INSTITUTE OF ACTUARIAL & QUANTITATIVE STUDIES

Subject: LOAN SCHEDULES

Chapter:

Category:

1. A housing loan is to be repaid with a 15-year monthly annuity-immediate of \$2,000 at a nominal rate of 6% per year. After 20 payments, the borrower requests for the installments to be stopped for 12 months.

Calculate the revised installment when the borrower starts to pay back again, so that the loan period remains unchanged.

What is the difference in the interest paid due to the temporary stoppage of installments?

Solution

The loan is to be repaid over 15 × 12 = 180 payments. After 20 payments, the loan still has 160 installments to be paid. Using the prospective method, the balance of the loan after 20 payments is

$$2{,}000 \times a_{\overline{160}|0.005} = \$219{,}910.$$

Due to the delay in payments, the loan balance 12 months after the 20th payment is

$$219{,}910\,(1.005)^{12} = \$233{,}473,$$

which has to be repaid with a 148-payment annuity-immediate. Hence, the revised installment is

$$\begin{aligned} \frac{233{,}473}{a_{\overline{148}|0.005}} &= \frac{233{,}473}{104.401} \\ &= \$2{,}236.31. \end{aligned}$$

The difference in the interest paid is

$$2{,}236.31 \times 148 - 2{,}000 \times 160 = \$10{,}973.$$

2. A loan of £3,000 is repayable over 5 years by level quarterly instalments of £170.07, calculated using a rate of interest of 5% pa effective.

(i) Calculate the capital content of the sixth repayment.

(ii) Calculate how much interest is paid in the second year.

(i) ***Capital content***

The capital outstanding immediately after the 5th quarterly repayment is given by the present value at that time of the future repayments:

$$4 \times 170.07 a_{\overline{3.75}|}^{(4)} = £2,317$$

The interest content of the sixth repayment (quarter of a year later) is:

$$2,317 \times (1.05^{0.25} - 1) = £28.44$$

The capital content of the sixth repayment can be found by subtraction:

$$170.07 - 28.44 = £141.63$$

(ii) ***Interest***

The capital outstanding at the end of the first year (*ie* with 4 years still to run) is:

$$4 \times 170.07 a_{\overline{4}|}^{(4)} = 4 \times 170.07 \times 3.6118 = £2,457.01$$ [½]

Similarly, the capital outstanding at the end of the second year (*ie* with 3 years still to run) is:

$$4 \times 170.07 a_{\overline{3}|}^{(4)} = 4 \times 170.07 \times 2.7738 = £1,886.95$$ [½]

So the capital repaid during the second year is:

$$2,457.01 - 1,886.95 = £570.06$$ [1]

Subtracting this from the total payment made during the year gives the total interest paid during the second year:

$$4 \times 170.07 - 570.06 = £110.22$$ [1]

3. A loan is to be repaid by a series of instalments payable annually in arrears for 20 years. The first instalment is £1,400 and payments increase thereafter by £300 per year. Repayments are calculated using an annual effective interest rate of 7%.

(i) Calculate the amount of the loan. [3]

(ii) Calculate the capital outstanding immediately after the third instalment has been paid. [2]

(iii) Explain your answer to (ii). [2]

Immediately after the third instalment has been paid, it is decided to restructure the loan, so that level payments are made quarterly in arrears for the remaining term of the loan. The interest rate on the restructured loan is 9% pa convertible half-yearly.

(iv) Calculate the amount of the quarterly payment. [3]

(v) Calculate the total interest paid over the whole term of the loan.

Solution

(i) £38,084.33

(ii)

Working prospectively, the capital outstanding immediately after the third instalment has been paid is equal to the present value at time 3 years of the remaining instalments. The fourth instalment (due in one year's time) is £2,300 and thereafter the payments increase by £300 each year. So the capital outstanding immediately after the third instalment has been paid is:

$$2{,}000a_{\overline{17}|} + 300(Ia)_{\overline{17}|} \qquad [1]$$

Using values from the *Tables* at an annual effective interest rate of 7%, this gives:

$$2{,}000 \times 9.7632 + 300 \times 72.3555 = £41{,}233.05 \qquad [1]$$

(iii) Explanation

The capital outstanding after the third instalment has been paid is greater than the original loan amount. This is because the first few repayments are not enough to pay the interest accruing on the loan, so the capital outstanding initially increases.

For example, the interest in the first year is 7% of £38,084.33, which is £2,665.90. This is greater than the first instalment of £1,400, so the unpaid interest is added to the capital outstanding, causing it to increase.

