Lecture

INSTITUTE OF ACTUARIAL & QUANTITATIVE STUDIES

Class: MSc

Subject : Fixed Income Products

Subject Code:

Chapter: Unit 2 Chapter 1

Chapter Name: Fixed Income Valuation

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1 Introduction

- The fixed-income market is a key source of financing for businesses and governments. In fact, the total market value outstanding of corporate and government bonds is significantly larger than that of equity securities.

- Similarly, the fixed-income market, which is also called the debt market or bond market, represents a significant investing opportunity for institutions as well as individuals.

- Clearly, understanding how to value fixed-income securities is important to investors, issuers, and financial analysts.

2 Bond Pricing with Market Discount Rate

- The value of a bond is equal to the present value of its coupon payments plus the present value of the maturity value.
- The higher the discount rate, the lower a cash flow's present value
- Since the value of a security is the present value of the cash flows, the higher the discount rate, the lower a security's value.

2.1 Formula

- Formula for calculating bond price using market discount rate:

$$PV = \frac{PMT}{(1+r)^1} + \frac{PMT}{(1+r)^2} + \cdots + \frac{PMT + FV}{(1+r)^N}$$

Where,

PV= present value, or the price of the bond
PMT= coupon payment per period
FV= future value paid at maturity, or the par value of the bond
r= market discount rate, or required rate of return per period
N= number of evenly spaced periods to maturity

2.1 Example

- A 1-year, semi-annual-pay bond has a Rs10,000 face value and a 10% coupon.
- At a discount rate of 8%, the bond value is Rs10,190 (premium).
- At a discount rate of 10%, the bond value is $10,000 (par).
- At a discount rate of 12%, the bond value is $9820 (discount

2.1 Example

The coupon rate on a bond is 4% and the payment is made once a year. If the time-to-maturity is five years and the market discount rate is 6%, The price of the bond per 100 of par value will be:

The par value is the amount of principal on the bond:

$$\frac{4}{(1.06)^1} + \frac{4}{(1.06)^2} + \frac{4}{(1.06)^3} + \frac{4}{(1.06)^4} + \frac{104}{(1.06)^5}$$

= 3.774 + 3.560 + 3.358 + 3.168 + 77.715
= 91.575

2.2 Yield-to-Maturity

The yield-to-maturity is the internal rate of return on the cash flows—the uniform interest rate such that when the future cash flows are discounted at that rate, the sum of the present values equals the price of the bond. It is the implied market discount rate.

The yield-to-maturity is the rate of return on the bond to an investor given three critical assumptions:

1. The investor holds the bond to maturity.
2. The issuer makes all of the coupon and principal payments in the full amount on the scheduled dates. Therefore, the yield-to-maturity is the promised yield—the yield assuming the issuer does not default on any of the payments.
3. The investor is able to reinvest coupon payments at that same yield. This is a characteristic of an internal rate of return.

2.2 Example

A Japanese institutional investor owns a three-year, 2.5% semiannual payment bond having a par value of JPY 100 million. The bond currently is priced at JPY 98.175,677. Calculate the yield per semiannual period(r)

$$98.175677 = \frac{1.25}{(1+r)^1} + \frac{1.25}{(1+r)^2} + \frac{1.25}{(1+r)^3} + \frac{1.25}{(1+r)^4} + \frac{1.25}{(1+r)^5} + \frac{101.25}{(1+r)^6}$$

The yield per semiannual period turns out to be 1.571% (r= 0.01571), which can be annualized to be 3.142% (0.01571 × 2 = 0.03142).

In general, a three-year, 2.5% semiannual bond for any amount of par value has an annualized yield-to-maturity of 3.142% if it is priced at y 98.175677% of par value.

