INSTITUTE OF ACTUARIAL & QUANTITATIVE STUDIES

Subject: Fixed Income Products

Chapter: Unit 2

Category: Practice questions solutions

Answer 1:

$$\frac{4}{(1.03)^1}+\frac{4}{(1.03)^2}+\frac{4}{(1.03)^3}+\frac{4}{(1.03)^4}+\frac{4}{(1.03)^5}+\frac{104}{(1.03)^6}=105.417$$

Answer 2:

$$103.75=\frac{3.5}{(1+r)^1}+\frac{3.5}{(1+r)^2}+\frac{3.5}{(1+r)^3}+\frac{103.5}{(1+r)^4}, \qquad r=0.02503$$

Answer 3:

Solution to 1: Bond D will go up in price the most on a percentage basis because it has the lowest coupon rate (the coupon effect) and the longer time-to-maturity (the maturity effect). There is no exception to the maturity effect in these bonds because there are no low-coupon bonds trading at a discount.

Solution to 2: Bond C will go down in price the least on a percentage basis because it has the highest coupon rate (the coupon effect) and the shorter time-to-maturity (the maturity effect). There is no exception to the maturity effect because Bonds C and F are priced at a premium above par value.

Answer 4:

Given the 30/360 day-count convention assumption, there are 89 days between the last coupon on 19 March 2015 and the settlement date on 18 June 2015 (11 days between 19 March and 30 March, plus 60 days for the full months of April and May, plus 18 days in June). Therefore, the fraction of the coupon period that has gone by is assumed to be 89/180. At the beginning of the period, there are 11.5 years (and 23 semiannual periods) to maturity.

(A) Stated annual yield-to-maturity of 5.80%, or 2.90% per semiannual period:

$$PV=\frac{3}{(1.0290)^1}+\frac{3}{(1.0290)^2}+\cdots+\frac{103}{(1.0290)^{23}}=101.661589$$

$PV^{Full} = 101.661589 \times (1.0290)^{89/180} = 103.108770$

$$AI=\frac{89}{180}\times 3=1.4833333$$

$$PV^{Flat}=103.108770-1.483333=101.625437$$

Answer 5:

B is correct. The first step is to determine the yields-to-maturity on the observed bonds. The required yield on the three-year, 5.50% bond priced at 107.500 is 2.856%.

$$107.500=\frac{5.50}{(1+r)^1}+\frac{5.50}{(1+r)^2}+\frac{105.50}{(1+r)^3}, \quad r=0.02856$$

The required yield on the five-year, 4.50% bond priced at 104.750 is 3.449%.

$$104.750=\frac{4.50}{(1+r)^1}+\frac{4.50}{(1+r)^2}+\frac{4.50}{(1+r)^3}+\frac{4.50}{(1+r)^4}+\frac{104.50}{(1+r)^5}, \quad r=0.03449$$

The estimated market discount rate for a four-year bond having the same credit quality is the average of two required yields:

$$\frac{0.02856+0.03449}{2}=0.031525$$

Given an estimated yield-to-maturity of 3.1525%, the estimated price of the illiquid four-year, 4.50% annual coupon payment corporate bond is 104.991 per 100 of par value.

$$\frac{4.50}{(1.031525)^1}+\frac{4.50}{(1.031525)^2}+\frac{4.50}{(1.031525)^3}+\frac{104.50}{(1.031525)^4}=104.991$$

Answer 6:

Solution to 1:

Current Yield for Bond A

$$\frac{8}{90}=0.08889$$

Yield-to-Maturity for Bond A

$$90=\frac{4}{(1+r)^1}+\frac{4}{(1+r)^2}+\cdots+\frac{104}{(1+r)^{10}}, \quad r=0.05315, \quad \times 2=0.10630$$

Current Yield for Bond B

$$\frac{12}{105}=0.11429$$

Yield-to-Maturity for Bond B

$$105=\frac{3}{(1+r)^1}+\frac{3}{(1+r)^2}+\cdots+\frac{103}{(1+r)^{20}}, \quad r=0.02674, \quad \times 4=0.10696$$

Solution to 2:

The yield-to-maturity on Bond A of 10.630% is an annual rate for compounding semiannually. The yield-to-maturity on Bond B of 10.696% is an annual rate for compounding quarterly. The difference in the yields is not 6.6 bps (0.10696 – t 0.10630 = 0.00066). It is essential to compare the yields for the same periodicity to make a statement about relative value.

