Lecture

INSTITUTE OF ACTUARIAL & QUANTITATIVE STUDIES

Class: MSc

Subject : Fixed Income Products

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Chapter: Unit 4 Chapter 1

Chapter Name: Fixed income risk and return – 2

Today's Agenda

1. Percentage change in price
   1. Convexity impact on price

2. Term structure of yield volatility affects the interest rate risk of a bond.

2. Interest rate risk, Macaulay duration & Investment horizon

2. Changes in credit spread and liquidity affect yield-to-maturity

2. Empirical duration and analytical duration

1 Percentage change in price

By taking account of both a bond's duration (the primary measure of risk arising from a change in the yield-to-maturity.) and convexity (the secondary risk measure), we can improve an estimate of the effects of a change in yield on a bond's value, especially for larger changes in yield.

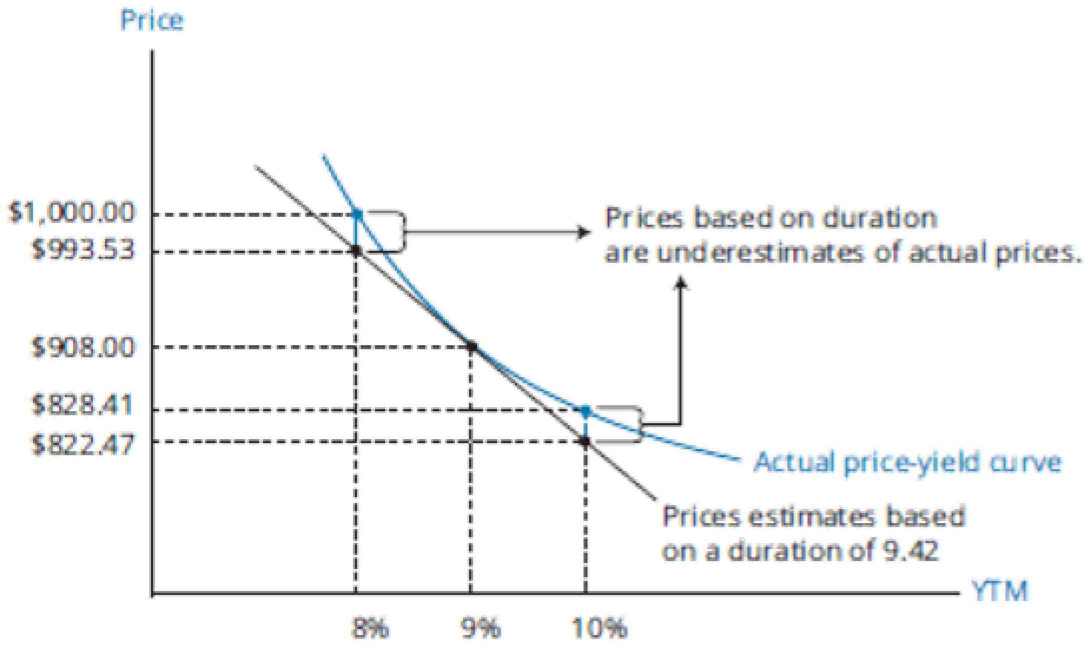
Change in full bond price = –annual modified duration(ΔYTM) + ½ annual convexity($\Delta YTM)^2$

1 Percentage change in price

The convexity adjustment to the price change is the same for both an increase and a decrease in yield.

As illustrated in diagram,

- The duration-only based estimate of the increase in price resulting from a decrease in yield is too low for a bond with positive convexity, and is improved by a positive adjustment for convexity.
- The duration-only based estimate of the decrease in price resulting from an increase in yield is larger than the actual decrease, so it's also improved by a positive adjustment for convexity.



1.1 Convexity Impact on Price (due to change in Yield)

$$\%\Delta PV^{Full} \approx$$

$$(-\text{AnnModDur} \times \Delta\text{Yield}) + \left[\frac{1}{2} \times \text{AnnConvexity} \times (\Delta\text{Yield})^2\right]$$

$$\text{ApproxCon} = \frac{(PV_-) + (PV_+) - [2 \times (PV_0)]}{(\Delta\text{Yield})^2 \times (PV_0)}$$

$$\text{ApproxModDur} = \frac{(PV_-) - (PV_+)}{2 \times (\Delta\text{Yield}) \times (PV_0)}$$

1 Example

A bond has a convexity of 114.6. Calculate the convexity effect, if the yield decreases by 110 basis points

Solution:

Convexity effect $= 1/2 \times \text{convexity} \times (\Delta YTM)^2$

$= (0.5)(114.6)(0.011)^2$

$= 0.00693 = 0.693\%$

2

Term structure of yield volatility affects the interest rate risk of a bond.