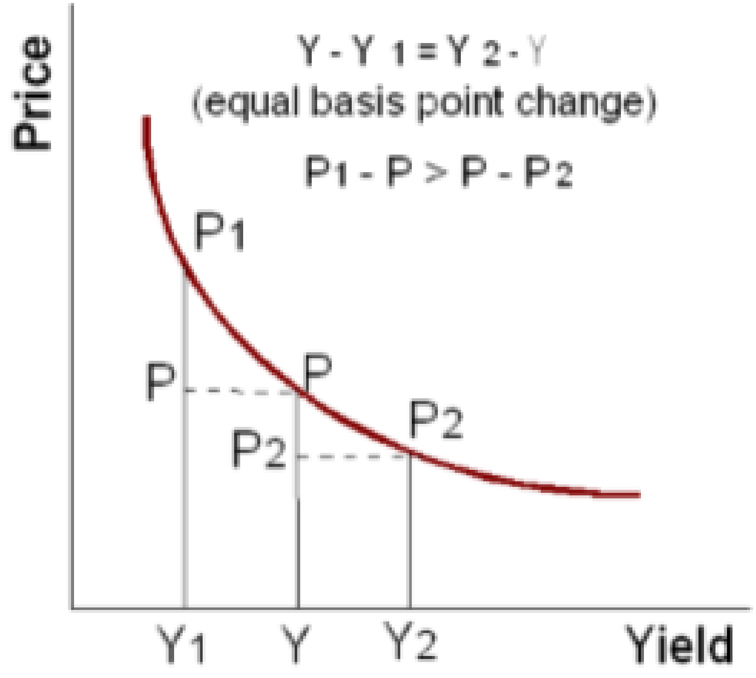
2.3 The Bond Price and Bond Characteristics

The price of a fixed-rate bond will change whenever the market discount rate changes. Four relationships about the change in the bond price given the market discount rate are:

1. The bond price is inversely related to the market discount rate. When the market discount rate increases, the bond price decreases (the inverse effect).
2. For the same coupon rate and time-to-maturity, the percentage price change is greater (in absolute value, meaning without regard to the sign of the change) when the market discount rate goes down than when it goes up (the convexity effect).
3. For the same time-to-maturity, a lower-coupon bond has a greater percentage price change than a higher-coupon bond when their market discount rates change by the same amount (the coupon effect).
4. Generally, for the same coupon rate, a longer-term bond has a greater percentage price change than a shorter-term bond when their market discount rates change by the same amount (the maturity effect).

2.3.1 Convexity

- The degree of price change is not always the same for a particular bond.
- The price/yield relationship for an option free bond is convex.
- Not a straight-line relationship.
- For a given change in yield, the price increases by more than it decreases. P1 – P > P – P2.

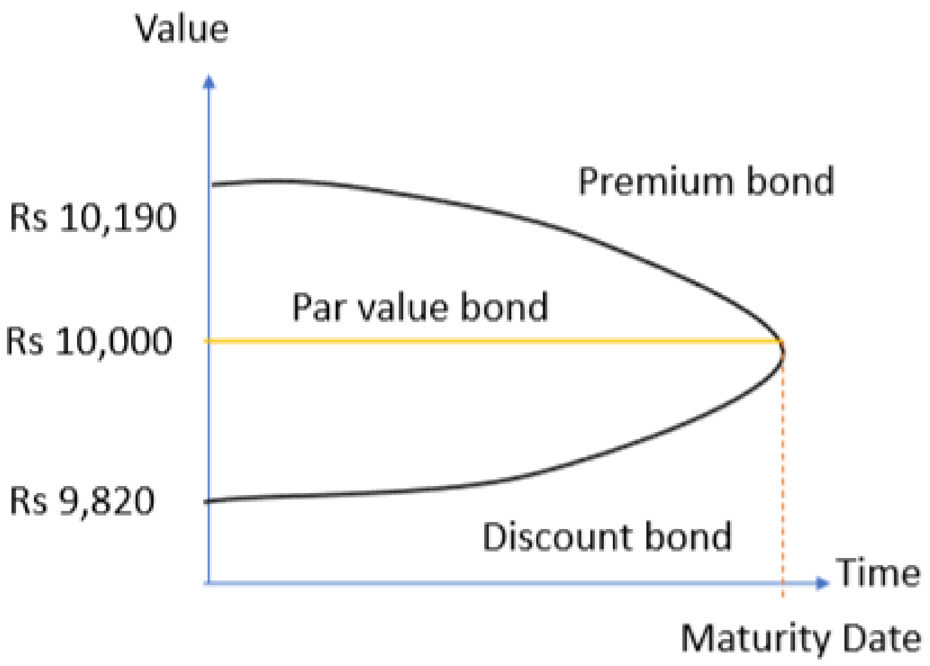


2.3.2 Constant-Yield Price Trajectory

- Bond prices change as time passes even if the market discount rate remains the same. As time passes, the bondholder comes closer to receiving the par value at maturity. The constant-yield price trajectory illustrates the change in the price of a fixed-income bond over time.