(iv) ***Quarterly payment***

The interest rate of 9% *pa* convertible half-yearly is equivalent to an annual effective interest rate of:

$$i=\left(1+\frac{i^{(2)}}{2}\right)^2-1=1.045^2-1=9.2025\% \qquad [1]$$

After the third instalment has been paid, the outstanding loan amount is £41,233.05 and 17 years of repayments remain. Letting Q denote the new quarterly repayment:

$$41{,}233.05=4Qa_{\overline{17}|}^{(4)} \text{ @}9.2025\% \qquad [1]$$

Now:

$$a_{\overline{17}|}^{(4)}=\frac{1-1.092025^{-17}}{4\left(1.092025^{0.25}-1\right)}=8.71932$$

So:

$$Q=\frac{41{,}233.05}{4\times 8.71932}=£1{,}182.23 \qquad [1]$$

(v) ***Total interest paid***

The total interest paid is equal to the difference between the total repayments made and the total capital to be repaid.

There are 3 years of annual increasing repayments, followed by 17 years of level quarterly repayments, so the total of all the repayments made is:

$$1{,}400+1{,}700+2{,}000+17\times 4\times 1{,}182.23=£85{,}491.64 \qquad [1]$$

The capital to be repaid is the original loan amount of £38,084.33. So the total interest paid is:

$$85{,}491.64-38{,}084.33=£47{,}407.31 \qquad [1]$$

4. Construct an amortization schedule of a loan of $5,000 to be repaid over 6 years with a 6-payment annuity-immediate at effective rate of interest of 6% per year.

Solution:

Annual Payment = $1,016.8

Year	Installment	Interest payment	Principal payment	Outstanding balance
0				5,000.00
1	1,016.81	300.00[a]	716.81[b]	4,283.19[c]
2	1,016.81	256.99[d]	759.82	3,523.37
3	1,016.81	211.40	805.41	2,717.96
4	1,016.81	163.08	853.73	1,864.23
5	1,016.81	111.85	904.96	959.26
6	1,016.81	57.55	959.26	0.00
Total	6,100.86	1,100.86	5,000.00	

5. A person takes a $12,000 loan and has to make monthly payments for the first year at the interest rate of 10% compounded monthly, and for the following 2 years with the interest rate of 12% compounded monthly. The first payment is due one month from now. Find a monthly payment on this loan and the total amount of payment.

SOLUTION Let $\$P$ be a monthly payment. The effective monthly interest rate for the 1st year is $i_1 = 0.10/12 = 0.0083$, while for the next two years, it is $i_2 = 0.12/12 = 0.01$. The amount of monthly payment $\$P$ solves the equation

$$\$12,000 = \$P\left[a_{\overline{12}|i_1} + (1+i_1)^{-12}a_{\overline{24}|i_2}\right]$$

$$= \$P\left[\frac{1-(1+i_1)^{-12}}{i_1} + (1+i_1)^{-12}\frac{1-(1+i_2)^{-24}}{i_2}\right] = (30.6043)\$P,$$

Thus, $\$P = \$392.10$ and the total amount paid is $(\$392.10)(36) = \$14,115.60$.

6. A loan is amortized over five years with monthly payments at a nominal interest rate of 9% compounded monthly. The first payment is 1000 and is to be paid one month from the date of the loan. Each succeeding monthly payment will be 2% lower than the prior payment. Calculate the outstanding loan balance immediately after the 40th payment is made.

The point of this question is to test whether a student can determine the outstanding balance of a loan when the payments are not level.

Monthly payment at time t = $1000(0.98)^{t-1}$

Since the actual amount of the loan is not given, the outstanding balance must be calculated prospectively,

OB_{40} = present value of payments at time 41 to time 60

$= 1000(0.98)^{40}(1.0075)^{-1} + 1000(0.98)^{41}(1.0075)^{-2} + \ldots + 1000(0.98)^{59}(1.0075)^{-20}$

This is the sum of a finite geometric series, with

first term, $a = 1000(0.98)^{40}(1.0075)^{-1}$

common ratio, $r = (0.98)(1.0075)^{-1}$

number of terms, n = 20

Thus, the sum

$= a\,(1 - r^n)/(1 - r)$

$= 1000(0.98)^{40}(1.0075)^{-1}\,[1 - (0.98/1.0075)^{20}]/[1 - (0.98/1.0075)]$

$= 6889.11$

7. A young couple, Mr. & Mrs. Jones took a personal loan for an amount of INR 20,000 to refurbish their home. The loan is being repaid with payments of INR 427.90 made monthly in arrears for 5 years.

i) Calculate the Annual Percentage Rate (APR) of the couple's loan.