The term structure of yield volatility refers to the relation between the volatility of bond yields and their times to maturity.

The volatility of a bond's price has two components:

- The sensitivity of the bond's price to a given change in yield
- The volatility of the bond's yield.

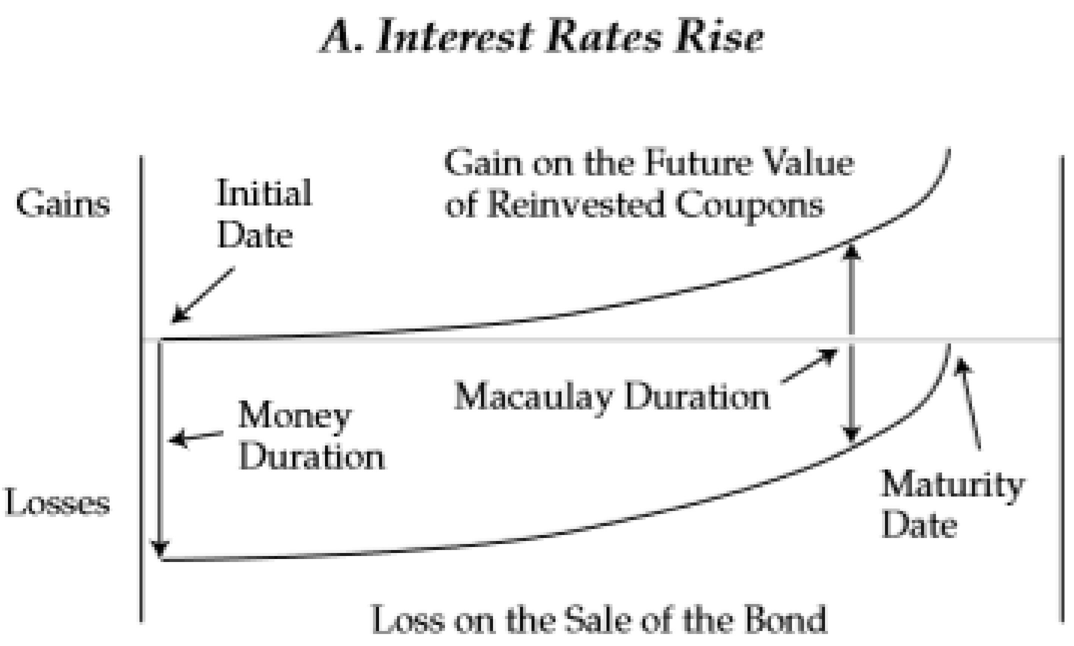
3

Interest rate risk, Macaulay duration & Investment horizon

Macaulay duration has an interesting application in matching a bond to an investor's investment horizon. When the investment horizon and the bond's Macaulay duration are matched, a parallel shift in the yield curve prior to the first coupon payment will not (or will minimally) affect the investor's horizon return.

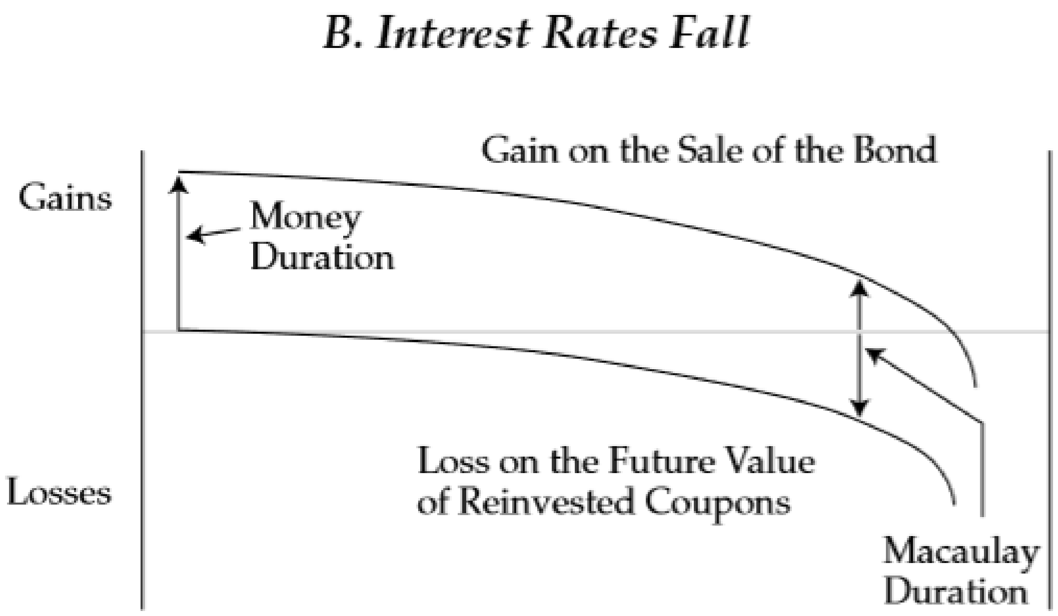
3 Interest rate risk, Macaulay duration & Investment horizon

