



INSTITUTE OF ACTUARIAL
& QUANTITATIVE STUDIES

Subject: Financial
Engineering II

Chapter: Unit 4

Category: Practice Question
Solution

1. The waterfall defines how the cash flows from the underlying assets are allocated to the tranches. In a typical arrangement, cash flows are first used to pay the senior tranches their promised return. The cash flows (if any) that are left over are used to provide the mezzanine tranches with their promised returns. The cash flows (if any) that are left over are then used to provide the equity tranches with their promised returns. Any residual cash flows are used to pay down the principal on the senior tranches.

2.

- (i) Merton's model assumes that a corporate entity has issued both equity and debt such that its total value at time t is of $F(t)$. $F(t)$ varies over time as a result of actions by the corporate entity which does not pay dividends on its equity or coupons on its bonds. Part of the corporate entity's value is zero-coupon debt with a promised repayment amount of L at a future time T . At time T the remainder of the value of the corporate entity will be distributed amongst the equity holders and the corporate entity will be wound up.

The corporate entity will default if the total value of its assets, $F(T)$ is less than the promised debt repayment at time T i.e. $F(T) < L$. In this situation, the bond holders will receive $F(T)$ instead of L and the equity holders will receive nothing. This can be regarded as treating the equity holders of the corporate entity as having a European call option on the assets of the company with maturity T and a strike price equal to the value of the debt.

The Merton model can be used to estimate either the risk-neutral probability that the company will default or the credit spread on the debt.

- (ii) We assume the Merton model, so the value of the company is the value of a call on the assets. The underlying is the gross value and the strike is the debt.

Thus $S_0 = 10.009$, $\sigma = 0.2$, $T = 1$, $K = 8$, and 2.9428 is the value of the call (at time 0).

So, $2.9428 = 10.009\Phi((\ln(10.009/8) + .02 + r)/0.2) - 8e^{-r}\Phi((\ln(10.009/8) - .02 + r)/0.2) = 10.009\Phi(1.2202 + 5r) - 8e^{-r}\Phi(1.0202 + 5r)$. This is a differentiable and increasing function of r so interpolation should get a solution.

Setting $r = 10\%$, we get $10.009\Phi(1.2202 + 5r) - 8e^{-r}\Phi(1.0202 + 5r) = 10.009\Phi(1.7202) - 8e^{-.1}\Phi(1.5202) = 10.009 \times 0.95730 - 8e^{-.1} \times 0.93577 = 2.80786$, so we need to increase r .

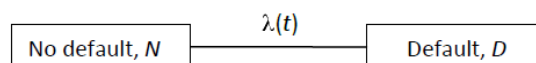
Setting $r = 15\%$, we get $10.009\Phi(1.2202 + 5r) - 8e^{-r}\Phi(1.0202 + 5r) = 10.009\Phi(1.9702) - 8e^{-.15}\Phi(1.7702) = 10.009 \times 0.97559 - 8e^{-.15} \times 0.96166 = 3.14301$, so we need to decrease r .

Interpolating gives $r = 10 + 5X(2.9428 - 2.80786)/(3.14301 - 2.80786)\% = 12\%$.

If we try $r = 12\%$, we get $10.009\Phi(1.2202 + 5r) - 8e^{-r}\Phi(1.0202 + 5r) = 10.009\Phi(1.8202) - 8e^{-.12}\Phi(1.6202) = 10.009 \times 0.96564 - 8e^{-.12} \times 0.94740 = 2.9429$, so $r = 12\%$.

3.

- (i) The model is a continuous time Markov with two states: N (not previously defaulted) and D (previously defaulted). Under this simple model it is assumed that the default-free interest rate term structure is deterministic with $r(t) = r$ for all t . If the transition intensity, under the real-world measure P , from N to D at time t is denoted by $\lambda(t)$, this model can be represented as:



and D is an absorbing state.

If $X(t)$ is the state at time t , the transition intensity, $\lambda(t)$, can be interpreted as:

$$\begin{aligned} \Pr_P(X(t+dt) = N \mid X(t) = N) &= 1 - \lambda(t) dt + o(dt) & \text{as } dt \rightarrow 0, \\ \Pr_P(X(t+dt) = D \mid X(t) = N) &= \lambda(t) dt + o(dt) & \text{as } dt \rightarrow 0. \end{aligned}$$

4. A number of participants are involved in the securitization process. In most cases, the process begins with the originator. This is the institution that owns the pool of assets such as mortgages, auto-loans, or receivables which it wishes to repackage and sell.

