INSTITUTE OF ACTUARIAL & QUANTITATIVE STUDIES

Subject: Fixed Income Products

Chapter: Unit 1 & 2

Category: Assignment 1 solutions

Answer 1.

Bond A

$$\frac{2}{(1.03)^1}+\frac{2}{(1.03)^2}+\frac{2}{(1.03)^3}+\frac{2}{(1.03)^4}+\frac{2}{(1.03)^5}+\frac{102}{(1.03)^6}=94.583$$

Bond A is trading at a discount. Its price is below par value because the coupon rate per period (2%) is less than the required yield per period (3%). The deficiency per period is the coupon rate minus the market discount rate, times the par value: (0.02 – 0.03) × 100 = –1. The present value of deficiency is –5.417, discounted using the required yield (market discount rate) per period.

$$\frac{-1}{(1.03)^1}+\frac{-1}{(1.03)^2}+\frac{-1}{(1.03)^3}+\frac{-1}{(1.03)^4}+\frac{-1}{(1.03)^5}+\frac{-1}{(1.03)^6}=-5.417$$

The amount of the deficiency can be used to calculate the price of the bond; the price is 94.583 (= 100 – 5.417).

Bond B

$$\frac{6}{(1.04)^1}+\frac{6}{(1.04)^2}+\frac{6}{(1.04)^3}+\frac{106}{(1.04)^4}=107.260$$

Bond B is trading at a premium because the coupon rate per period (6%) is greater than the market discount rate per period (4%). The excess per period is the coupon rate minus market discount rate, times the par value: (0.06 – 0.04) × 100 =+2. The present value of excess is+7.260, discounted using the required yield per period.

$$\frac{2}{(1.04)^1}+\frac{2}{(1.04)^2}+\frac{2}{(1.04)^3}+\frac{2}{(1.04)^4}=7.260$$

The price of the bond is 107.260 (= 100 + 7.260).

Bond C

$$\frac{5}{(1.05)^1}+\frac{5}{(1.05)^2}+\frac{5}{(1.05)^3}+\frac{5}{(1.05)^4}+\frac{105}{(1.05)^5}=100.000$$

Bond C is trading at par value because the coupon rate is equal to the market discount rate. The coupon payments are neither excessive nor deficient given the risk of the bond.

Bond D

$$\frac{100}{(1.02)^{10}}=82.035$$

Bond D is a zero-coupon bond, which always will trade at a discount below par value (as long as the required yield is greater than zero). The deficiency in the coupon payments is –2 per period: (0 – 0.02) × 100 = –2.

$$\frac{-2}{(1.02)^1}+\frac{-2}{(1.02)^2}+\frac{-2}{(1.02)^3}+\frac{-2}{(1.02)^4}+\frac{-2}{(1.02)^5}+$$
$$\frac{-2}{(1.02)^6}+\frac{-2}{(1.02)^7}+\frac{-2}{(1.02)^8}+\frac{-2}{(1.02)^9}+\frac{-2}{(1.02)^{10}}=-17.965$$

The price of the bond is 82.035 (= 100 – 17.965).

Answer 2.

(i)

$$96.50=\frac{2.25}{(1+r)^1}+\frac{2.25}{(1+r)^2}+\frac{2.25}{(1+r)^3}+\frac{2.25}{(1+r)^4}+$$
$$\frac{2.25}{(1+r)^5}+\frac{102.25}{(1+r)^6},\qquad r=0.02894$$

Bond B is trading at a discount, so the yield-to-maturity per period (2.894%) must be higher than the coupon rate per period (2.25%).

(ii)

$$22.375=\frac{100}{(1+r)^{60}},\qquad r=0.02527$$

Bond C is a zero-coupon bond trading at a significant discount below par value. Its yield-to-maturity is 2.527% per period.

Answer 3:

Spot Rates A

$$\frac{3}{(1.0039)^1}+\frac{3}{(1.0140)^2}+\frac{3}{(1.0250)^3}+\frac{103}{(1.0360)^4}=$$
$$2.988+2.918+2.786+89.412=98.104$$

Given spot rates A, the four-year, 3% bond is priced at 98.104.

$$98.104=\frac{3}{(1+r)^1}+\frac{3}{(1+r)^2}+\frac{3}{(1+r)^3}+\frac{103}{(1+r)^4},\qquad r=0.03516$$

The yield-to-maturity is 3.516%.

