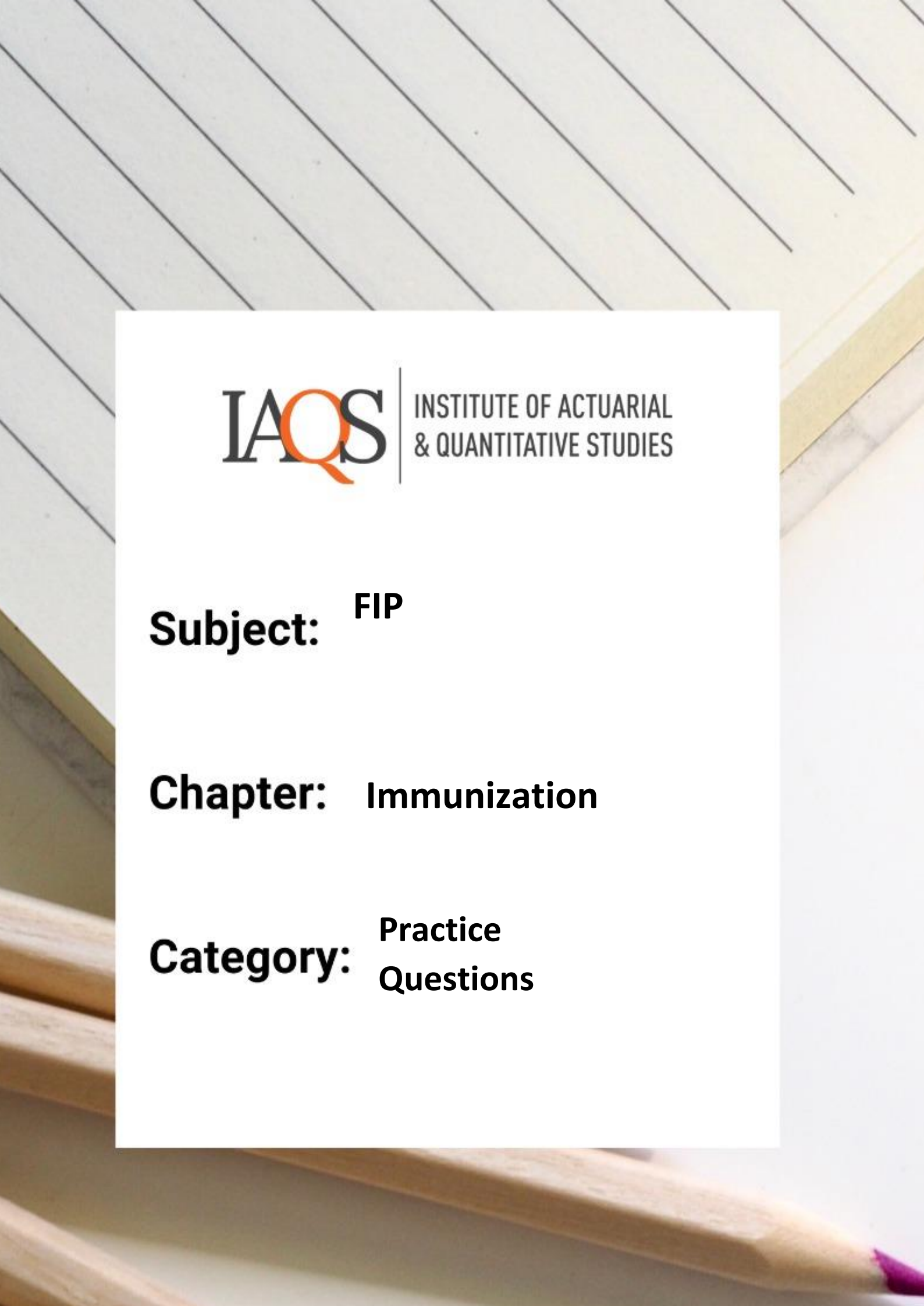

IAQS | INSTITUTE OF ACTUARIAL & QUANTITATIVE STUDIES

Subject: FIP

Chapter: Immunization

Category: Practice Questions

1. CT1 April 2018 Q8

An investment fund has liabilities of £20 million due in 8 years' time and £15 million due in 12 years' time.

