IAQS | INSTITUTE OF ACTUARIAL & QUANTITATIVE STUDIES

**Subject:** Fixed Income Products

**Chapter:** Unit 3 & 4

**Category:** Assignment 2

**1.** Assume that the 3.75% US Treasury bond that matures on 15 August 2041 is priced to yield 5.14% for settlement on 15 October 2014. Coupons are paid semiannually on 15 February and 15 August. The yield-to-maturity is stated on a street-convention semiannual bond basis. This settlement date is 61 days into a 184-day coupon period, using the actual/actual day-count convention. Compute the approximate modified duration and the approximate Macaulay duration for this Treasury bond assuming a 5 bp change in the yield-to-maturity.

**2.** Defined-benefit pension schemes typically pay retirees a monthly amount based on their wage level at the time of retirement. The amount could be fixed in nominal terms or indexed to inflation. These programs are referred to as "defined-benefit pension plans" when US GAAP or IFRS accounting standards are used. In Australia, they are called "superannuation funds."

A British defined-benefit pension scheme seeks to measure the sensitivity of its retirement obligations to market interest rate changes. The pension scheme manager hires an actuarial consultancy to model the present value of its liabilities under three interest rate scenarios: (1) a base rate of 5%; (2) a 100 bp increase in rates, up to 6%; and (3) a 100 bp drop in rates, down to 4%. The actuarial consultancy uses a complex valuation model that includes assumptions about employee retention, early retirement, wage growth, mortality, and longevity. The following chart shows the results of the analysis.

| Interest Rate Assumption | Present Value of Liabilities |
|---|---|
| 4% | GBP973.5 million |
| 5% | GBP926.1 million |
| 6% | GBP871.8 million |

Compute the effective duration of the pension scheme's liabilities.

**3.** A hedge fund specializes in investments in emerging market sovereign debt. The fund manager believes that the implied default probabilities are too high, which means that the bonds are viewed as "cheap" and the credit spreads are too high. The hedge fund plans to take a position on one of these available bonds.

| Bond | Time-to-Maturity | Coupon Rate | Price | Yield-to-Maturity |
|---|---|---|---|---|
| (A) | 10 years | 10% | 58.075279 | 20% |
| (B) | 20 years | 10% | 51.304203 | 20% |
| (C) | 30 years | 10% | 50.210636 | 20% |

The coupon payments are annual. The yields-to-maturity are effective annual rates. The prices are per 100 of par value.

1) Compute the approximate modified duration of each of the three bonds using a 1 bp change in the yield-to-maturity and keeping precision to six decimals (because approximate duration statistics are very sensitive to rounding).

2) Which of the three bonds is expected to have the highest percentage price increase if the yield-to-maturity on each decreases by the same amount—for instance, by 10 bps from 20% to 19.90%?

**4.** An investment fund owns the following portfolio of three fixed-rate government bonds:

| | Bond A | Bond B | Bond C |
|---|---|---|---|
| Par value | EUR25,000,000 | EUR25,000,000 | EUR50,000,000 |
| Coupon rate | 9% | 11% | 8% |
| Time-to-maturity | 6 years | 8 years | 12 years |
| Yield-to-maturity | 9.10% | 9.38% | 9.62% |
| Market value | EUR24,886,343 | EUR27,243,887 | EUR44,306,787 |
| Macaulay duration | 4.761 | 5.633 | 7.652 |

The total market value of the portfolio is EUR96,437,017. Each bond is on a coupon date so that there is no accrued interest. The market values are the full prices given the par value. Coupons are paid semiannually. The yields-to-maturity are stated on a semiannual bond basis, meaning an annual rate for a periodicity of 2. The Macaulay durations are annualized.

1) Calculate the average (annual) modified duration for the portfolio using the shares of market value as the weights.

2) Estimate the percentage loss in the portfolio's market value if the (annual) yield-to-maturity on each bond goes up by 20 bps.

**5.** An Italian bank holds a large position in a 7.25% annual coupon payment corporate bond that matures on 4 April 2029. The bond's yield-to-maturity is 7.44% for settlement on 27 June 2014, stated as an effective annual rate. That settlement date is 83 days into the 360-day year using the 30/360 method of counting days.

