



Subject: Fixed Income Products

Chapter: Unit 4

**Category: Practice questions
solutions**

1. Solution

a. The price value of a basis point is:

$$\$114.1338 - \$114.0051 = \$0.1287$$

b. the approximate percentage price change for a 1 basis point increase in interest rates (i.e., $\Delta y = 0.0001$), ignoring the negative sign is:

$$11.28 \times (0.0001) \times 100 = 0.1128\%$$

Given the initial price of 114.1338, the dollar price change estimated using duration is:

$$0.1128\% \times 114.1338 = \$0.1287$$

2. Solution

a. For a 25 basis point rate shock, the duration formula is:

$$\text{duration} = \frac{V_- - V_+}{2V_0(0.0025)}$$

		5%, 4 year	5%, 25 year	8%, 4 year	8%, 25 year
Initial value	V_0	100.0000	100.0000	110.7552	142.5435
Value at 4.75%	V_-	100.9011	103.6355	111.7138	147.2621
Value at 5.25%	V_+	99.1085	96.5416	109.8066	138.0421
Duration		3.59	14.19	3.44	12.94

b. For a 50 basis point rate shock, the duration formula is:

$$\text{duration} = \frac{V_- - V_+}{2V_0(0.0050)}$$

		5%, 4 year	5%, 25 year	8%, 4 year	8%, 25 year
Initial value	V_0	100.0000	100.0000	110.7552	142.5435
Value at 4.50%	V_-	101.8118	107.4586	112.6826	152.2102
Value at 5.50%	V_+	98.2264	93.2507	108.8679	133.7465
Duration		3.59	14.21	3.44	12.95

3. Solution

a. For a 10 basis point change:

duration for 8% 4-year bond = 3.44

duration for 8% 25-year bond = 12.94

$$\Delta y^* = 0.0010$$

For the 8% 4-year bond: approximate percentage price change for 10 basis point change in yield ($\Delta y^* = 0.0010$):

10 basis point increase:

$$\text{approximate percentage price change} = -3.44 \times (0.0010) \times 100 = -0.34\%$$

10 basis point decrease:

$$\text{approximate percentage price change} = -3.44 \times (-0.0010) \times 100 = +0.34\%$$

For the 8% 25-year bond: approximate percentage price change for 10 basis point change in yield (0.0010):

10 basis point increase:

$$\text{approximate percentage price change} = -12.94 \times (0.0010) \times 100 = -1.29\%$$

10 basis point decrease:

$$\text{approximate percentage price change} = -12.94 \times (-0.0010) \times 100 = +1.29\%$$

b. For the 4-year bond, the estimated percentage price change using duration is excellent for a 10 basis point change, as shown below:

	Duration estimate	Actual change
10 bp increase	-0.34%	-0.34%
10 bp decrease	+0.34%	+0.35%

For the 25-year bond, the estimated percentage price change using duration is excellent for a 10 basis point change, as shown below:

	Duration estimate	Actual change
10 bp increase	-1.29%	-1.28%
10 bp decrease	+1.29%	+1.31%

4. Solution

a. For a 200 basis point change:

$$\text{duration for 8\% 4-year bond} = 3.44$$

$$\text{duration for 8\% 25-year bond} = 12.94$$

$$\Delta y^* = 0.02$$

For the 8% 4-year bond: approximate percentage price change for 200 basis point change in yield ($\Delta y^* = 0.02$):

200 basis point increase:

$$\text{approximate percentage price change} = -3.44 \times (0.02) \times 100 = -6.89\%$$

200 basis point decrease:

$$\text{approximate percentage price change} = -3.44 \times (-0.02) \times 100 = +6.89\%$$

For the 8% 25-year bond: approximate percentage price change for 200 basis point shock:

200 basis point increase:

$$\text{approximate percentage price change} = -12.94 \times (0.02) \times 100 = -25.88\%$$

200 basis point decrease:

$$\text{approximate percentage price change} = -12.94 \times (-0.02) \times 100 = +25.88\%$$

b. For the 4-year bond, the estimated percentage price change using duration is excellent for a 200 basis point change, as shown below:

	Duration estimate	Actual change
200 bp increase	-6.88%	-6.61%
200 bp decrease	+6.88%	+7.19%

For the 25-year bond, the estimated percentage price change using duration is excellent for a 200 basis point change, as shown below:

	Duration estimate	Actual change
200 bp increase	-25.88%	-21.62%
200 bp decrease	+25.88%	+31.54%

5. Solution

a. The convexity adjustment for the two 25-year bonds is:

For the 5% 25-year bond:

$$C = 141.68$$

$$\Delta y^* = 0.02$$

$$\text{convexity adjustment to percentage price change} = 141.68 \times (0.02)^2 \times 100 = 5.67\%$$

For the 8% 25-year bond:

$$C = 121.89$$

$$\text{convexity adjustment to percentage price change} = 121.89 \times (0.02)^2 \times 100 = 4.88\%$$

b. Estimated price change using duration and convexity adjustment.

For the 5% 25 year bond:

$$\text{duration} = 14.19$$

$$\Delta y^* = 0.02$$

$$\text{approximate percentage price change based on duration} = -14.19 \times 0.02 \times 100 = -28.38\%$$

$$\text{convexity adjustment} = 5.67\%$$

Therefore,

Yield Change (Δy_*)	+200 bps
Estimated change using duration	-28.38%
Convexity adjustment	5.67%
Total estimated percentage price change	-22.71%
Yield Change (Δy_*)	-200 bps
Estimated change using duration	28.38%
Convexity adjustment	5.67%
Total estimated percentage price change	34.05%

For the 8% 25-year bond:

duration = 12.94

$\Delta y * = 0.02$

approximate percentage price change based on duration = $-12.94 \times 0.02 \times 100 = -25.88\%$

convexity adjustment = 4.88%

Yield Change (Δy_*)	+200 bps
Estimated change using duration	-25.88%
Convexity adjustment	4.88%
Total estimated percentage price change	-21.00%
Yield Change (Δy_*)	-200 bps
Estimated change using duration	25.88%
Convexity adjustment	4.88%
Total estimated percentage price change	30.76%

c. For a large change in rates of 200 basis points, duration with the convexity adjustment does a pretty good job of estimating the actual percentage price change, as shown below.

	Duration/convexity estimate	Actual change
For 5% 25-year bond		
200 bp increase	-22.71%	-23.46%
200 bp increase	+34.05%	+35.00%
For 8% 25-year bond		
200 bp increase	-21.00%	-21.62%
200 bp increase	+30.76%	+31.54%