The manager wishes to immunise the fund against small changes in the rate of interest and seeks to achieve this by purchasing two zero-coupon bonds. One bond is for a term of exactly 7 years and the other bond is for a term of exactly 14 years. The current interest rate is 4.5% per annum effective.

(i) Calculate the amount that should be invested in each bond, demonstrating that all three Redington conditions are met. [9]

(ii) Explain, without performing any further calculations, how the relative values of the assets and the liabilities will change if the interest rate changes immediately to 4.7% per annum effective. [2]
[Total 11]

2. CT1 September 2018 Q10 (part)

(i) Describe the characteristics of a repayment mortgage. [3]

A bank has just granted a loan of $10,000 to a business to be repaid in ten equal instalments, annually in arrears. The rate of interest is 4% per annum effective.

(ii) (a) Calculate the amount of the annual repayment.
(b) Calculate the duration (discounted mean term) of the repayments.
[5]

The bank wishes to immunise itself from changes in interest rates in relation to this particular asset. For this purpose, the bank has issued two zero-coupon bonds. The first bond is of nominal amount $5,000 and has a term to redemption of two years.

(iii) Determine the nominal amount of the second zero-coupon bond and its term to redemption such that the present value and durations of the assets and liabilities are equal.

3. CT1 April 2017 Q5

An investment fund has liabilities of £11 million due in 7 years' time and £8.084 million in 11 years' time.

The manager of the fund will meet the liabilities by investing in zero-coupon bonds. The manager is able to buy zero-coupon bonds for whatever term is required and there are adequate funds at the manager's disposal.
(i) Explain whether it is possible for the manager to immunise the fund against small changes in the rate of interest by purchasing a single zero-coupon bond. [2]

The manager decides to purchase two zero-coupon bonds, one paying £15.363 million in 7.5 years' time and the other paying £3.787 million in 14.25 years' time. The current interest rate is 5.5% per annum effective.

(ii) Determine whether the investment fund satisfies the necessary conditions to be immunised against small changes in the rate of interest. [7]
[Total 9]

4. CT1 September 2017 Q10

An insurance company has liabilities of £100 million due in 10 years' time and £200 million due in 20 years' time.

The company's assets consist of a zero-coupon bond and a level annuity paid annually in arrear. The zero coupon bond will pay £144.054 million in 15 years' time. The current interest rate is 3% per annum effective at all terms to redemption.

Redington's first two conditions for immunisation against small changes in the rate of interest have been satisfied for this insurance company.

(i) (a) Calculate the present value of the liabilities.
(b) Calculate the discounted mean term of the liabilities. [4]

(ii) Show that the term of the annuity is 41 years. [6]

(iii) Determine the annual rate of payment of the annuity. [1]

(iv) State Redington's third condition for immunisation, explaining whether you think it is fulfilled. [2]

The insurance company decides to sell the zero-coupon bond it holds and invest the proceeds in another zero-coupon bond with a shorter term to maturity.

(v) Explain the risks of implementing this decision. [2] [Total 15]

5. CT1 April 2016 Q2

An insurance company has liabilities of £6 million due in exactly 8 years' time and a further £11 million due in exactly 15 years' time.

The assets held by the insurance company consist of:
- a 5-year zero-coupon bond of nominal amount £5.5088 million; and
- a 20-year zero-coupon bond of nominal amount £13.7969 million.

The current rate of interest is 8% per annum effective at all durations.

