INSTITUTE OF ACTUARIAL & QUANTITATIVE STUDIES

Subject: Financial mathematics

Chapter:

Category: Assignment 1

1. How many days does one need to hold a 364 days Government Bond redeemable at INR 100 if he buys at INR 96.5 and sells at INR98.0 after achieving a return of 4% per annum effective.

Answer:

Solution 7: Option B

Workings only for reference:

Return earned = 98/96.5-1 = 1.55%

We note that: (1+4%)^(144/365) = 1.55%. Therefore, Option B.

144 days

2. A 91-day treasury bill was purchased for INR 103 at the time of issue and later sold to another investor for INR 104 who held the bill to maturity. The bill was redeemable at INR 105. The rate of return received by the initial purchaser was 5% p.a. effective.

i) Calculate the length of time for which the initial purchaser held the bill.

ii) Calculate the annual rate of return achieved by the second investor.

Answer:

i) Option B

Workings for reference (no marks to be awarded for showing steps as this is an MCQ)

Number of days the bill is held by the first purchaser = t

$103\ x\ (1.05)^t = 104$

$t = log1.009709/log1.05 = 0.198$

$Converted\ to\ days, this\ is\ 0.198\ x\ 365 = 72.2809\ or\ 72\ days$

ii) Option B

Workings for reference (no marks to be awarded for showing steps as this is an MCQ)

Number of days the bill was held by second investor = 91 – 72 = 19 days

$$104\ x\ (1 + \frac{19}{365}\ i) = 105$$

Solving this, we get i = 0.1875 = 19% return

3. Describe the following:

i) Describe the cashflows for an investor who purchases an index-linked bond.

ii) Describe main features of an endowment assurance contract.

ASSIGNMENT 1

iii) Describe the effective rates of interest and discount. Calculate annual effective rate of interest that is equivalent to a simple interest rate of 4% over 5 years.

Answer 3:

i) At the outset the investor has a negative cashflow [½]
In return the investor receives a series of regular interest payments linked to an index [½] reflecting the effects of inflation [½]. A final capital repayment [½], which is also linked to an index [½] and may be subjected to a lag.

ii) An endowment assurance is a contract that provides a survival benefit at the end of the term, but it also provides a lump sum benefit on death before the end of the term. [1]
The benefits are provided in return for a series of regular premiums (or a single premium). [½]

The sum assured payable on death or survival need not be the same, although generally they often are. [½] The term of the contract could be fixed or up to certain age. [½]

iii) An investor may lend an amount of INR 1 at time 0 in return for a repayment of (1+i) at time 1. In this case, i is the interest paid at the end of the year and hence considered to be the effective rate of interest per unit time. [1]

In case an investor lends and amount of (1-d) at time 0 in return for a payment of 1 at time 1. In this case d is called the effective rate of discount per unit time. [½]

The accumulation factor for 4% pa simple interest rate over 5 years is:

$A(5) = 1+5*0.04 = 1.2$ [½]

The accumulation factor for effective interest rate over 5 years is:

$A(5) = (1+i)^{\wedge 5}$ [½]

If the two rates are equivalent, then they result in the same accumulation factors:

$(1+i)^{\wedge 5} = 1.2$

⇨ 0.03714 [½]

4. i) Calculate the effective monthly rate of interest corresponding to:

a) Nominal rate of interest of 6% p.a. convertible quarterly.

b) Nominal rate of interest of 10% p.a. convertible six times a year.

ii) The rate of interest at time t is given by :

$$\delta(t) = \begin{cases} .05+.005\,t & 0<=t<5 \\ .08t - .01 & 5<=t<10 \\ 0.10 & 10<=t \end{cases}$$

Calculate the present value at time 2 of a payment of 5000 at time 15 years.

Answer:

ASSIGNMENT 1

i)

a) $i^{(4)}$ = 6% p.a.

Let i be the effective monthly rate of interest

$(1+i)^{12} = (1+ i^{(4)}/4)^{4}$

i = 0.498%

b) $i^{(6)}$ = 10% p.a.

Let i be the effective monthly rate of interest

$(1+i)^{12} = (1+ i^{(6)}/6)^{6}$

i = 0.830%

ii)

given that

$$\delta(t) = \begin{cases} .05+.005\,t & 0<=t<5 \\ .08t - .01 & 5<=t<10 \\ .10 & 10<=t \end{cases}$$

The present value at time 2 of a payment of 5000 at time 15 years will be

$$PV= 5000*e^{-\int_2^5(.05+.005t)dt}*e^{-\int_5^{10}(.08t-.01)dt}*e^{-\int_{10}^{15}(.10)dt}$$

$$= 5000*e^{-[.05t+\frac{.005t^2}{2}]_2^5}*e^{-[\frac{.08t^2}{2}-.01t]_5^{10}}*e^{-[.10t]_{10}^{15}}$$

$$= 5000*e^{-[.05(5-2)+\frac{.005}{2}(25-4)]}*e^{-[.04(100-25)-.01(10-5)]}*e^{-[.10(15-10)]}$$

$$= 5000*e^{-[.2025]}*e^{-[2.95]}*e^{-[.5]}$$

= 129.63

Hence the present value is Rs. 129.63

5. A Government issues are 91 day treasury bill at a simple rate of discount 7% per annum. Calculate the rate of return per annum convertible half yearly received by an investor who purchases the Bill and holds it to maturity.