UNIT 4

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An issuer will then acquire these assets. The issuer is often a firm specially set up to facilitate the securitization process. It is usually set up as an SPV that is domiciled offshore. For the sale to be considered a “true sale,” the issuer must have a legal identity that is distinct from the originator. The assets being securitized permanently shift from the originator’s balance sheet to that of the SPV.

The SPV is set up such that it is bankruptcy-remote, meaning that the underlying asset pool is held separately from the other assets of the originator.

In effect, the SPV and its assets are protected from insolvency or bankruptcy of the originator. In other words, the originator’s credit rating and financial status become almost irrelevant to the bondholders when assets are held within an SPV framework.



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5.

i) Delinquency rate/ratio refers to the percentage of loans within a financial institution's loan portfolio whose payments are delinquent.

When analyzing and investing in loans, the delinquency rate is an important metric to follow; it is easy to find comprehensive statistics on the delinquencies of all types of loans. Typically, a lender will not report a loan as being delinquent until the borrower has missed two consecutive payments, after which a lender will report to the credit reporting agencies, or "credit bureaus," that the borrower is 60 days late in their payment. If late payments persist, then each month that the borrower is late, the lender may continue reporting the delinquency to the credit agencies for as long as 270 days.

ii) The default rate/ratio is the percentage of all outstanding loans that a lender has written off as unpaid after a prolonged period of missed payments.

The term default rate—also called penalty rate—may also refer to the higher interest rate imposed on a borrower who has missed regular payments on a loan.

An individual loan is typically declared in default if payment is 270 days late. Defaulted loans are typically written off from an issuer's financial statements and transferred to a collection agency.

iii) The debt-service coverage ratio (DSCR) is a measure of the cash flow available to pay current debt obligations.

The debt-service coverage ratio applies to corporate, government, and personal finance. In the context of corporate finance, the debt-service coverage ratio (DSCR) is a measurement of a firm's available cash flow to pay current debt obligations. The DSCR shows investors whether a company has enough income to pay its debts.

$$DSCR = (\text{Net Operating Income}) / (\text{Total Service Debt})$$

where: Net Operating Income = Revenue – COE & COE = Certain operating expenses

Total Debt Service = Current debt obligations

iv) The weighted average coupon (WAC) is a measurement of the rate of return on a pool of mortgages that is sold to investors as a mortgage-backed security (MBS).

The underlying mortgages are repaid at different lengths of time, so the WAC represents its return at the time it was issued and may differ from its WAC later.

The WAC represents the average interest rate of different pools of mortgages with varying interest rates. In the weighted average calculation, the principal balance of each underlying mortgage is used as the weighting factor.

v) Weighted average maturity (WAM) is the weighted average amount of time until the maturities on mortgages in a mortgage-backed security (MBS).

This term is used more broadly to describe maturities in a portfolio of debt securities, including corporate debt and municipal bonds. The higher the WAM, the longer it takes for all of the mortgages or bonds in the portfolio to mature. WAM is used to manage debt portfolios and to assess the performance of debt portfolio managers. WAM is closely related to weighted average loan age (WALA).

vi) The weighted average life (WAL) is the average length of time that each dollar of unpaid principal on a loan, a mortgage, or an amortizing bond remains outstanding.

Calculating WAL shows an investor, an analyst, or a portfolio manager how many years it will take to receive roughly half of the amount of the outstanding principal. The formula is useful in measuring the credit risk associated with fixed-income securities.

The time weightings used in weighted average life calculations are based on payments to the principal. In many loans, such as mortgages, each payment consists of payments to principal and payments to interest. In WAL, only the principal payments are considered and these payments tend to get larger over time, with early payments of a mortgage going mostly to interest, while payments made towards the end of the loan are applied mostly to the principal balance of the loan.

6. A constant prepayment rate (CPR) is an estimate of the percentage of a loan pool's principal that is likely to be paid off prematurely.

The estimate is calculated based on a number of factors, such as historical prepayment rates for previous loans similar to the ones in the pool and future economic outlooks. These calculations are important for investors in evaluating assets like mortgage-backed securities or other securitized bundles of loans.

