INSTITUTE OF ACTUARIAL & QUANTITATIVE STUDIES

Financial Engineering II

Subject:

Unit 2

Chapter:

Practice Questions

Category:

1.

Simply evaluate the Black–Scholes formula with $s = \$10$, $k = \$12$, $\mu = 0.15$, $\sigma = 0.20$, $r = 0.05$ and $T = 1$. The option value is \$0.325.

2.

In this case X is \$1 if $S_T > \$10$, and is zero otherwise, where T is 1. Hence by the derivative pricing formula

$$V_0 = \mathbb{E}_{\mathbb{Q}}(B_T^{-1}X) = e^{-rT}\mathbb{Q}(S_T > \$10) = e^{-rT}\Phi\left(\frac{rT - \frac{1}{2}\sigma^2 T}{\sigma\sqrt{T}}\right).$$

This has the numerical value of \$0.532.

3.

The price of an option or other derivative when expressed in terms of the price of the underlying stock is independent of risk preferences. Options therefore have the same value in a risk-neutral world as they do in the real world. We may therefore assume that the world is risk neutral for the purposes of valuing options. This simplifies the analysis. In a risk-neutral world all securities have an expected return equal to risk-free interest rate. Also, in a risk-neutral world, the appropriate discount rate to use for expected future cash flows is the risk-free interest rate.

4.

In this case $S_0 = 52$, $K = 50$, $r = 0.12$, $\sigma = 0.30$ and $T = 0.25$.

$$d_1 = \frac{\ln(52/50) + (0.12 + 0.3^2/2)0.25}{0.30\sqrt{0.25}} = 0.5365$$

$$d_2 = d_1 - 0.30\sqrt{0.25} = 0.3865$$

The price of the European call is

$$\begin{aligned} &52N(0.5365) - 50e^{-0.12\times 0.25}N(0.3865) \\ &= 52\times 0.7042 - 50e^{-0.03}\times 0.6504 \\ &= 5.06 \end{aligned}$$

or \$5.06.

UNIT 1

PRACTICE QUESTIONS

5. :

Black's approach in effect assumes that the holder of option must decide at time zero whether it is a European option maturing at time t_n (the final ex-dividend date) or a European option maturing at time T. In fact the holder of the option has more flexibility than this. The holder can choose to exercise at time t_n if the stock price at that time is above some level but not otherwise. Furthermore, if the option is not exercised at time t_n, it can still be exercised at time T.

It appears that Black's approach should understate the true option value. This is because the holder of the option has more alternative strategies for deciding when to exercise the option than the two strategies implicitly assumed by the approach. These alternative strategies add value to the option.

However, this is not the whole story! The standard approach to valuing either an American or a European option on a stock paying a single dividend applies the volatility to the stock price less the present value of the dividend. (The procedure for valuing an American option is explained in Chapter 21.) Black's approach when considering exercise just prior to the dividend date applies the volatility to the stock price itself. Black's approach therefore assumes more stock price variability than the standard approach in some of its calculations. In some circumstances it can give a higher price than the standard approach.

6. :

The answer is no. If markets are efficient they have already taken potential dilution into account in determining the stock price. This argument is explained in Business Snapshot 15.3.

Business Snapshot 15.3 Warrants, Employee Stock Options, and Dilution

Consider a company with 100,000 shares each worth \$50. It surprises the market with an announcement that it is granting 100,000 stock options to its employees with a strike price of \$50. If the market sees little benefit to the shareholders from the employee stock options in the form of reduced salaries and more highly motivated managers, the stock price will decline immediately after the announcement of the employee stock options. If the stock price declines to \$45, the dilution cost to the current shareholders is \$5 per share or \$500,000 in total.

Suppose that the company does well so that by the end of three years the share price is \$100. Suppose further that all the options are exercised at this point. The payoff to the employees is \$50 per option. It is tempting to argue that there will be further dilution in that 100,000 shares worth \$100 per share are now merged with 100,000 shares for which only \$50 is paid, so that (a) the share price reduces to \$75 and (b) the payoff to the option holders is only \$25 per option. However, this argument is flawed. The exercise of the options is anticipated by the market and already reflected in the share price. The payoff from each option exercised is \$50.

This example illustrates the general point that when markets are efficient the impact of dilution from executive stock options or warrants is reflected in the stock price as soon as they are announced and does not need to be taken into account again when the options are valued.