- When interest rates rise, duration measures the immediate drop in value. In particular, the money duration indicates the change in price.
- Then as time passes, the bond price is "pulled to par."
- The gain in the future value of reinvested coupons starts small but builds over time as more coupons are received.
- The curve indicates the additional future value of reinvested coupons because of the higher interest rate.
- At some point in the lifetime of the bond, those two effects off set each other and the gain on reinvested coupons is equal to the loss on the sale of the bond. That point in time is the Macaulay duration statistic.



3 Interest rate risk, Macaulay duration & Investment horizon

- When interest rates fall, which leads to a reduction in the bond yield and the coupon reinvestment rate.
- There is an immediate jump in the bond price, as measured by the money duration, but then the "pulled to par" effect brings the price down as time passes.
- The impact from reinvesting at a lower rate starts small but then becomes more significant over time. The loss on reinvested coupons is with respect to the future value if interest rates had not fallen.
- Once again, the bond's Macaulay duration indicates the point in time when the two effects off set each other and the gain on the sale of the bond matches the loss on coupon reinvestment.



3

Interest rate risk, Macaulay duration & Investment horizon

A statement of the general relationships among interest rate risk, the Macaulay duration, and the investment horizon.

1. When the investment horizon is greater than the Macaulay duration of a bond, coupon reinvestment risk dominates market price risk. The investor's risk is to lower interest rates.
2. When the investment horizon is equal to the Macaulay duration of a bond, coupon reinvestment risk off sets market price risk.
3. W hen the investment horizon is less than the Macaulay duration of the bond, market price risk dominates coupon reinvestment risk. The investor's risk is to higher interest rates.

3 Interest rate risk, Macaulay duration & Investment horizon

The difference between the Macaulay duration of a bond and the investment horizon is called the duration gap .

- A positive duration gap (Macaulay duration greater than the investment horizon) exposes the investor to market price risk from increasing interest rates.
- A negative duration gap (Macaulay duration less than the investment horizon) exposes the investor to reinvestment risk from decreasing interest rates.

4 Changes in credit spread and liquidity affect yield-to-maturity

The benchmark yield curve's interest rates have two components;

1. the real rate of return and
2. expected inflation.

A bond's spread to the benchmark curve also has two components,

1. A premium for credit risk
2. A premium for lack of liquidity relative to the benchmark securities.

Because we are treating the yields associated with each component as additive, a given increase or decrease in any of these components of yield will increase or decrease the bond's YTM by the same amount.

4

Changes in credit spread and liquidity affect yield-to-maturity

With a direct relationship between a bond's yield spread to the benchmark yield curve and its YTM, we can estimate the impact on a bond's value of a change in spread using the formula we introduced earlier for the price effects of a given change in YTM.

$$\%\Delta \text{ bond value} = -\text{duration}(\Delta \text{ spread}) + \tfrac{1}{2} \text{ convexity}(\Delta \text{ spread})^2$$

5 Empirical duration and analytical duration

The duration measures we have introduced in this reading, based on mathematical analysis, are often referred to as analytical durations.
A different approach is to estimate empirical durations using the historical relationship between benchmark yield changes and bond price changes.

When we estimate corporate bond durations based on a shift in the benchmark (government) yield curve, we implicitly assume that the credit spread for the corporate bond remains unchanged (i.e., changes in the benchmark yield curve and a bond's yield spread are uncorrelated). When this assumption is not justified, estimates of empirical duration, based ont he actual relationship between changes in the benchmark yield curve and bond values, may be more appropriate.