- Assuming that the discount rate does not change, a bond's value:
  1. decreases over time if the bond is selling at a premium
  2. increases over time if the bond is selling at a discount
  3. is unchanged if the bond is selling at par value

2.3.2 Constant-Yield Price Trajectory



At the maturity date, the bond's value is equal to its par value ("pull to par value").

2.4 Pricing Bonds with Spot Rates

Spot rates are yields-to-maturity on zero-coupon bonds maturing at the date of each cash flow. Sometimes these are called “zero rates.”

Bond price (or value) determined using the spot rates is sometimes referred to as the bond’s “no-arbitrage value.”

- The arbitrage-free approach values a bond as a package of cash flows,
- Each cash flow viewed as a zero-coupon bond and discounted at its own unique discount rate.
- These spot rates are used to discount cash flows to get the arbitrage-free value of a bond.

2.4.1 Formula

Formula for calculating a bond price given the sequence of spot rates:

$$PV = \frac{PMT}{(1+Z_1)^1} + \frac{PMT}{(1+Z_2)^2} + \cdots + \frac{PMT + FV}{(1+Z_N)^N}$$

where ,

Z_1 = spot rate, or the zero-coupon yield, or zero rate, for Period 1

Z_2 = spot rate, or the zero-coupon yield, or zero rate, for Period 2

Z_N = spot rate, or the zero-coupon yield, or zero rate, for Period N

2.4 Example

The one-year spot rate is 2%, the two-year spot rate is 3%, and the three-year spot rate is 4%. Then, the price of a three-year bond with par value 100 is:

$$\frac{5}{(1.02)^1} + \frac{5}{(1.03)^2} + \frac{105}{(1.04)^3} =$$

= 4.902 + 4.713 + 93.345
= 102.960

3 Prices & Yields

When investors purchase shares, they pay the quoted price. For bonds, however, there can be a difference between the quoted price and the price paid.
How does this difference arise ? & How to calculate these prices will be explained in detail in this section.

3.1 Flat Price, Accrued Interest, and the Full Price

When a bond is between coupon payment dates, its price has two parts:

The flat price (PV^{Flat}) i.e the quoted or “clean” price and the accrued interest (AI). The sum of the parts is the full price (PV^{Full}) i.e the invoice or “dirty” price.

$$PV^{Flat} + AI = PV^{Full}$$

3.1 Flat Price, Accrued Interest, and the Full Price

Accrued interest is the proportional share of the next coupon payment. Assume that the coupon period has " T" days between payment dates and that "t" days have gone by since the last payment. The accrued interest is calculated:

$$AI = \frac{t}{T} \times PMT$$

where ,
t = number of days from the last coupon payment to the settlement date
T= number of days in the coupon period
t/T = fraction of the coupon period that has gone by since the last payment
PMT= coupon payment per period

3.1 Flat Price, Accrued Interest, and the Full Price

The full price of a fixed-rate bond between coupon payments given the market discount rate per period (r) can be calculated :

$$PV^{Full} = \frac{PMT}{(1+r)^{1-t/T}} + \frac{PMT}{(1+r)^{2-t/T}} + \cdots + \frac{PMT + FV}{(1+r)^{N-t/T}}$$

The next coupon payment (PMT) is T discounted for the remainder of the coupon period, which is 1 – t/T .
The second coupon payment is discounted for that fraction plus another full period, 2 – t/T.