Spot Rates B

$$\frac{3}{(1.0408)^1}+\frac{3}{(1.0401)^2}+\frac{3}{(1.0370)^3}+\frac{103}{(1.0350)^4}=$$
$$2.882+2.773+2.690+89.759=98.104$$

$$98.104=\frac{3}{(1+r)^1}+\frac{3}{(1+r)^2}+\frac{3}{(1+r)^3}+\frac{103}{(1+r)^4}, \qquad r=0.03516$$

Given spot rates B, the four-year, 3% bond is again priced at 98.104 to yield 3.516%.

This example demonstrates that two very diff erent sequences of spot rates can result in the same bond price and yield-to-maturity. Spot rates A are increasing for longer maturities, whereas spot rates B are decreasing.

Answer 4:

Given the 30/360 day-count convention assumption, there are 89 days between the last coupon on 19 March 2015 and the settlement date on 18 June 2015 (11 days between 19 March and 30 March, plus 60 days for the full months of April and May, plus 18 days in June). Therefore, the fraction of the coupon period that has gone by is assumed to be 89/180. At the beginning of the period, there are 11.5 years (and 23 semiannual periods) to maturity.

(A) Stated annual yield-to-maturity of 6.00%, or 3.00% per semiannual period:

The price at the beginning of the period is par value, as expected, because the coupon rate and the market discount rate are equal.

$$PV=\frac{3}{(1.0300)^1}+\frac{3}{(1.0300)^2}+\cdots+\frac{103}{(1.0300)^{23}}=100.000000$$

PV^{Full} = 100.000000 × $(1.0300)^{89/180}$ = 101.472251

$$AI=\frac{89}{180}\times 3=1.4833333$$

$$PV^{Flat}=101.472251-1.483333=99.988918$$

The flat price of the bond is a little below par value, even though the coupon rate and the yield-to-maturity are equal, because the accrued interest does not take into account the time value of money. The accrued interest is the interest earned by the owner of the bond for the time between the last coupon payment and the settlement date, 1.483333 per 100 of par value. However, that interest income is not received until the next coupon date. In theory, the accrued interest should be the present value of 1.483333. In practice, however, accounting and financial reporting need to consider issues of practicality and materiality. For those reasons, the calculation of accrued interest in practice neglects the time value of money. Therefore, compared to theory, the reported accrued interest is a little "too high" and the fl at price is a little "too low." The full price, however, is correct because it is the sum of the present values of the future cash flows, discounted using the market discount rate.

(B) Stated annual yield-to-maturity of 6.20%, or 3.10% per semiannual period:

$$PV = \frac{3}{(1.0310)^1} + \frac{3}{(1.0310)^2} + \cdots + \frac{103}{(1.0310)^{23}} = 98.372607$$

PV ^{Full} = 98.372607 × (1.0310) $^{89/180}$ = 99.868805

$$AI = \frac{89}{180} \times 3 = 1.4833333$$

$$PV^{Flat} = 99.868805 - 1.483333 = 98.385472$$

The accrued interest is the same in each case because it does not depend on the yield-to-maturity. The differences in the flat prices indicate the differences in the rate of return that is required by investors.

Answer 5:

The stated annual yield-to-maturity on a semiannual bond basis is 4.96% (0.0248 × 2 = 0.0496).

$$98 = \frac{2.25}{(1+r)^1} + \frac{2.25}{(1+r)^2} + \frac{2.25}{(1+r)^3} + \frac{2.25}{(1+r)^4} + \frac{2.25}{(1+r)^5} + \frac{2.25}{(1+r)^6} + \frac{2.25}{(1+r)^7} + \frac{2.25}{(1+r)^8} + \frac{2.25}{(1+r)^9} + \frac{102.25}{(1+r)^{10}}, \quad r = 0.0248$$

A. Convert 4.96% from a periodicity of two to a periodicity of four:

$$\left(1 + \frac{0.0496}{2}\right)^2 = \left(1 + \frac{APR_4}{4}\right)^4, \quad APR_4 = 0.0493$$

The annual percentage rate of 4.96% for compounding semiannually compares with 4.93% for compounding quarterly. That makes sense because increasing the frequency of compounding lowers the annual rate.