The higher the CPR, the more prepayments are expected and the less interest the investor is likely to receive in total. This is called prepayment risk.

7. Credit risk is the possibility of a loss resulting from a borrower's failure to repay a loan or meet contractual obligations. Traditionally, it refers to the risk that a lender may not receive the owed principal and interest, which results in an interruption of cash flows and increased costs for collection.

Credit risk is usually ignored with respect to payments by a sovereign government in its own currency, but needs to be accommodated for if an obligation is met in a currency issued by a third-party (such as corporate obligations, obligations by a government in a currency it does not control).

8. Corporate entities issuing bonds consist mainly of large companies and banks. Each lender has its own method for analyzing a borrower's creditworthiness.

The use of the five C's—character, capacity, capital, collateral, and conditions is very common for credit analysis

Character - Character more specifically refers to credit history, which is a borrower's reputation or track record for repaying debts.

This information appears on the borrower's credit reports. Generated by credit bureaus, credit reports contain detailed information about how much an applicant has borrowed in the past and whether they have repaid loans on time.

Capacity - Capacity measures the borrower's ability to repay a loan by comparing income against recurring debts and assessing the borrower's debt-to-income (DTI) ratio. Lenders calculate DTI by adding a borrower's total monthly debt payments and dividing that by the borrower's gross monthly income.

The lower an applicant's DTI, the better the chance of qualifying for a new loan.

Capital - Lenders also consider any capital the borrower puts toward a potential investment. A large contribution by the borrower decreases the chance of default. Borrowers who can put a down payment on a home, for example, typically find it easier to receive a mortgage.

Down payments indicate the borrower's level of seriousness, which can make lenders more comfortable extending credit. Down payment size can also affect the rates and terms of a borrower's loan. Generally speaking, larger down payments result in better rates and terms.

Collateral - Collateral can help a borrower secure loans. It gives the lender the assurance that if the borrower defaults on the loan, the lender can get something back by repossessing the collateral. The collateral is often the object one is borrowing the money for: Auto loans, for instance, are secured by cars, and mortgages are secured by homes.

Conditions - In addition to examining income, lenders look at the length of time an applicant has been employed at their current job and future job stability. Lenders may also consider conditions that are outside of the borrower's control, such as the state of the economy, industry trends, or pending legislative changes.

9. Credit rating refers to a quantified assessment of a borrower's creditworthiness in general terms or with respect to a particular debt or financial obligation. A credit rating can be assigned to any entity that seeks to borrow money—an individual, a corporation, a state or provincial authority, or a sovereign government.

A borrower's credit rating should play a role in determining which lenders to apply to for a loan. The right lender for someone with great credit likely will be different than for someone with good or even poor credit.

Credit ratings also play a large role in a potential investor's decision as to whether or not to purchase bonds. A poor credit rating is a risky investment. That's because it indicates a larger probability that the company will be unable to make its bond payments.

The ratings are used in structured finance transactions such as asset-backed securities, mortgage-backed securities, and collateralized debt obligations. Rating agencies focus on the type of pool underlying the security and the proposed capital structure to rate structured financial products. The issuers of the structured products pay rating agencies to not only rate them, but also to advise them on how to structure the tranches.

Rating agencies also give ratings to sovereign borrowers, who are the largest borrowers in most financial markets. Sovereign borrowers include national governments, state governments, municipalities, and other sovereign-supported institutions. The sovereign ratings given by a rating agency shows a sovereign's ability to repay its debt.

10. Credit Risk can be majorly classified as Credit Default Risk, Concentration Risk and Country Risk.

Credit Default Risk: Credit default risk occurs when the borrower is unable to pay

the loan obligation in full or when the borrower is already 90 days past the due date of the loan repayment. The credit default risk may affect all credit-sensitive financial transactions such as loans, bonds, securities, and derivatives.

Concentration Risk: Concentration risk is the level of risk that arises from exposure to a single counterparty or sector, and it offers the potential to produce large amounts of losses that may threaten the lender's core operations. The risk results from the observation that more concentrated portfolios lack diversification, and therefore, the returns on the underlying assets are more correlated.