By multiplying the numerator and denominator by the expression $(1 + r)^{t/T}$ in the above equation we get:

$$PV^{Full} = \left[\frac{PMT}{(1+r)^1} + \frac{PMT}{(1+r)^2} + \cdots + \frac{PMT + FV}{(1+r)^N}\right] \times (1+r)^{t/T}$$

$= PV \times (1 + r)^{t/T}$

3.1 Example

A 5% semiannual coupon payment government bond that matures on 15 February 2024. Accrued interest on this bond uses the actual/actual day-count convention. The coupon payments are made on 15 February and 15 August of each year. The bond is to be priced for settlement on 14 May 2015. That date is 88 days into the 181-day period.

There are actually 88 days from the last coupon on 15 February to 14 May and 181 days between 15 February and the next coupon on 15 August.
The annual yield-to-maturity is stated to be 4.80%. That corresponds to a market discount rate of 2.40% per semiannual period.
As of the beginning of the coupon period on 15 February 2015, there would be 18 evenly spaced semiannual periods until maturity.

3.1 Example

Here,
PMT= 2.5, N= 18, FV= 100, and r= 0.0240.

$$PV = \frac{2.5}{(1.0240)^1} + \frac{2.5}{(1.0240)^2} + \cdots + \frac{102.5}{(1.0240)^{18}} = 101.447790$$

The price of the bond would be 101.447790 per 100 of par value if its yield-to-maturity is 2.40% per period on the last coupon payment date. This is not the actual price for the bond on that date. It is a “what-if” price using the required yield that corresponds to the settlement date of 14 May 2015.

The full price for the bond:

$PV^{Full} = 101.447790 \times (1.0240)^{88/181}$

$= 102.624323$

3.1 Example

The accrued interest:

$$AI = 88/181 \times 2.5$$
$$= 1.215470$$

The flat price:

$$PV^{Flat} = PV^{Full} - AI$$
$$= 102.624323 - 1.215470$$
$$= 101.408853$$

3.2 Matrix Pricing

- Some fixed-rate bonds are not actively traded. Therefore, there is no market price available to calculate the rate of return required by investors. The same problem occurs for bonds that are not yet issued. In these situations, it is common to estimate the market discount rate and price based on the quoted or flat prices of more frequently traded comparable bonds.

- These comparable bonds have similar times-to-maturity, coupon rates, and credit quality. This estimation process is called matrix pricing.

- It attempts to categorize bonds with similar features (e.g., type of issuer, credit rating, coupon, maturity, etc.) and apply a general yield level to the entire category of bonds. Typically a required yield over the benchmark rate is estimated.

- It then calculates the approximate price of a specific bond within a category using the derived yield level.

- It represents an educated guess and not an actual offer or trade price.

3.3 Yield measures for fixed rate bonds

- There are many ways to measure the rate of return on a fixed-rate bond investment

- Yield measures typically are annualized. For bonds maturing in more than one year, investors want an annualized and compounded yield-to-maturity. Money market rates on instruments d maturing in one year or less typically are annualized but not compounded.

- An annualized and compounded yield on a fixed-rate bond depends on the assumed number of periods in the year, which is called the periodicity of the annual rate.

- A general formula to convert an annual percentage rate for m periods per year, denoted APR_m, to an annual percentage rate for n periods per year, APR_n

$$\left(1+\frac{APR_m}{m}\right)^m=\left(1+\frac{APR_n}{n}\right)^n$$

3.3 Example

A three-year, 5% semiannual coupon payment corporate bond is priced at 104 per 100 of par value. Its yield-to-maturity is 3.582%, quoted on a semiannual bond basis for a periodicity of two :

$r = 0.01791 \times 2 = 0.03582$

$$104 = \frac{2.5}{(1+r)^1} + \frac{2.5}{(1+r)^2} + \frac{2.5}{(1+r)^3} + \frac{2.5}{(1+r)^4} + \frac{2.5}{(1+r)^5} + \frac{102.5}{(1+r)^6},$$

To compare this bond with others, an analyst converts this annualized yield-to-maturity to quarterly and monthly compounding. That entails using Equation mentioned before to convert from a periodicity of m= 2 to periodicities of n = 4 and n= 12.

$$\left(1+\frac{0.03582}{2}\right)^2 = \left(1+\frac{APR_4}{4}\right)^4, \quad APR_4 = 0.03566$$

$$\left(1+\frac{0.03582}{2}\right)^2 = \left(1+\frac{APR_{12}}{12}\right)^{12}, \quad APR_{12} = 0.03556$$

3.3 Yield measures for fixed rate bonds

- Yield measures that neglect weekends and holidays are quoted on what is called **street convention** . The street convention yield-to-maturity is the internal rate of return on the cash flows assuming the payments are made on the scheduled dates.