10.630% for a periodicity of two converts to 10.492% for a periodicity of four:

$$\left(1+\frac{0.10630}{2}\right)^2=\left(1+\frac{APR_4}{4}\right)^4, \quad APR_4=0.10492$$

10.696% for a periodicity of four converts to 10.839% for a periodicity of two:

$$\left(1+\frac{0.10696}{4}\right)^4=\left(1+\frac{APR_2}{2}\right)^2, \quad APR_2=0.10839$$

The additional compensation for the greater risk in Bond B is 20.9 bps (0.10839 – 0.10630 = 0.00209) when the yields are stated on a semiannual bond basis. The additional compensation is 20.4 bps (0.10696 – 0.10492 = 0.00204) when both are annualized for quarterly compounding.

Answer 7:

$$\frac{(\text{Index}+QM)\times FV}{m}=\frac{(0.0110+0.0075)\times 100}{4}=0.4625$$

$$95.50=\frac{0.4625}{\left(1+\frac{0.0110+DM}{4}\right)^1}+\frac{0.4625}{\left(1+\frac{0.0110+DM}{4}\right)^2}+\cdots+\frac{0.4625+100}{\left(1+\frac{0.0110+DM}{4}\right)^{20}}$$

$$95.50=\frac{0.4625}{(1+r)^1}+\frac{0.4625}{(1+r)^2}+\cdots+\frac{0.4625+100}{(1+r)^{20}}, \quad r=0.007045$$

$$0.007045=\frac{0.0110+DM}{4}, \quad DM=0.01718$$

Answer 8:

Solution for A:

Use Equation 9 to get the price per 100 of par value, where FV= 100, Days= 180, Year = 360, and DR = 0.0433.

$$PV=100\times\left(1-\frac{180}{360}\times 0.0433\right)=97.835$$

Equation 12 to get the bond equivalent yield, where Year = 365, Days = 180, FV= 100, and PV = 97.835.

$$AOR=\left(\frac{365}{180}\right)\times\left(\frac{100-97.835}{97.835}\right)=0.04487$$

The bond equivalent yield for Bond A is 4.487%.

Solution for B:

First, determine the redemption amount per 100 of principal (PV = 100), where Days = 180, Year= 360, and AOR= 0.0435.

$$FV = 100 + \left(100 \times \frac{180}{360} \times 0.0435\right) = 102.175$$

Use Equation 12 to get the bond equivalent yield, where Year = 365, Days = 180, FV= 102.175, and PV = 100.

$$AOR = \left(\frac{365}{180}\right) \times \left(\frac{102.175 - 100}{100}\right) = 0.04410$$

The bond equivalent yield for Bond C is 4.410%. Another way to get the bond equivalent yield for Bond C is to observe that the AOR of 4.35% for a 360-day year can be obtained using Equation 12 for Year = 360, Days = 180, FV= 102.175, and PV= 100.

$$AOR = \left(\frac{360}{180}\right) \times \left(\frac{102.175 - 100}{100}\right) = 0.0435$$

Therefore, an add-on rate for a 360-day year only needs to be multiplied by the factor of 365/360 to get the 365-day year bond equivalent yield.

$$\frac{365}{360} \times 0.0435 = 0.04410$$

Answer 9:

The one-year par rate is 5.263%.

$$100 = \frac{PMT + 100}{(1.05263)^1}, \quad PMT = 5.263$$

The two-year par rate is 5.606%.