1) Calculate the full price of the bond per 100 of par value.

2) Calculate the approximate modified duration and approximate convexity using a 1 bp increase and decrease in the yield-to-maturity.

3) Calculate the estimated convexity-adjusted percentage price change resulting from a 100 bp increase in the yield-to-maturity.

4) Compare the estimated percentage price change with the actual change, assuming the yield-to-maturity jumps to 8.44% on that settlement date.

**6.** A fixed-income analyst is asked to rank three bonds in terms of interest rate risk. Interest rate risk here means the potential price decrease on a percentage basis given a sudden change in financial market conditions. The increases in the yields-to-maturity represent the "worst case" for the scenario being considered.

| Bond | Modified Duration | Convexity | ΔYield |
|---|---|---|---|
| A | 3.72 | 12.1 | 25 bps |
| B | 5.81 | 40.7 | 15 bps |
| C | 12.39 | 158.0 | 10 bps |

The modified duration and convexity statistics are annualized. ΔYield is the increase in the annual yield-to-maturity. Rank the bonds in terms of interest rate risk



**7.** An insurance company has liabilities of £6 million due in 8 years' time and £11 million due in 15 years' time. The assets consist of two zero-coupon bonds, one paying £X in 5 years' time and the other paying £Y in 20 years' time. The current interest rate is 8% per annum effective. The insurance company wishes to ensure that it is immunised against small changes in the rate of interest.

(i) Determine the values of £X and £Y such that the first two conditions for Redington's immunisation are satisfied.

(ii) Demonstrate that the third condition for Redington's immunisation is also satisfied.

**8.** A company has the following liabilities:

• annuity payments of £200,000 per annum to be paid annually in arrear for the next 20 years

• a lump sum of £300,000 to be paid in 15 years

The company wishes to invest in two fixed-interest securities in order to immunise its liabilities.

Security A has a coupon rate of 9% per annum and a term to redemption of 12 years.

Security B has a coupon rate of 4% per annum and a term to redemption of 30 years.

Both securities are redeemable at par and pay coupons annually in arrears. The rate of interest is 8% per annum effective.

(i) Calculate the present value of the liabilities.

(ii) Calculate the discounted mean term of the liabilities.

(iii) Calculate the nominal amount of each security that should be purchased so that Redington's first two conditions for immunisation against small changes in the rate of interest are satisfied for this company.

(iv) Describe the further calculations that will be necessary to determine whether the company is immunised against small changes in the rate of interest.

**9.** A company has liabilities of £10 million due in three years' time and £20 million due in six years' time. The investment manager for the company is able to buy zero coupon bonds for whatever term he requires and has adequate monies at his disposal.

(i) Explain whether it is possible for the investment manager to immunise the fund against small changes in the rate of interest by purchasing a single zero coupon bond.

The investment manager decides to purchase two zero-coupon bonds, one for a term of four years and the other for a term of 20 years. The current interest rate is 4% per annum effective.

(ii) Calculate the amount that must be invested in each bond in order that the company is immunised against small changes in the rate of interest. You should demonstrate that all three Redington conditions are met.

**10.** (i) State the conditions that are necessary for an insurance company to be immunised from small, uniform changes in the rate of interest.

An insurance company has liabilities to pay £100m annually in arrear for the next 40 years. In order to meet these liabilities, the insurance company can invest in zero coupon bonds with terms to redemption of five years and 40 years.

(ii) (a) Calculate the present value of the liabilities at a rate of interest of 4% per annum effective.

(b) Calculate the duration of the liabilities at a rate of interest of 4% per annum effective.

(iii) Calculate the nominal amount of each bond that the fund needs to hold so that the first two conditions for immunisation are met at a rate of interest of 4% per annum effective.

(iv) (a) Estimate, using your calculations in (ii) (b), the revised present value of the liabilities if there were a reduction in interest rates by 1.5% per annum effective.

(b) Calculate the present value of the liabilities at a rate of interest of 2.5% per annum effective.

(c) Comment on your results to (iv) (a) and (iv) (b).