- The **true yield-to-maturity** is the internal rate of return on the cash flows using the actual calendar of weekends and bank holidays.

- A **government equivalent yield** is quoted for a corporated bond. A government equivalent yield restates a yield-to-maturity based on 30/360 day-count to one based on actual/actual.

- A yield measure that is commonly quoted for fixed-income bonds is the **current yield**, also called the income or interest yield. The current yield is the sum of the coupon payments received over the year divided by the flat price.

- Sometimes the **simple yield** on a bond is quoted. It is the sum of the coupon payments plus the straight-line amortized share of the gain or loss, divided by the flat price.

3.3 Yield measures for fixed rate bonds

- The **yield-to-worst** is a commonly cited yield measure for fixed-rate callable bonds used by bond dealers and investors.

- The value of the embedded call option is added to the flat price of the bond to get the option-adjusted price.

- The option-adjusted price is used to calculate the **option-adjusted yield** . The option-adjusted yield is the required market discount rate where by the price is adjusted for the value of the embedded option.

3.4 Yield measures for Floating Rate Notes

A floating rate note is an instrument on which interest payments are not fixed they vary from period to period depending on the current level of a reference interest rate.

- A specified yield spread i.e **quoted margin** on the FRN is added to, or subtracted from, the reference rate. The role of the quoted margin is to compensate the investor for the difference in the credit risk of the issuer and that implied by the reference rate.

- The required margin i.e the **discount margin** is the yield spread over, or under, the reference rate such that the FRN is priced at par value on a rate reset date.

3.4.1 Formula

$$PV = \frac{\frac{(\text{Index} + QM) \times FV}{m}}{\left(1 + \frac{\text{Index} + DM}{m}\right)^1} + \frac{\frac{(\text{Index} + QM) \times FV}{m}}{\left(1 + \frac{\text{Index} + DM}{m}\right)^2} + \cdots + \frac{\frac{(\text{Index} + QM) \times FV}{m} + FV}{\left(1 + \frac{\text{Index} + DM}{m}\right)^N}$$

Where,
PV= present value, or the price of the floating-rate note
Index = reference rate, stated as an annual percentage rate
QM= quoted margin, stated as an annual percentage rate
FV= future value paid at maturity, or the par value of the bond
m= periodicity of the floating-rate note, the number of payment periods per year
DM= discount margin, the required margin stated as an annual percentage rate
N= number of evenly spaced periods to maturity

3.4.1 Example

A two-year FRN pays six-month Libor plus 0.50%. Currently, six-month Libor is 1.25%. In Equation given earlier, Index = 0.0125, QM= 0.0050, and m= 2.

$$\frac{(\text{Index} + QM) \times FV}{m} = \frac{(0.0125 + 0.0050) \times 100}{2} = 0.875$$

Suppose that the yield spread required by investors is 40 bps over the reference rate, DM= 0.0040. The assumed discount rate per period is

$$\frac{\text{Index} + DM}{m} = \frac{0.0125 + 0.0040}{2} = 0.00825$$

For N= 4, the FRN is priced at 100.196 per 100 of par value.

$$\frac{0.875}{(1+0.00825)^1} + \frac{0.875}{(1+0.00825)^2} + \frac{0.875}{(1+0.00825)^3} + \frac{0.875+100}{(1+0.00825)^4} = 100.196$$

3.5 Yield measures for Money Market Instruments

There are several important differences in yield measures between the money market and the bond market:

1. Bond yields-to-maturity are annualized and compounded. Yield measures in the money market are annualized but not compounded. Instead, the rate of return on a money market instrument is stated on a simple interest basis.
2. Bond yields-to-maturity can be calculated using standard time-value-of-money analysis and with formulas programmed into a financial calculator. Money market instruments often are quoted using nonstandard interest rates and require different pricing equations than those used for bonds.
3. Bond yields-to-maturity usually are stated for a common periodicity for all times-to-maturity. Money market instruments having different times-to-maturity have different periodicities for the annual rate.