Country Risk: Country risk is the risk that occurs when a country freezes foreign currency payments obligations, resulting in a default on its obligations. The risk is associated with the country's political instability and macroeconomic performance, which may adversely affect the value of its assets or operating profits. The changes in the business environment will affect all companies operating within a particular country

11. Exposure at default (EAD) is the total value a bank is exposed to when a loan defaults.

Using the internal ratings-based (IRB) approach, financial institutions calculate their risk. Banks often use internal risk management default models to estimate respective EAD systems. Outside of the banking industry, EAD is known as credit exposure.

EAD, along with loss given default (LGD) and the probability of default (PD), are used to calculate the credit risk capital of financial institutions.

A bank may calculate its expected loss by multiplying the variable, EAD, with the PD and the LGD:

$$\text{EAD} \times \text{PD} \times \text{LGD} = \text{Expected Loss}$$

12. Concentration risk is the potential for a loss in value of an investment portfolio or a financial institution when an individual or group of exposures move together in an unfavorable direction. The implication of concentration risk is that it generates such a significant loss that recovery is unlikely. The portfolio will be liquidated or the institution will face bankruptcy.

It can be analysed by:

- Use concentration indices (e.g., concentration portfolio, Gini coefficient,

Herfindahl-Hirschman index, Hannah-Kay index, Hall-Tideman index, and Theil entropy index) to measure the level of concentration in the portfolio

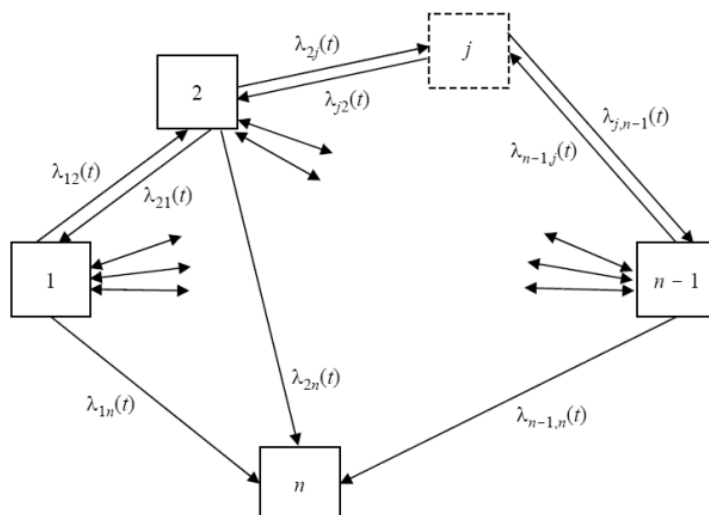
- Use transaction cost analysis to estimate market impact
- Incorporate turnover constraints into portfolio analysis
- Determine the impact from “what if” scenarios
- Stress test the extreme events

13. Risk adjusted pricing in the credit market refers to the offering of different interest rates and loan terms to different consumers based on their creditworthiness.

Risk-based pricing methodologies allow lenders to use credit profile characteristics to charge borrowers interest rates that vary by credit quality. Thus, not all borrowers for a single product will receive the same interest rate and credit terms.

14.

(i)



The n states represent $n - 1$ credit ratings plus default.

$\lambda_{ij}(t)$ are the deterministic transition intensities from state i to state j at time t under the real world measure P .

(ii) (a) $h'(t) = 2p_1'(t) - p_2'(t) = -3p_1(t) + 3/2p_2(t) = -3/2h(t).$

Similarly

$$k'(t) = 2p_1'(t) + p_2'(t) = -p_1(t) - 1/2p_2(t) = -1/2k(t).$$

- (b) Solving these linear differential equations with initial conditions $h(0) = -1$ and $k(0) = 1$ we get $h(t) = -e^{-3/2t}$ and $k(t) = e^{-1/2t}$.

It follows that $p_1(t) = 1/4(h(t) + k(t)) = 1/4(e^{-1/2t} - e^{-3/2t})$
while $p_2(t) = 1/2(k(t) - h(t)) = 1/2(e^{-1/2t} + e^{-3/2t})$.

Now, since $p_3(t) = 1 - p_1(t) - p_2(t)$,
we obtain $p_3(t) = 1 - 3/4e^{-1/2t} - 1/4e^{-3/2t}$.

And so $p_3(2) = .71164$.

- (iv) (a) The bond price is thus $e^{-.04} (1 - p_3(2))£100 + p_3(2)£60 = £68.729$.