1. Two twins Lauren & Mallory both will save $2000 at 12% compounded annually. Mallory begins at age 20 and deposits $2000 a year till age 29, for a total of 10 deposits, then does nothing till retirement at age 65 (36 years). How much will Mallory have at age 65? Lauren begins at age 29 depositing $2000 a year until retirement at age 65 (37 deposits). How much will Lauren have at retirement?

Mallory: First determine the accumulation of the 10 deposits.

$$A = \frac{P\left(\left(1+\frac{r}{m}\right)^{mt}-1\right)}{r/m} = \frac{2000\left((1+0.12)^{10}-1\right)}{0.12} = 35097.47$$ then this is compounded annually for 36 years $\Rightarrow$ $A = 35097.47(1+0.12)^{36} = 2{,}075{,}509.03$.

Lauren: $$A = \frac{P\left(\left(1+\frac{r}{m}\right)^{mt}-1\right)}{r/m} = \frac{2000\left((1+0.12)^{37}-1\right)}{0.12} = 1{,}087{,}197.38.$$

2. Find the present value at time 0 of an annuity–immediate such that the payments start at 1, each payment thereafter increases by 1 until reaching 10, and they remain at that level until 25 payments in total are made. The effective annual rate of interest is 4%.

Solution: The cashflow is

Payments	1	2	3	...	10	10	...	10
Time	1	2	3	...	10	11	...	25

The present value at time 0 of the perpetuity is

$$\begin{aligned}&(Ia)_{\overline{10}|0.04} + (1+0.04)^{-10}10a_{\overline{15}|0.04}\\ =&\frac{\ddot{a}_{\overline{n}|4\%} - 10(1+0.04)^{-10}}{0.04} + (1+0.04)^{-10}(10)a_{\overline{15}|0.04}\\ =&41.99224806 + 75.11184164 = 117.1040897.\end{aligned}$$

3. An perpetuity–immediate provides annual payments. The first payment of 13000 is one year from now. Each subsequent payment is 3.5% more than the one preceding it. The annual effective rate of interest is i = 6%. Find the present value of this perpetuity.

Ans – 520000

4. A 15 year annuity pays 1000 at the end of year 1 and increases by 1000 each year until the payment is 8000 at the end of year 8. Payments then decrease by 1000 each year until a payment of 1000 is paid at the end of year 15. The annual effective interest rate of 6.5%. Compute the present value of this annuity.

Ans - 39482.626.

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5. Chris makes annual deposits into a bank account at the beginning of each year for 10 years. Chris initial deposit is equal to 100, with each subsequent deposit k% greater than the previous year deposit. The bank credits interest at an annual effective rate of 4.5%. At the end of 10 years, the accumulated amount in Chris account is equal to 1657.22. Calculate k

Ans – 6%

6. An annuity provides for 20 annuals payments, the first payment a year hence being $4500. The payments increase in such a way that each payment is 4.5% greater than the previous one. The annual effective rate of interest is 4.5%. Find the present value of this annuity.

Ans - 86124.40191

7. An annuity provides for 10 annuals payments, the first payment a year hence being $2600. The payments increase in such a way that each payment is 3% greater than the previous one. The annual effective rate of interest is 4%. Find the present value of this annuity.

Ans - 23945.54454.

8. An annuity-immediate consists of a first payment of $100, with subsequent payments increased by 10% over the previous one until the 10th payment, after which subsequent payments decreases by 5% over the previous one. If the effective rate of interest is 10% per payment period, what is the present value of this annuity with 20 payments?

Solution: The present value of the first 10 payments is (note that $k = i$)

$$100 \times 10(1.1)^{-1} = \$909.09.$$

For the next 10 payments, $k = -0.05$ and their present value at time 10 is (note that the payment at time 11 is $100(1.10)^9(0.95)$)

$$100(1.10)^9(0.95) \times \frac{1 - \left(\frac{0.95}{1.1}\right)^{10}}{0.1 + 0.05} = 1{,}148.64.$$

Hence, the present value of the 20 payments is

$$909.09 + 1{,}148.64(1.10)^{-10} = \$1{,}351.94.$$

9. A perpetuity-due pays $1000 for the first year and payments increase by 3% for each subsequent year until the 20th payment. After that the payments are the same as the 20th. Find the present value if the effective annual interest rate is 5%.

Ans - $30,641.46

10. Suppose a deposit of $1000 is made on the first of the months of January, February and March, $1,200 at the beginning of each month in the second quarter, $1,400 at the beginning of each month in the third quarter and $1,600 each month in the fourth quarter. If the account has a nominal 8% rate of interest compounded quarterly, what is the balance at the end of the year?

Ans - $16,226.10

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11.

2. **Saving for an Upgrade** A corporation sets up a sinking fund to replace some aging machinery. It deposits $100,000 into the fund at the end of each month for 10 years. The annuity earns 12% interest compounded monthly. The equipment originally cost $13 million. However, the cost of the equipment is rising 6% each year. Will the annuity be adequate to replace the equipment? If not, how much additional money is needed?

AV of deposits = 23003868.9

AV of machine costs = 23281020.1

Extra amount required = 277151

12. An investor receives payments half-yearly in arrears for 20 years. The first payment is £250, and each payment is 2% higher than the previous one. The interest rate is 6% pa effective for the first 10 years and 4% pa effective for the final 10 years. Calculate, showing all workings, the present value of the payments. [6]

We will work in years, with $v_1 = \dfrac{1}{1.06}$ denoting the one-year discount factor applicable in each of the first 10 years, and $v_2 = \dfrac{1}{1.04}$ denoting the one-year discount factor applicable in each of the final 10 years.

The total present value of the payments received is therefore:

$$250v_1^{0.5} + 250(1.02)v_1 + 250(1.02)^2 v_1^{1.5} + \cdots + 250(1.02)^{19} v_1^{10}$$

$$+250(1.02)^{20} v_1^{10} v_2^{0.5} + 250(1.02)^{21} v_1^{10} v_2 + \cdots + 250(1.02)^{39} v_1^{10} v_2^{10} \qquad [2]$$

The terms in the first line of the present value expression form a geometric progression of 20 terms, with first term $250v_1^{0.5}$ and common ratio $(1.02)v_1^{0.5}$, so:

$$250v_1^{0.5} + 250(1.02)v_1 + 250(1.02)^2 v_1^{1.5} + \cdots + 250(1.02)^{19} v_1^{10}$$

$$= 250v_1^{0.5} \frac{\left(1 - \left(1.02v_1^{0.5}\right)^{20}\right)}{1 - 1.02v_1^{0.5}}$$

$$= 4{,}450.86 \qquad [1½]$$

The terms in the second line of the present value expression form a geometric progression of 20 terms, with first term $250(1.02)^{20}v_1^{10}v_2^{0.5}$ and common ratio $(1.02)v_2^{0.5}$, so:

$$250(1.02)^{20}v_1^{10}v_2^{0.5}+250(1.02)^{21}v_1^{10}v_2+\cdots+250(1.02)^{39}v_1^{10}v_2^{10}$$

$$=250(1.02)^{20}v_1^{10}v_2^{0.5}\frac{\left(1-\left(1.02v_2^{0.5}\right)^{20}\right)}{1-1.02v_2^{0.5}}$$

$$=4{,}075.60$$ [1½]

So the total present value of the payments is:

$$4{,}450.86+4{,}075.60=£8{,}526.46$$ [1]

[Total 6]

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FM Annuities

Unit 2