3.5.1 Pricing formula on discount rate basis

The pricing formula for money market instruments quoted on a discount rate basis:

$$PV = FV \times \left(1 - \frac{Days}{Year} \times DR\right)$$

Where ,
PV= present value, or the price of the money market instrument
FV = future value paid at maturity, or the face value of the money market instrument
Days = number of days between settlement and maturity
Year = number of days in the year
DR= discount rate, stated as an annual percentage rate

On rearranging the equation DR can be calculated using the following formula:

$$DR = \left(\frac{Year}{Days}\right) \times \left(\frac{FV - PV}{FV}\right)$$

3.5.1 Example

Calculate the price of a 91-day US Treasury bill (T-bill) with a face value of USD10 million is quoted at a discount rate of 2.25% for an assumed 360-day year. Enter FV = 10,000,000, Days = 91, Year= 360, and DR= 0.0225.

$$PV = 10{,}000{,}000 \times (1 - 91/360 \times 0.0225)$$
$$= 9{,}943{,}125$$

3.5.2 Pricing formula on add-on rate basis.

The pricing formula for money market instruments quoted on an add-on rate basis:

$$PV = \frac{FV}{\left(1 + \frac{\text{Days}}{\text{Year}} \times AOR\right)}$$

Where ,
PV= present value, principal amount, or the price of the money market instrument
FV = future value, or the redemption amount paid at maturity including interest
Days = number of days between settlement and maturity
Year = number of days in the year
AOR = add-on rate, stated as an annual percentage rate

3.5.2 Example

A Canadian pension fund buys a 180-day banker's acceptance (BA) with a quoted add-on rate of 4.38% for a 365-day year. If the initial principal amount is CAD 10 million. What will be the redemption amount due at maturity if PV = 10,000,000, Days = 180, Year = 365, and AOR = 0.0438.

$$FV = 10{,}000{,}000 + (10{,}000{,}000 \times 180/365 \times 0.0438)$$
$$= 10{,}216{,}000$$

3.5.3 Investment Analysis

Investment analysis is made difficult for money market securities because:

(1) some instruments are quoted on a discount rate basis and others on an add-on rate basis and
(2) some are quoted for a 360-day year and others for a 365-day year. Another difference is that the "amount" of a money market instrument quoted on a discount rate basis typically is the face value paid at maturity. However, the "amount" when quoted on an add-on rate basis usually is the principal, the price at issuance.

To make money market investment decisions, it is essential to compare instruments on a common basis

4 The Maturity structure of interest rates

There are many reasons why the yields-to-maturity on any two bonds are different. The following are some possible reasons for the difference between the yields:

- Currency—Bond X could be denominated in a currency with a higher expected rate of inflation than the currency in which Bond Y is denominated.
- Credit risk—Bond X could have a non-investment-grade rating of BB, and Bond Y could have an investment-grade rating of AA.
- Liquidity—Bond X could be illiquid, and Bond Y could be actively traded.
- Tax status—Interest income on Bond X could be taxable, whereas interest income on Bond Y could be exempt from taxation.
- Periodicity—Bond X could make a single annual coupon payment, and its yield-to-maturity could be quoted for a periodicity of one. Bond Y could make monthly coupon payments, and its yield-to-maturity could be annualized for a periodicity of 12.

This factor explaining the differences in yields is called the **maturity structure** , or term structure , of interest rates.

4.1 Defining a few terms

A **yield curve** shows yields by maturity. Yield curves are constructed for yields of various types and it's very important to understand exactly which yield is being shown. The term structure of interest rates refers to the yields at different maturities (terms) for like securities or interest rates.

The **spot rate yield curve** is also referred to as the zero curve or strip curve . Spot rates are the appropriate yields, and therefore appropriate discount rates, for single payments to be made in the future. Spot rates are usually quoted on a semiannual bond basis.

A **yield curve for coupon bonds** shows the YTMs for coupon bonds at various maturities. Yields are calculated for several maturities and yields for bonds with maturities between these are estimated by linear interpolation. Yields are expressed on a semiannual bond basis.

