1. At six-month interval, A deposited ₹2000 in a saving account which credit interest at 10% p.a. compounded semi-annually. The first deposit was made when A' s son was six-month-old and the last deposit was made when his son was 8 years old. The money remained in the account and was presented to the son on his 10th birthday. How much did he receive?

Ans - 57511.68

2. An annuity of ₹500 p.a. is flowing continuously for 10 years. Find its future value if the rate of interest is 10% compounded continuously

Solⁿ:- 6.

F.V of an annuity when interest rate is compounding continuously @10%.

$$F.V = \int_0^n R e^{rt}\, dt$$

$$= \int_0^{10} 500 e^{\cdot 10 \times t}\, dt$$

$$= \left[500 \frac{e^{\cdot 10}}{\cdot 10} \right]_0^{10}$$

$$= \frac{500}{\cdot 10} \left[e^{\cdot 10 \times 10} - e^{\cdot 10 \times 0} \right]$$

$$= 5000 \left[2.7183 - 1 \right]$$

$$= 5000 \times 1.7183 = ₹\ 8591.5$$

3. Calculate the present value of an annuity-immediate of amount $100 paid annually for 5 years at the rate of interest of 9% per annum.

Table 2.1: Present value of annuity

Year	Payment ($)	Present value ($)
1	100	$100\,(1.09)^{-1} = 91.74$
2	100	$100\,(1.09)^{-2} = 84.17$
3	100	$100\,(1.09)^{-3} = 77.22$
4	100	$100\,(1.09)^{-4} = 70.84$
5	100	$100\,(1.09)^{-5} = 64.99$
Total		388.97

4. Calculate the present value of an annuity-immediate of amount $100 payable quarterly for 10 years at the annual rate of interest of 8% convertible quarterly. Also calculate its future value at the end of 10 years.

Solution: Note that the rate of interest per payment period (quarter) is $\frac{8}{4}\% = 2\%$, and there are $4 \times 10 = 40$ payments. Thus, from (2.1) the present value of the annuity-immediate is

$$100\, a_{\overline{40}|0.02} = 100 \times \left[\frac{1-(1.02)^{-40}}{0.02}\right] = \$2{,}735.55,$$

and the future value of the annuity-immediate is

$$2{,}735.55 \times (1.02)^{40} = \$6{,}040.20.$$

5. A man borrows a loan of $20,000 to purchase a car at annual rate of interest of 6%. He will pay back the loan through monthly installments over 5 years, with the first installment to be made one month after the release of the loan. What is the monthly installment he needs to pay?

Solution: The rate of interest per payment period is $\frac{6}{12}\% = 0.5\%$. Let P be the monthly installment. As there are $5 \times 12 = 60$ payments, from (2.1) we have

$$\begin{aligned} 20{,}000 &= P\, a_{\overline{60}|0.005} \\ &= P \times \left[\frac{1-(1.005)^{-60}}{0.005}\right] \\ &= P \times 51.7256, \end{aligned}$$

so that

$$P = \frac{20{,}000}{51.7256} = \$386.66.$$

6. A company wants to provide a retirement plan for an employee who is aged 55 now. The plan will provide her with an annuity-immediate of $7,000 every year for 15 years upon her retirement at the age of 65. The company is funding this plan with an annuity-due of 10 years. If the rate of interest is 5%, what is the amount of installment the company should pay?

Solution: We first calculate the present value of the retirement annuity. This is equal to

$$7{,}000\, a_{\overline{15}|} = 7{,}000 \times \left[\frac{1-(1.05)^{-15}}{0.05}\right] = \$72{,}657.61.$$

This amount should be equal to the future value of the company's installments P, which is $P\ddot{s}_{\overline{10}|}$. Now from (2.4), we have

$$\ddot{s}_{\overline{10}|} = \frac{(1.05)^{10}-1}{1-(1.05)^{-1}} = 13.2068,$$

so that

$$P = \frac{72{,}657.61}{13.2068} = \$5{,}501.53.$$

7. Find the present value of an annuity-due of $200 per quarter for 2 years, if interest is compounded monthly at the nominal rate of 8%.

Solution: This is the situation where the payments are made less frequently than interest is converted. We first calculate the effective rate of interest per quarter, which is

$$\left[1+\frac{0.08}{12}\right]^3-1=2.01\%.$$

As there are $n=8$ payments, the required present value is

$$200\,\ddot{a}_{\overline{8}|0.0201}=200\times\left[\frac{1-(1.0201)^{-8}}{1-(1.0201)^{-1}}\right]=\$1{,}493.90.$$

8. Al Bundy says he paid $25,000 down on a new house and will pay $525 per month for 30 years. If interest is 7.8% compounded monthly, what was the selling price of the house?

First calculate the present value of the loan ($ borrowed)

$$V=\frac{P\left(1-\left(1+\frac{r}{m}\right)^{-mt}\right)}{r/m}=\frac{525\left(1-\left(1+\frac{0.078}{12}\right)^{-360}\right)}{0.078/12}=72{,}929.78$$

Then add the down payment: 72929.78 + 25000 = 97929.78.

9. An perpetuity–immediate provides annual payments. The first payment of 13000 is one year from now. Each subsequent payment is 3.5% more than the one preceding it. The annual effective rate of interest is i = 6%. Find the present value of this perpetuity.

Ans – 520000

10. Find the present value of a 15–year decreasing annuity–immediate paying 150000 the first year and decreasing by 10000 each year thereafter. The effective annual interest rate of 4.5%.

Ans - 946767.616.

11. Frasier is 33 years old and just received an inheritance from his parents' estate. He wants to invest an amount of money today such that he can receive $5,000 at the end of every month for 15 years when he retires at age 65. If he can earn 9% compounded annually until age 65 and then 5% compounded annually when the fund is paying out, how much money must he invest today?

Step 2: Ordinary General Annuity (Payment Stage):

Calculate the equivalent periodic interest rate that matches the payment interval (i_{eq}, Formula 9.6), number of annuity payments (n, Formula 11.1), and present value of the ordinary general annuity (PV_{ORD}, Formula 11.3A).

$$i = \frac{I/Y}{C/Y} = \frac{5\%}{1} = 5\%$$

$$i_{eq} = (1+i)^{\frac{C/Y}{P/Y}} - 1 = (1+0.05)^{\frac{1}{12}} - 1 = 0.004074124 \text{ per month}$$

$$n = P/Y \times (\text{Number of Years}) = 12 \times 15 = 180 \text{ payments}$$

$$\begin{aligned} PV_{ORD} &= PMT\left[\frac{1-(1+i)^{-n}}{i}\right] \\ &= \$5{,}000\left[\frac{1-(1+0.004074124)^{-180}}{0.004074124}\right] \\ &= \$5{,}000\left[\frac{0.518982921}{0.004074124}\right] \\ &= \$636{,}925.79 \end{aligned}$$

Step 3: Deferral Period (Accumulation Stage):

Discount the principal of the annuity (PV_{ORD}) back to today (Age 33). Calculate the periodic interest rate (i, Formula 9.1), number of single payment compound periods (n, Formula 9.2A), and present value of a single payment (PV, Formula 9.2B), rearranged.

$$i = \frac{I/Y}{C/Y} = 9\%1 = 9\%$$

$$n = C/Y \times (\text{Number of Years}) = 1 \times 32 = 32 \text{ compounds}$$

$$\begin{aligned} PV &= \frac{FV}{(1+i)^n} \\ &= \frac{\$636{,}925.79}{(1+0.09)^{32}} \\ &= \$40{,}405.54 \end{aligned}$$