However, the couple have been struggling to repay the loan for last few months, since Mr. Jones lost his job. After exactly one year, the loan company offers to help the couple by restructuring their loan with new monthly payments of INR 274.49 made in arrears.

ii) Assuming the APR remains unchanged from the original loan; calculate the term of the new loan.

iii) Mr. & Mrs. Jones' initial reaction towards the restructured loan is positive, both in terms of the monthly repayment amount and the term of the loan. However, they are unsure if this may mean that they are paying too much as compared to the original loan. Calculate how much more interest in total the couple will pay on their restructured loan than on the original loan.

Solution:

(i) APR is the interest rate that solves equation of value. Hence, APR, I, is
$20{,}000 = 12 \times 427.90\, a_{5}^{(12)} \Rightarrow a_{5}^{(12)} = 3.89499$

APR is usually double the flat rate of interest. Hence,
Flat Rate $= \dfrac{total\ interest}{loan \times total\ years} = \dfrac{5\times12\times427.90-20000}{20000\times5} = 5.67\%$

Thus, APR is likely to be around 11%.

At i=11%, $a_{5}^{(12)} = 3.87872$

At i= 10%, $a_{5}^{(12)} = 3.96154$

Interpolating, APR, I is

$$\frac{i-10\%}{3.89499-3.96154} = \frac{11\%-10\%}{3.87872-3.96154}$$

i = 10.8%

At i = 10.8%, $12 \times 427.90\, a_{5}^{(12)} = 20{,}000.2$

At i = 10.9%, $12 \times 427.90\, a_{5}^{(12)} = 19{,}958.3$

i = 10.8% is more close. Hence, APR is 10.8%

(ii) APR = 10.8%

After 1 year, capital left to pay = $12 \times 427.90\, a_{4@10.8\%}^{(12)} = 16{,}775.98$

Let t denote the term of the new loan, thus

$$16{,}775.98 = 12 \times 274.49\, a_{t}^{(12)}$$

$16{,}775.98 = 3293.88\, \dfrac{1-1.108^{-t}}{12\times(1.108^{\frac{1}{12}}-1)}$

$1.108^{-t} = 0.4754$

t = 7.25 years

The original loan had 4 years of payments remaining, thus

Total Interest = $4 \times 12 \times 427.90 - 16775.98 = 3763.22$

New loan has term of 7.25 years, hence

Total Interest = $7.25 \times 12 \times 274.49 - 16775.98 = 7104.65$

Thus, Mr. & Mrs. Jones will pay 3341.43 (= 7104.65 – 3763.22) more interest under the restructured loan.

8. A loan is repayable by an increasing annuity payable annually in arrears for 15 years. The repayment at the end of the first year is Rs.3,000 and subsequent payments increase by Rs.200 each year. The repayments were calculated using a rate of interest of 8% per annum effective.

a) Calculate the original amount of the loan.

b) Construct the capital/interest schedule for years nine (after the eighth payment) and ten, showing the outstanding capital at the beginning of the year, the interest element and the capital repayment.

c) Immediately after the tenth payment of interest and capital, the interest rate on the outstanding loan is reduced to 6% per annum effective.

Calculate the amount of the eleventh payment if subsequent payments continue to increase by Rs.200 each year, and the loan is to be repaid by the original date, i.e. 15 years from commencement.

Solution:

a) Loan amount = 35255.76

b)

Capital o/s after 8th payment

$$= 200(Ia)_{\overline{7}|} + 4,400a_{\overline{7}|} \quad @ \ 8\%$$

$$(Ia)_{\overline{7}|} = \frac{\ddot{a}_{\overline{7}|} - 7v^7}{0.08} = \frac{1.08 \times 5.2064 - 7 \times 0.58349}{0.08}$$

$$= 19.2310$$

$$\Rightarrow \text{Cap o/s} = 200 \times 19.2310 + 4,400 \times 5.2064$$

$$= 26,754.36$$

Year	*Loan o/s at start*	*Repayment*	*Interest element*	*Capital element*
9	26,754.36	4,600	2,140.35	2,459.65
10	24,294.71	4,800	1,943.58	2,856.42

c)

Loan o/s after 10th payment

$$= 24,294.71 - 2,856.42 = 21,438.29$$

Let 11th payment be X then

$$200(Ia)_{\overline{5}|} + (X - 200)a_{\overline{5}|} = 21,438.29 \quad @ \ 6\%$$

$$(Ia)_{\overline{5}|} = \frac{1.06 \times 4.2124 - 5v^5}{0.06} = 12.1476$$

Hence $200 \times (12.1476 - 4.2124) + X \times 4.2124 = 21,438.29$

$$\Rightarrow X = 4,712.57$$