A **par bond yield curve**, or par curve, is not calculated from yields on actual bonds but is constructed from the spot curve. The yields reflect the coupon rate that a hypothetical bond at each maturity would need to have to be priced at par.

4.1 Defining a few terms

Forward rates are yields for future periods. The rate of interest on a 1-year loan that would be made two years from now is a forward rate. A **forward yield curve** shows the future rates for bonds or money market securities for the same maturities for annual periods in the future.
1y1y is the rate for a 1-year loan one year from now; 2y1y is the rate for a 1-year loan to be made two years from now; 3y2y is the 2-year forward rate three years from now; and so on.

4.2 Short-Term Forward Rates and Spot Rates

A general formula for the relationship between the two spot rates and the forward rate.

$$(1 + z_A)^A \times (1 + IFR_{A,B-A})^{B-A} = (1 + z_B)B$$

5 Yield spreads

A yield spread, in general, is the difference in yield between different fixed income securities.

5.1 Yield Spreads over Benchmark Rates

5.1 Yield Spreads over Benchmark Rates

Benchmark yield:

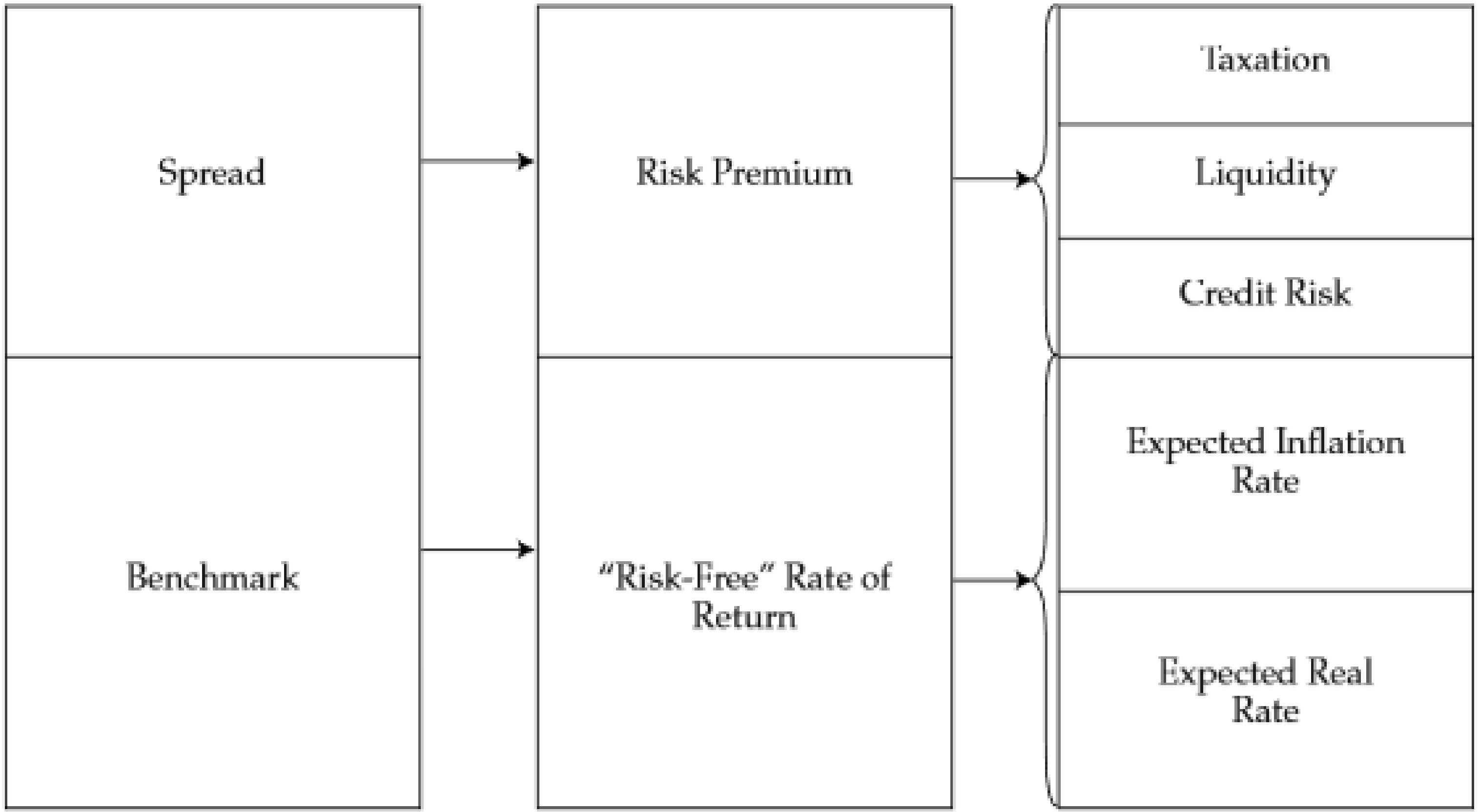
- The benchmark yield for a fixed-income security with a given time-to-maturity is the base rate, often a government bond yield.
- The benchmark captures the macroeconomic factors: the expected rate of inflation in the currency in which the bond is denominated, general economic growth and the business cycle, foreign exchange rates, and the impact of monetary and fiscal policy.
- Changes in those factors impact all bonds in the market, and the effect is seen mostly in changes in the benchmark yield.
- The benchmark is often called the risk-free rate of return. Also, the benchmark can be broken down into the expected real rate and the expected inflation rate in the economy.