(i) Show that the first two conditions of Redington's theory for immunisation against small changes in the rate of interest are satisfied. [5]

(ii) Explain, without doing any further calculations, whether the insurance company will be immunised against small changes in the rate of interest. [2]
[Total 7]

6. CT1 April 2014 Q6

An insurance company has liabilities of £10 million due in 10 years' time and £20 million due in 15 years' time. The company's assets consist of two zero-coupon bonds. One pays £7.404 million in 2 years' time and the other pays £31.834 million in 25 years' time. The current interest rate is 7% per annum effective.

(i) Show that Redington's first two conditions for immunisation against small changes in the rate of interest are satisfied for this insurance company. [6]

(ii) Calculate the present value of profit that the insurance company will make if the interest rate increases immediately to 7.5% per annum effective. [2]

(iii) Explain, without any further calculation, why the insurance company made a profit as a result of the change in the interest rate. [2]
[Total 10]

7. CT1 April 2013 Q7

An insurance company has liabilities of £6 million due in 8 years' time and £11 million due in 15 years' time. The assets consist of two zero-coupon bonds, one paying £*X* in 5 years' time and the other paying £*Y* in 20 years' time. The current interest rate is 8% per annum effective. The insurance company wishes to ensure that it is immunised against small changes in the rate of interest.

(i) Determine the values of £*X* and £*Y* such that the first two conditions for Redington's immunisation are satisfied. [8]

(ii) Demonstrate that the third condition for Redington's immunisation is also satisfied. [2]
[Total 10]

8. CT1 April 2012 Q10

A company has the following liabilities:
• annuity payments of £200,000 per annum to be paid annually in arrear for the next 20 years
• a lump sum of £300,000 to be paid in 15 years

The company wishes to invest in two fixed-interest securities in order to immunise its liabilities.
Security A has a coupon rate of 9% per annum and a term to redemption of 12 years.
Security B has a coupon rate of 4% per annum and a term to redemption of 30 years.
Both securities are redeemable at par and pay coupons annually in arrear. The rate of interest is 8% per annum effective.

(i) Calculate the present value of the liabilities. [3]

(ii) Calculate the discounted mean term of the liabilities. [4]

(iii) Calculate the nominal amount of each security that should be purchased so that Redington's first two conditions for immunisation against small changes in the rate of interest are satisfied for this company. [8]

(iv) Describe the further calculations that will be necessary to determine whether the company is immunised against small changes in the rate of interest. [2]
[Total 17]

9. CT1 April 2011 Q8

A company has liabilities of £10 million due in three years' time and £20 million due in six years' time. The investment manager for the company is able to buy zerocoupon bonds for whatever term he requires and has adequate monies at his disposal.

(i) Explain whether it is possible for the investment manager to immunise the fund against small changes in the rate of interest by purchasing a single zerocoupon bond. [2]

The investment manager decides to purchase two zero-coupon bonds, one for a term of four years and the other for a term of 20 years. The current interest rate is 4% per annum effective.

(ii) Calculate the amount that must be invested in each bond in order that the company is immunised against small changes in the rate of interest. You should demonstrate that all three Redington conditions are met. [10]
[Total 12]

10. CT1 September 2011 Q8

(i) State the conditions that are necessary for an insurance company to be immunised from small, uniform changes in the rate of interest. [2]

An insurance company has liabilities to pay £100m annually in arrear for the next 40 years. In order to meet these liabilities, the insurance company can invest in zero coupon bonds with terms to redemption of five years and 40 years.

(ii) (a) Calculate the present value of the liabilities at a rate of interest of 4% per annum effective.
(b) Calculate the duration of the liabilities at a rate of interest of 4% per annum effective. [5]

(iii) Calculate the nominal amount of each bond that the fund needs to hold so that the first two conditions for immunisation are met at a rate of interest of 4% per annum effective. [5]

(iv) (a) Estimate, using your calculations in (ii) (b), the revised present value of the liabilities if there were a reduction in interest rates by 1.5% per annum effective.
(b) Calculate the present value of the liabilities at a rate of interest of 2.5% per annum effective.
(c) Comment on your results to (iv) (a) and (iv) (b). [6]
[Total 18]