INSTITUTE OF ACTUARIAL & QUANTITATIVE STUDIES

Subject: Fixed Income Products

Chapter: Unit 3

Category: Practice questions solutions

Answer 1:

Any capital gain or loss is based on the bond's carrying value at the time of sale, when it has 15 years (30 semiannual periods) to maturity. The carrying value is calculated using the bond's YTM at the time the investor purchased it.

N = 30; I/Y = 3; PMT = 2.5; FV = 100; CPT → PV = –90.20

Because the selling price of 91.40 is greater than the carrying value of 90.20, the investor realizes a capital gain of 91.40 – 90.20 = 1.20 per 100 of face value.

Answer 2:

1) A - The decrease in the YTM to 5.5% will decrease the reinvestment income over the life of the bond so that the investor will earn less than 6%, the YTM at purchase.

2) B - The interest portion of a bond's return is the sum of the coupon payments and interest earned from reinvesting coupon payments over the holding period.

N = 18; PMT = 50 ; PV = 0; I/Y = 5%; CPT → FV = –1,406.62

3) A - The price of the bond after three years that will generate neither a capital gain nor a capital loss is the price if the YTM remains at 7.3%. After three years, the present value of the bond is 800,000 / 1.073^{12} = 343,473.57, so she will have a capital gain relative to the bond's carrying value.

4) A - Key rate duration refers to the sensitivity of a bond or portfolio value to a change in one specific spot rate.

5) C - Other things equal, Macaulay duration is less when yield is higher and when maturity is shorter. The bond with the highest yield and shortest maturity must have the lowest Macaulay duration.

Answer 3:

First we need to find the YTM of the bond:

N = 20; PV = –101.39; PMT = 6; FV = 100; CPT→I/Y = 5.88

Now we need the values for the bond with YTMs of 5.89 and 5.87.

I/Y = 5.89; CPT → PV = –101.273 (V+)

I/Y = 5.87; CPT → PV = –101.507 (V–)

PVBP (per $100 of par value) = (101.507 – 101.273) / 2 = 0.117

For the $1 million par value bond, each 1 basis point change in the yield to maturity will change the bond's price by 0.117 × $1 million × 0.01 = $1,170.

Answer 4:

The duration effect is –9.42 × 0.003 = 0.02826 = –2.826%.

The convexity effect is 0.5 × 68.33 × (0.003)2 = 0.000307 = 0.0307%.

The expected change in bond price is (–0.02826 + 0.000307) = –2.7953%.

Answer 5:

$$PV_0 = 58.075279$$

$$PV_+ = 58.047598$$

$$\frac{10}{(1.2001)^1} + \frac{10}{(1.2001)^2} + \cdots + \frac{110}{(1.2001)^{10}} = 58.047598$$

$$PV_- = 58.102981$$

$$\frac{10}{(1.1999)^1} + \frac{10}{(1.1999)^2} + \cdots + \frac{110}{(1.1999)^{10}} = 58.102981$$

The approximate modified duration of Bond A is 4.768.

$$\text{ApproxModDur} = \frac{58.102981 - 58.047598}{2 \times 0.0001 \times 58.075279} = 4.768$$

Answer 6:

1. The money duration is the annual modified duration times the full price of the bond per 100 of par value.

$$\left(\frac{2.4988}{1+\frac{0.052617}{2}}\right) \times \text{USD}99.650 = \text{USD}242.62$$

2. For each 1 bp increase in the yield-to-maturity, the loss is estimated to be USD0.024262 per 100 of par value: USD242.62 × 0.0001 = USD0.024262.

Given a position size of USD10 million in par value, the estimated loss per basis-point increase in the yield is USD2,426.20. The money duration is per 100 of par value, so the position size of USD10 million is divided by 100.

$$\text{USD}0.024262 \times \frac{\text{USD}10{,}000{,}000}{100} = \text{USD}2{,}426.20$$

Answer 7:

In the calculations, the yield per semiannual period goes up by 2.5 bps to 2.025% and down by 2.5 bps to 1.975%. The 30-year bond has an approximate modified duration of 17.381 and an approximate convexity of 420.80.

$$PV_{+} = \frac{2}{(1.02025)^{1}} + \cdots + \frac{102}{(1.02025)^{60}} = 99.136214$$

$$PV_{-} = \frac{2}{(1.01975)^{1}} + \cdots + \frac{102}{(1.01975)^{60}} = 100.874306$$

$$\text{ApproxModDur} = \frac{100.874306 - 99.136214}{2 \times 0.0005 \times 100} = 17.381$$

$$\text{ApproxCon} = \frac{100.874306 + 99.136214 - (2 \times 100)}{(0.0005)^{2} \times 100} = 420.80$$

The 100-year century bond has an approximate modified duration of 24.527 and an approximate convexity of 1,132.88.

$$PV_{+} = \frac{2}{(1.02025)^{1}} + \cdots + \frac{102}{(1.02025)^{200}} = 98.787829$$

$$PV_{-} = \frac{2}{(1.01975)^{1}} + \cdots + \frac{102}{(1.01975)^{200}} = 101.240493$$

$$\text{ApproxModDur} = \frac{101.240493 - 98.787829}{2 \times 0.0005 \times 100} = 24.527$$

$$\text{ApproxCon} = \frac{101.240493 + 98.787829 - (2 \times 100)}{(0.0005)^{2} \times 100} = 1{,}132.88$$

The century bond offers a higher modified duration—24.527 compared with 17.381— and a much greater degree of convexity—1,132.88 compared with 420.80.

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