of forward rate agreements. The calculated value is

$$(2.95-2.5)e^{-0.1182\times 2/12} + (3.0-2.5)e^{-0.1182\times 5/12}$$
$$+(3.0-2.5)e^{0.1182\times 8/12} + (3.0-2.5)e^{-0.1182\times 11/12}$$
$$+(3.0-2.5)e^{-0.1182\times 14/12} = \$2.263 \text{ million}$$

which is in agreement with the answer obtained using the first approach.

10. :

The payoff from the swaption is a series of five cash flows equal to $\max[0.076 - s_T$ millions of dollars where s_T is the five-year swap rate in four years. The value of a that provides \$1 per year at the end of years 5, 6, 7, 8, and 9 is

$$\sum_{i=5}^{9} \frac{1}{1.08^i} = 2.9348$$

The value of the swaption in millions of dollars is therefore

$$2.9348[0.076N(-d_2) - 0.08N(-d_1)]$$

where

$$d_1 = \frac{\ln(0.08/0.076) + 0.25^2 \times 4/2}{0.25\sqrt{4}} = 0.3526$$

and

$$d_2 = \frac{\ln(0.08/0.076) - 0.25^2 \times 4/2}{0.25\sqrt{4}} = -0.1474$$

The value of the swaption is

$$2.9348[0.076N(0.1474) - 0.08N(-0.3526)] = 0.03955$$

11. :

It is instructive to consider two different ways of valuing this instrument. From the perspective of a sterling investor it is a cash or nothing put. The variables are $S_0 = 1/1.48 = 0.6757$, $K = 1/1.50 = 0.6667$, $r = 0.08$, $q = 0.04$, $\sigma = 0.12$, and $T = 1$. The derivative pays off if the exchange rate is less than 0.6667. The value of the derivative is $10{,}000N(-d_2)e^{-0.08\times1}$ where

$$d_2 = \frac{\ln(0.6757/0.6667) + (0.08 - 0.04 - 0.12^2/2)}{0.12} = 0.3852$$

Since $N(-d_2)$=0.3500, the value of the derivative is $10{,}000\times0.3500\times e^{-0.08}$ or 3,231. In dollars this is $3{,}231 \times 1.48 = \$4782$

From the perspective of a dollar investor the derivative is an asset or nothing call. The variables are $S_0 = 1.48$, $K = 1.50$, $r = 0.04$, $q = 0.08$, $\sigma = 0.12$ and $T = 1$. The value is $10{,}000N(d_1)e^{-0.08\times1}$ where

$$d_1 = \frac{\ln(1.48/1.50) + (0.04 - 0.08 + 0.12^2/2)}{0.12} = -0.3852$$

$N(d_1) = 0.3500$ and the value of the derivative is as before $10{,}000 \times 1.48 \times 0.3500 \times e^{-0.08} = 4782$ or \$4,782.

12. :

In an Asian option the payoff becomes more certain as time passes and the delta always approaches zero as the maturity date is approached. This makes delta hedging easy. Barrier options cause problems for delta hedgers when the asset price is close to the barrier because delta is discontinuous.

13. :

This is a cash-or-nothing call. The value is $100N(d_2)e^{-0.08\times 0.5}$ where

$$d_2 = \frac{\ln(960/1000) + (0.08 - 0.03 - 0.2^2/2)\times 0.5}{0.2\times\sqrt{0.5}} = -0.1826$$

Since $N(d_2) = 0.4276$ the value of the derivative is $41.08.

The income yield is relevant because the income would lead to a reduction in the price of the bond during the life of the derivative and hence change the probability of achieving the strike price.

14.

- Suppose the first two factors are used to model rate moves.
- Using the data in 1st PCA Table, the exposure to the first factor (measured in millions of dollars per factor score basis point) is

7 x -0.032838 + 6 x 0.031877 + 5 x -0.009330 - 4 x 0.219963 - 2 x 0.533525 = -2.032156

- and the exposure to the second factor is

7 x 0.067482 + 6 x 0.062923 + 5 x 0.078643 - 4 x 0.048486 - 2 x 0.126723 = 0.795731

- Suppose that f1 and f2 are the factor scores (measured in basis points). The change in the portfolio value is, to a good approximation, given by

$$P = -2.032156\ f1 + 0.795731\ f2$$

- The factor scores are uncorrelated and have the standard deviations given in 2nd PCA table. The standard deviation of P is therefore:

$\sqrt{(-2.032156)^2 x(3.54)^2 + (0.795731)^2 x(2.23)^2}$ = 51.75122 + 3.14877973
$= 54.89999$

15. :

It is instructive to consider two different ways of valuing this instrument. From the perspective of a sterling investor it is a cash or nothing put. The variables are $S_0 = 1/1.48 = 0.6757$, $K = 1/1.50 = 0.6667$, $r = 0.08$, $q = 0.04$, $\sigma = 0.12$, and $T = 1$. The derivative pays off if the exchange rate is less than 0.6667. The value of the derivative is $10,000 N(-d_2)e^{-0.08\times 1}$ where

$$d_2 = \frac{\ln(0.6757/0.6667) + (0.08 - 0.04 - 0.12^2/2)}{0.12} = 0.3852$$

Since $N(-d_2)$=0.3500, the value of the derivative is $10,000\times 0.3500\times e^{-0.08}$ or 3,231. In dollars this is $3,231 \times 1.48 = \$4782$

From the perspective of a dollar investor the derivative is an asset or nothing call. The variables are $S_0 = 1.48$, $K = 1.50$, $r = 0.04$, $q = 0.08$, $\sigma = 0.12$ and $T = 1$. The value is $10,000 N(d_1)e^{-0.08\times 1}$ where

$$d_1 = \frac{\ln(1.48/1.50) + (0.04 - 0.08 + 0.12^2/2)}{0.12} = -0.3852$$

$N(d_1) = 0.3500$ and the value of the derivative is as before $10,000 \times 1.48 \times 0.3500 \times e^{-0.08} = 4782$ or \$4,782.