B. Convert 4.96% from a periodicity of two to a periodicity of one:

$$\left(1 + \frac{0.0496}{2}\right)^2 = \left(1 + \frac{APR_1}{1}\right)^1, \quad APR_1 = 0.0502$$

The annual rate of 4.96% for compounding semiannually compares with an effective annual rate of 5.02%. Converting from more frequent to less frequent compounding entails raising the annual percentage rate.

Answer 6:

$$\frac{(\text{Index}+QM)\times FV}{m}=\frac{(0.0200+0.0125)\times 100}{4}=0.8125$$

The discount margin can be estimated by solving for DM in this equation.

$$98=\frac{0.8125}{\left(1+\frac{0.0200+DM}{4}\right)^1}+\frac{0.8125}{\left(1+\frac{0.0200+DM}{4}\right)^2}+\cdots+\frac{0.8125+100}{\left(1+\frac{0.0200+DM}{4}\right)^{16}}$$

$$98=\frac{0.8125}{(1+r)^1}+\frac{0.8125}{(1+r)^2}+\cdots+\frac{0.8125+100}{(1+r)^{16}},\qquad r=0.009478$$

$$0.009478=\frac{0.0200+DM}{4},\qquad DM=0.01791$$

The quoted margin is 125 bps over the Euribor reference rate. Using the simplified FRN pricing model, it is estimated that investors require a 179.1 bp spread for the fl oater to be priced at par value.

Answer 7:

Solution A:

Use Equation 9 to get the price per 100 of face value, where FV= 100, Days= 180, Year = 365, and DR = 0.0436.

$$PV=100\times\left(1-\frac{180}{365}\times 0.0436\right)=97.850$$

Use Equation 12 to get the bond equivalent yield, where Year = 365, Days = 180, FV = 100, and PV = 97.850.

$$AOR=\left(\frac{365}{180}\right)\times\left(\frac{100-97.850}{97.850}\right)=0.04456$$

The bond equivalent yield for Bond B is 4.456%.

Solution B

The quoted rate for Bond D of 4.45% is a bond equivalent yield, which is defined as an add-on rate for a 365-day year. If the risk of these money market instruments is the same, Bond A offers the highest rate of return on a bond equivalent yield basis, 4.487%.

Answer 8:

Solution to 1:

The "1y1y" implied forward rate is 3.419%. In Equation 14 , A = 2 (periods), B = 4 (periods), B – B A = 2 (periods), z_2 = 0.02548/2 (per period), and z_4 = 0.02983/2 (per period).

$$\left(1+\frac{0.02548}{2}\right)^2 \times \left(1+IFR_{2,2}\right)^2 = \left(1+\frac{0.02983}{2}\right)^4, \quad IFR_{2,2} = 0.017095,$$
$$\times 2 = 0.03419$$

The "2y1y" implied forward rate is 2.707%. In Equation 14, A = 4 (periods), B = 6 (periods), B – A = 2 (periods), z_4 = 0.02983/2 (per period), and z_6 = 0.02891/2 (per period).

$$\left(1+\frac{0.02983}{2}\right)^4 \times \left(1+IFR_{4,2}\right)^2 = \left(1+\frac{0.02891}{2}\right)^6, \quad IFR_{4,2} = 0.013536,$$
$$\times 2 = 0.02707$$

Solution to 2:

B is correct. The investor's view is that the one-year yield in two years will be greater than or equal to 2.707%. The "2y1y" implied forward rate of 2.707% is the breakeven reinvestment rate. If the investor expects the one-year rate in two years to be less than that, the investor would prefer to buy the three-year zero. If the investor expects the one-year rate in two years to be greater than 2.707%, the investor might prefer to buy the two-year zero and reinvest the cash flow.

Answer 9:

A. An indenture is the contract between the company and its bondholders and contains the bond's covenants.

Answer 10:

C. This pattern describes a deferred-coupon bond. The first payment of $229.25 is the value of the accrued coupon payments for the first three years.

Answer 11:

B. In a best-efforts offering, the investment bank or banks do not underwrite (i.e., purchase all of) a bond issue, but rather sell the bonds on a commission basis. Bonds sold by auction are offered directly to buyers by the issuer (typically a government).

Answer 12:

B. The repo rate is the percentage difference between the repurchase price and the amount borrowed. The repo margin or haircut is the percentage difference between the amount borrowed and the value of the collateral.