$$100 = \frac{PMT}{(1.05263)^1} + \frac{PMT + 100}{(1.05616)^2}, \quad PMT = 5.606$$

The three-year and four-year par rates are 6.306% and 6.899%, respectively.

$$100 = \frac{PMT}{(1.05263)^1} + \frac{PMT}{(1.05616)^2} + \frac{PMT + 100}{(1.06359)^3}, \quad PMT = 6.306$$

$$100 = \frac{PMT}{(1.05263)^1} + \frac{PMT}{(1.05616)^2} + \frac{PMT}{(1.06359)^3} + \frac{PMT + 100}{(1.07008)^4}, \quad PMT = 6.899$$

Answer 10:

Solution to 1:

The yield-to-maturity for the corporate bond is 5.932%.

$$100.125 = \frac{6}{(1+r)^1} + \frac{106}{(1+r)^2}, \quad r = 0.05932$$

The yield-to-maturity for the government benchmark bond is 3.605%.

$$100.750 = \frac{4}{(1+r)^1} + \frac{104}{(1+r)^2}, \qquad r = 0.03605$$

The G-spread is 232.7 bps: 0.05932 – 0.03605 = 0.02327.

Solution to 2:

Solve for the value of the corporate bond using z_1= 0.0210, z_2 = 0.03635, and Z = 0.023422:

$$\frac{6}{(1+0.0210+0.023422)^1} + \frac{106}{(1+0.03635+0.023422)^2}$$
$$= \frac{6}{(1.044422)^1} + \frac{106}{(1.059772)^2} = 100.125$$

Answer 11:

There are several important differences in yield measures between the money market and the bond market:

1. Bond yields-to-maturity are annualized and compounded. Yield measures in the money market are annualized but not compounded. Instead, the rate of return on a money market instrument is stated on a simple interest basis.
2. Bond yields-to-maturity can be calculated using standard time-value-of-money analysis and with formulas programmed into a financial calculator. Money market instruments often are quoted using nonstandard interest rates and require different pricing equations than those used for bonds.
3. Bond yields-to-maturity usually are stated for a common periodicity for all times-to-maturity. Money market instruments having different times-to-maturity have different periodicities for the annual rate.

Answer 12:

The yield-to-maturity on the bond is calculated as:

N = 20; PMT = 30; FV = 1,000; PV = –1,020; CPT → I/Y = 2.867%

2 × 2.867 = 5.734% = YTMT

To calculate the yield-to-first call, we calculate the yield-to-maturity using the number of semiannual periods until the first call date (10) for N and the call price (1,020) for FV:

N = 10; PMT = 30; FV = 1,020; PV = –1,020; CPT → I/Y = 2.941%

2 × 2.941 = 5.882% = yield-to-first call

To calculate the yield-to-first par call (second call date), we calculate the yield-to-maturity using the number of semiannual periods until the first par call date (16) for N and the call price (1,000) for FV: N = 16; PMT = 30; FV = 1,000; PV = –1,020; CPT → I/Y = 2.843%

2 × 2.843 = 5.686% = yield-to-first par call

The lowest yield, 5.686%, is realized if the bond is called at par on January 1, 2022, so the yield-to-worst is 5.686%.

Answer 13:

$$1.\ \text{annual yield} = \left(1 + \frac{0.10}{2}\right)^2 - 1 = 1.05^2 - 1 = 0.1025 = 10.25\%$$

$$2.\ \text{annual yield} = \left(1 + \frac{0.10}{4}\right)^4 - 1 = 1.025^4 - 1 = 0.1038 = 10.38\%$$

Answer 14:

$$S_3 = [(1.02)(1.03)(1.04)]^{1/3} - 1 = 2.997\%$$

This can be interpreted to mean that a dollar compounded at 2.997% for three years would produce the same ending value as a dollar that earns compound interest of 2% the first year, 3% the next year, and 4% for the third year