7. :

With the notation in the text

$$D_1 = D_2 = 1.50, \quad t_1 = 0.3333, \quad t_2 = 0.8333, \quad T = 1.25, \quad r = 0.08 \quad \text{and} \quad K = 55$$

$$K\left[1 - e^{-r(T-t_2)}\right] = 55(1 - e^{-0.08\times0.4167}) = 1.80$$

Hence

$$D_2 < K\left[1 - e^{-r(T-t_2)}\right]$$

Also:

$$K\left[1 - e^{-r(t_2-t_1)}\right] = 55(1 - e^{-0.08\times0.5}) = 2.16$$

Hence:

$$D_1 < K\left[1 - e^{-r(t_2-t_1)}\right]$$

It follows from the conditions established in Section 15.12 that the option should never be exercised early.

The present value of the dividends is

$$1.5e^{-0.3333\times0.08} + 1.5e^{-0.8333\times0.08} = 2.864$$

The option can be valued using the European pricing formula with:

$$S_0 = 50 - 2.864 = 47.136, \quad K = 55, \quad \sigma = 0.25, \quad r = 0.08, \quad T = 1.25$$

$$d_1 = \frac{\ln(47.136/55) + (0.08 + 0.25^2/2)1.25}{0.25\sqrt{1.25}} = -0.0545$$

$$d_2 = d_1 - 0.25\sqrt{1.25} = -0.3340$$

$$N(d_1) = 0.4783, \quad N(d_2) = 0.3692$$

and the call price is

$$47.136\times0.4783 - 55e^{-0.08\times1.25}\times0.3692 = 4.17$$

or $4.17.

8. :

Lower bound if option is European is

$$(F_0 - K)e^{-rT} = (47 - 40)e^{-0.1\times2/12} = 6.88$$

Lower bound if option is American is

$$F_0 - K = 7$$

9. :

The rate received will be less than 6.5% when LIBOR is less than 7%. The corporation requires a three-month call option on a Eurodollar futures option with a strike price of 93. If three-month LIBOR is greater than 7% at the option maturity, the Eurodollar futures quote at option maturity will be less than 93 and there will be no payoff from the option. If the three-month LIBOR is less than 7%, one Eurodollar futures options provide a payoff of $25 per 0.01%. Each 0.01% of interest costs the corporation $125 (=5,000,000×0.0001×0.25). A total of 125/25 = 5 contracts are therefore required.

10. :

The final fixed payment is in millions of dollars:

$$[(4\times1.0415+4)\times1.0415+4]\times1.0415+4=17.0238$$

The final floating payment assuming forward rates are realized is

$$[(4.05\times1.041+4.05)\times1.041+4.05]\times1.041+4.05=17.2238$$

The value of the swap is therefore $-0.2000/(1.04^4)=-0.1710$ or $-\$171{,}000$.

11. :

The fixed side consists of four payments of USD 0.9 million. The present value in millions of dollars is

$$\frac{0.9}{1.05}+\frac{0.9}{1.05^2}+\frac{0.9}{1.05^3}+\frac{0.9}{1.05^4}=3.191$$

The forward Australian LIBOR rate is 10% with annual compounding. From Section 30.3 the quanto adjustment to the floating payment at time t_i+1 is

$$0.1\times0.3\times0.15\times0.25t_i=0.01125t_i$$

The value of the floating payments is therefore

$$\frac{1}{1.05}+\frac{1.01125}{1.05^2}+\frac{1.0225}{1.05^3}+\frac{1.03375}{1.05^4}=3.593$$

The value of the swap is 3.593 – 3.191 = 0.402 million.

12. :

In this case $L = 1000$, $\delta_k = 0.25$, $F_k = 0.12$, $R_K = 0.13$, $r = 0.115$, $\sigma_k = 0.12$, $t_k = 1.25$, $P(0, t_{k+1}) = 0.8416$.

$$L\delta_k = 250$$

$$d_1 = \frac{\ln(0.12/0.13) + 0.12^2 \times 1.25/2}{0.12\sqrt{1.25}} = -0.5295$$

$$d_2 = -0.5295 - 0.12\sqrt{1.25} = -0.6637$$

The value of the option is

$$250 \times 0.8416 \times [0.12N(-0.5295) - 0.13N(-0.6637)]$$

$$= 0.59$$

0.59$