5.1 Yield Spreads over Benchmark Rates

Spread:

- The spread is the difference between the yield-to-maturity and the benchmark.
- The spread captures the microeconomic factors specific to the bond issuer and the bond itself: credit risk of the issuer and changes in the quality rating on the bond, liquidity and trading in comparable securities, and the tax status of the bond.
- The yield spread is called the risk premium over the “risk-free” rate of return. The risk premium provides the investor with compensation for the credit and liquidity risks, and possibly the tax impact of holding a specific bond.

5.1 Yield Spreads over Benchmark Rates



5.1 Yield Spreads over Benchmark Rates

- This benchmark is usually the most recently issued government bond and is called the on-the-run security. The on-the-run government bond is the most actively traded security and has a coupon rate closest to the current market discount rate for that maturity. That implies that it is priced close to par value. Seasoned government bonds are called off –the run .

- The yield spread over a specific benchmark is referred to as the benchmark spread and is usually measured in basis points.

- The yield spread in basis points over an actual or interpolated government bond is known as the G-spread.

- The yield spread of a specific bond over the standard swap rate in that currency of the same tenor is known as the I-spread or interpolated spread to the swap curve.

5.2 Yield Spreads over the Benchmark Yield Curve

- A yield curve shows the relationship between yields-to-maturity and times-to-maturity for securities with the same risk profile.
- The swap yield curve shows the relationship between fixed Libor swap rates and their times-to-maturity.
- Benchmark yield curves tend to be upward-sloping because investors typically demand a premium for holding longer-term securities.
- Another approach is to calculate a constant yield spread over a government (or interest rate swap) spot curve instead. This spread is known as the zero volatility spread (Z-spread) of a bond over the benchmark rate.
- The Z-spread over the benchmark spot curve can be calculated using:

$$PV = \frac{PMT}{(1+z_1+Z)^1} + \frac{PMT}{(1+z_2+Z)^2} + \cdots + \frac{PMT+FV}{(1+z_N+Z)^N}$$

5.2 Yield Spreads over the Benchmark Yield Curve

- The benchmark spot rates—z_1, z_2, …, z_N—are derived from the government yield curve (or from fixed rates on interest rate swaps).

- Z is the Z-spread per period and is the same for all time Z periods. In Equation 15, N is an integer, so the calculation is on a coupon date when the accrued interest is zero. Sometimes, the Z-spread is called the static spread because it is constant (and has zero volatility). In practice, the Z-spread is usually calculated in a spreadsheet using a goal seek function or similar solver function.

- The Z-spread is also used to calculate the option-adjusted spread (OAS) on a callable bond. The OAS, like the option-adjusted yield, is based on an option-pricing model and an assumption about future interest rate volatility.

$$\text{OAS} = \text{Z-spread} - \text{Option value (in basis points per year)}$$