INSTITUTE OF ACTUARIAL & QUANTITATIVE STUDIES

Subject: Financial Engineering II

Chapter: Unit 3

Category: Practice Questions

1.

Consider the following stochastic differential equation for the instantaneous risk free rate (also referred to as the short-rate):

$$dr(t) = a\left(b - r(t)\right)dt + \sigma dW_t$$

i) a) State the main drawback of such a model for the short-rate.
b) State the name and stochastic differential equation of an alternative model for the short-rate that is not subject to the drawback.

2.

Discuss whether one-factor models are good models for the short-rate of interest (instantaneous risk free rate). Include discussion of extensions that may be considered to improve the model. Illustrate your discussion by defining and referring to particular models.

3.

i) List the desirable characteristics of a model for the term structure of interest rates.

ii) Write down the stochastic differential equation for the short rate r_t under Q in the Hull-White model.

iii) Indicate whether or not the Hull-White model shows the characteristics listed in (i).

4.

Consider a market with the following properties:

t	$F(t-1,t)$	$B(0,t)$	$R(0,t)$	$C(t)$
0	-	-	-	£100.00
1	2%	(b)	2.0%	£102.02
2	4%	£94.18	(c)	£106.18
3	3%	£91.39	3.0%	(d)
4	(a)	£86.94	3.5%	£115.03

Where:

- t is time.
- *F(s,t)* is the forward rate at time 0 from time *s* to time *t*.
- *B(s,t)* is the price of a zero coupon bond at time *s* maturing at time *t* with a nominal value of £100.
- *R(s,t)* is the spot rate of interest at time s for the period *s* to *t*.

- $C(t)$ is the value of a cash account at time t.

i) Calculate the values of (a), (b), (c) and (d) in the table above.

At time 0 an investor buys £1,000 nominal of zero-coupon bonds maturing at time 2, and £2,000 nominal of zero-coupon bonds maturing at time 4. At time 1 interest rate expectations have changed as set out below.

t	$F(t-1,t)$
1	-
2	5%
3	4%
4	6%

ii) Calculate the loss the investor will make if she sells the bonds at time 1.

The investor decides to keep the bonds rather than selling them at time 1.

iii) Comment on whether the investor can restructure her portfolio to recover her loss if interest rates remain unchanged

5.

i) List the desirable characteristics of a term structure model.

Let $B(t,T)$ be the price at time $t > 0$ of a zero-coupon bond which pays a value of 1 when it matures at time T.

Let $F(t, S,T)$ be the forward rate at time t for a deposit starting at time $S > t$ and expiring at time $T > S$.

Consider the following two investment strategies implemented at time t:

A	At time t: Purchase one zero-coupon bond maturing at time T. Continue to hold the bond to time T.
B	At time t: Purchase $\alpha = e^{-F(t,S,T)(T-S)}$ zero-coupon bonds maturing at time $S < T$. At time S: Invest the redemption amount from the bond at the forward rate $F(t,S,T)$ and continue to hold this deposit to time T.

ii) Show that: $B(t,T) = e^{(-F(t,S,T)(T-S))}B(t,S)$

iii) Derive an expression for $B(t,T)$ in terms of the instantaneous forward rate, using the result from part (ii).

6.

i) State the main potential drawback of the Vasicek model
ii) Discuss the extent to which this drawback may be a problem.
iii) Explain how the Cox-Ingersoll-Ross model avoids this drawback.

The Vasicek term structure model is described by the following stochastic differential equation:

$$dr_t = a(b - r_t)dt + \sigma dW_t,$$

And $a, b, \sigma > 0$

Under this model, the short rate r_t follows a Normal distribution with mean:

$$E(r_t) = r_0 e^{-at} + b(1 - e^{-at})$$

And variance $Var(r_t) = \frac{\sigma^2}{2a}(1 - e^{-2at})$

iv) Assess, using the information provided above, whether the model generates interest rates that are mean reverting and, if so, the value to which they revert.
v) Assess, using the information provided above, the relevance of the parameter a to any mean reversion.

7.

Consider a market with the following bonds in issue.

Principal value	*Expire (years) T*	*Coupon (annual*)*	*Price*	*Zero rate R(0, T)*	*Forward rate F(0, S, T)*
100	0.25	0	97.5	(a)	
100	0.5	0	94.9	(b)	F(0, 0.25, 0.5) = 10.81%
100	1	0	90.0	10.54%	F(0, 0.5, 1) = (d)
100	1.5	8%	(c)	10.68%	F(0, 1, 1.5) = (e)
(* half the stated coupon is paid every 6 months)					

i) Calculate the values of (a), (b), (c), (d), (e) in the table above.

8.
Explain how you would value a swap that is the exchange of a floating rate in one currency for a fixed rate in another currency.

9.
Companies A and B face the following interest rates (adjusted for the differential impact of taxes):

	Company A	*Company B*
U.S. dollars (floating rate)	LIBOR + 0.5%	LIBOR + 1.0%
Canadian dollars (fixed rate)	5.0%	6.5%

Assume that A wants to borrow U.S. dollars at a floating rate of interest and B wants to borrow Canadian dollars at a fixed rate of interest. A financial institution is planning to arrange a swap and requires a 50-basis-point spread. If the swap is equally attractive to A and B, what rates of interest will A and B end up paying?

10.
A currency swap has a remaining life of 15 months. It involves exchanging interest at 10% on £20 million for interest at 6% on $30 million once a year. The term structure of risk free interest rates in the United Kingdom is flat at 7% and the term structure of risk-free interest rates in the United States is flat at 4% (both with annual compounding). The current exchange rate (dollars per pound sterling) is 1.5500. What is the value of the swap to the party paying sterling? What is the value of the swap to the party paying dollars?

11.
A corporate treasurer tells you that he has just negotiated a five-year loan at a competitive fixed rate of interest of 5.2%. The treasurer explains that he achieved the 5.2% rate by borrowing at six-month LIBOR plus 150 basis points and swapping LIBOR for 3.7%. He goes on to say that this was possible because his company has a comparative advantage in the floating-rate market. What has the treasurer overlooked?