Subject: Financial Engineering II

Chapter:

Category: Practice Question Solution

1. A position in the underlying asset cannot be used to make the portfolio gamma-neutral. [1]
The gamma of the underlying asset is zero. [½]
This is because the value of the asset is the asset, and hence there is no second order dependency of the asset price to itself. [½]
Therefore any position in it will not change the gamma of the portfolio. [½]

2.
i)
The portfolio can be made gamma-neutral by buying $3,706.2/1.74 = 2,130$ of the traded options. [1]

As a result of buying these the delta of the hedged portfolio increases by $0.70 \times 2,130 = 1,491$. [1]

To regain delta-neutrality, 1,491 units of the underlying asset are required to be sold. [1]

ii) The delta-hedging portfolio could use options which have smaller negative gammas yet also provide the required delta hedging. [1]
Use options with different characteristics, e.g. different maturity dates... [½]
... or non-vanilla options. [½]

iii) In future, set risk tolerances on how low negative gamma can reach before taking action. [1]

3.
The price of an option or other derivative when expressed in terms of the price of the underlying stock is independent of risk preferences. Options therefore have the same value in a risk-neutral world as they do in the real world. We may therefore assume that the world is risk neutral for the purposes of valuing options. This simplifies the analysis. In a risk-neutral world all securities have an expected return equal to risk-free interest rate. Also, in a risk-neutral world, the appropriate discount rate to use for expected future cash flows is the risk-free interest rate.

4.

In this case, $F_0 = (125 - 10)e^{0.1\times1} = 127.09$, $K = 110$, $P(0,T) = e^{-0.1\times1}$, $\sigma_B = 0.08$, and $T = 1.0$.

$$d_1 = \frac{\ln(127.09/110) + (0.08^2/2)}{0.08} = 1.8456$$

$$d_2 = d_1 - 0.08 = 1.7656$$

From equation (28.2) the value of the put option is

$$110e^{-0.1\times1}N(-1.7656) - 127.09e^{-0.1\times1}N(-1.8456) = 0.12$$

or \$0.12.

5.

Black's approach in effect assumes that the holder of option must decide at time zero whether it is a European option maturing at time t_n (the final ex-dividend date) or a European option maturing at time T. In fact the holder of the option has more flexibility than this. The holder can choose to exercise at time t_n if the stock price at that time is above some level but not otherwise. Furthermore, if the option is not exercised at time t_n, it can still be exercised at time T.

It appears that Black's approach should understate the true option value. This is because the holder of the option has more alternative strategies for deciding when to exercise the option than the two strategies implicitly assumed by the approach. These alternative strategies add value to the option.

However, this is not the whole story! The standard approach to valuing either an American or a European option on a stock paying a single dividend applies the volatility to the stock price less the present value of the dividend. (The procedure for valuing an American option is explained in Chapter 21.) Black's approach when considering exercise just prior to the dividend date applies the volatility to the stock price itself. Black's approach therefore assumes more stock price variability than the standard approach in some of its calculations. In some circumstances it can give a higher price than the standard approach.

6.

In this case $L = 1000$, $\delta_k = 0.25$, $F_k = 0.12$, $R_K = 0.13$, $r = 0.115$, $\sigma_k = 0.12$, $t_k = 1.25$, $P(0, t_{k+1}) = 0.8416$.

$$L\delta_k = 250$$

$$d_1 = \frac{\ln(0.12/0.13) + 0.12^2 \times 1.25/2}{0.12\sqrt{1.25}} = -0.5295$$

$$d_2 = -0.5295 - 0.12\sqrt{1.25} = -0.6637$$

The value of the option is

$$250 \times 0.8416 \times [0.12N(-0.5295) - 0.13N(-0.6637)]$$

$$= 0.59$$

or \$0.59.

7.

Lower bound if option is European is

$$(F_0 - K)e^{-rT} = (47 - 40)e^{-0.1\times 2/12} = 6.88$$

Lower bound if option is American is

$$F_0 - K = 7$$

8.

The rate received will be less than 6.5% when LIBOR is less than 7%. The corporation requires a three-month call option on a Eurodollar futures option with a strike price of 93. If three-month LIBOR is greater than 7% at the option maturity, the Eurodollar futures quote at option maturity will be less than 93 and there will be no payoff from the option. If the three-month LIBOR is less than 7%, one Eurodollar futures options provide a payoff of \$25 per 0.01%. Each 0.01% of interest costs the corporation \$125 (=5,000,000×0.0001×0.25). A total of 125/25 = 5 contracts are therefore required.

9.

The final fixed payment is in millions of dollars:

$$[(4\times1.0415+4)\times1.0415+4]\times1.0415+4=17.0238$$

The final floating payment assuming forward rates are realized is

$$[(4.05\times1.041+4.05)\times1.041+4.05]\times1.041+4.05=17.2238$$

The value of the swap is therefore $-0.2000/(1.04^4)=-0.1710$ or $-\$171{,}000$.

10.

We prove this result by considering two portfolios. The first consists of the swap option to receive s_K; the second consists of the swap option to pay s_K and the forward swap. Suppose that the actual swap rate at the maturity of the options is greater than s_K. The swap option to pay s_K will be exercised and the swap option to receive s_K will not be exercised. Both portfolios are then worth zero since the swap option to pay s_K is neutralized by the forward swap. Suppose next that the actual swap rate at the maturity of the options is less than s_K. The swap option to receive s_K is exercised and the swap option to pay s_K is not exercised. Both portfolios are then equivalent to a swap where s_K is received and floating is paid. In all states of the world the two portfolios are worth the same at time T_1. They must therefore be worth the same today. This proves the result. When s_K equals the current forward swap rate $f=0$ and $V_1=V_2$. A swap option to pay fixed is therefore worth the same as a similar swap option to receive fixed when the fixed rate in the swap option is the forward swap rate.

11.

The quoted futures price corresponds to a forward rate is 8% per annum with quarterly compounding and actual/360. The parameters for Black's model are therefore: $F_k=0.08$, $K=0.08$, $R=0.075$, $\sigma_k=0.15$, $t_k=0.75$, and $P(0,t_{k+1})=e^{-0.075\times1}=0.9277$

$$d_1=\frac{0.5\times0.15^2\times0.75}{0.15\sqrt{0.75}}=0.0650$$

$$d_2=-\frac{0.5\times0.15^2\times0.75}{0.15\sqrt{0.75}}=-0.0650$$

and the call price, c, is given by

$$c=0.25\times1{,}000\times0.9277\,[0.08N(0.0650)-0.08N(-0.0.650)]=0.96$$

12.

- Suppose the first two factors are used to model rate moves.
- Using the data in 1st PCA Table, the exposure to the first factor (measured in millions of dollars per factor score basis point) is

8 x 0.972 + 6 x 0.993 + 5 x 0.991 - 5 x 0.939 - 3 x 0.868 - 2 x 0.686 = +10.018

- and the exposure to the second factor is

8 x (-0.227) + 6 x (-0.059) + 5 x 0.089 - 5 x 0.335 - 3 x 0.495 - 2 x 0.710 = -6.305

- Suppose that f_1 and f_2 are the factor scores (measured in basis points). The change in the portfolio value is, to a good approximation, given by

$$\Delta P = +10.018\, f_1 \; - 6.305\, f_2$$

- The factor scores are uncorrelated and have the standard deviations given in 2nd PCA table. The standard deviation of ΔP is therefore

$$\sqrt{10.018^2 \text{x} 9.54^2 + (-6.305)^2 \text{x} 3.23^2} = 97.717$$