Answer:

ASSIGNMENT 1

ii) $\left(1 - \frac{91}{365} \times 0.07\right) = \left(1 + \frac{i^{(2)}}{2}\right)^{\frac{-91}{182.5}}$

$$\Rightarrow i^{(2)} = (0.982548^{\frac{-182.5}{91}} - 1) \times 2 = 0.03594 \times 2$$

$$\Rightarrow i^{(2)} = 0.07188 = 7.19\% \text{ p.a. convertible half-yearly}$$

6. i) For a rate of interest of 7% per annum, convertible monthly, calculate:
a) The equivalent rate of interest per annum convertible half yearly, and
b) The equivalent rate of discount per annum convertible monthly

ii) The force of interest p.a. changes from δ_0 at time 0 to δ_m at time t = m years and will thereafter remain constant at δ_m. Calculate the value of accumulation when m =16, n = 39, $\delta_0 = \log_e(1.04)$ and $\delta_m = \log_e(1.03)$.

Answer:

i)

a) $(1 + \frac{i^{(2)}}{2})^2 = (1 + \frac{i^{\wedge}((12)}{12})^{12}$

$(1 + \frac{i^{(2)}}{2}) = (1 + \frac{0.07}{12})^6$

$i^{(2)} = 7.103\%$

b) $(1 - \frac{d^{(12)}}{12})^{12} = (1 + \frac{i^{\wedge}((12)}{12})^{-12}$

$(1 - \frac{d^{12}}{12})$ = $(1 + \frac{0.07}{12})^{-1}$

$d^{(12)} =$ 6.959%

ii) δ(t) = $\delta_0 + \frac{t}{m}(\delta_m - \delta_0)$ for 0 <= t <= m

δ(t)= δ_m for t>m

Required accumulated value

exp $(\int_0^n \delta(t)dt$

= Exp $\{\int_0^m [\delta_0 + \frac{t}{m}(\delta_m - \delta_0)]$ dt + $\delta_m\ (n-m)\}$

=exp $\{[\delta_0\, t + \frac{t^2}{2m}(\delta_m - \delta_0)]_{t=0}^{t=m} + \delta_m\ (n-m)\}$

= exp $[\delta_0\, m + \frac{m}{2}(\delta_m - \delta_0) + \delta_m\ (n-m)]$

= exp $[\frac{m}{2}(\delta_0 - \delta_m) + \delta_m\ n]$

= $[\exp(\delta_0 - \delta_m)]^{m/2}\, [exp(\delta_m)]^n$

= $(\frac{1.04}{1.03})^8\,(1.03)^{39}$ = 3.4215

7. A new super market has earned a simple rate of interest of 8% p.a. over the last calendar year based on the following cash flows:

i) Net investment income earned from above cash-flows over the year is Rs. 20,00,000. Assuming that all cash flows occur at the middle of the year, calculate the value of X.
ii) Also calculate the effective yield of above cashflows.

Answer:

i) 20,00,000 = 8% x[2,50,00,000 + 0.5 * (X- 22,00,000-7,50,000)]

= 18,82,000 +0.04 X

X = 29,50,000

Assets at the end of the year

= 2,50,00,000+29,50,000 +20,00,000-22,00,000-7,50,000

=2,70,00,000

ASSIGNMENT 1

ii) Hence effective yield is

2,70,00,000 =2,50,00,000 * (1+i) +(29,50,000- 22,00,000-7,50,000)*$(1+i)^{0.5}$

2,70,00,000 =2,50,00,000 * (1+i) +0*$(1+i)^{0.5}$

(1+i) = 1.08 => i = 8%

8. The force of interest at time t (where t is measured in years) is given by:

δ (t) = 0.07 for $0 \le t < 4$

δ (t) = 0.06 for $4 \le t < 8$

δ (t) = 0.05 for $8 \le t < 20$

i) Derive expressions for v(t), the present value of 1 due at time t.

ii) Calculate the accumulated value at time 15 of an investment of INR 5,000 made at time 3.

iii) What constant force of interest would produce the same accumulation as in (ii) for an investment of INR 5,000 over a period of 12 years?

iv) Calculate the effective annual rate of interest that would have the same effect as the varying force of interest given above, over a period of 20 years.

v) Calculate the present value at time 0 of an annuity of INR 1,000 per annum payable annually in advance for 10 years

Answer:

i) $v(t) = \exp(-\int_0^t \delta(s)\,ds)$

$v(t) = \exp(-\int_0^t 0.07\,ds) = \exp(-0.07t)$ for $0 \le t < 4$

$v(t) = \exp(-\int_0^4 0.07\,ds - \int_4^t 0.06\,ds) = \exp(-0.04 - 0.06t)$ for $4 \le t < 8$

$v(t) = \exp(-\int_0^4 0.07\,ds - \int_4^8 0.06\,ds - \int_8^t 0.05\,ds) = \exp(-0.08 - 0.04 - 0.05t)$

$v(t) = \exp(-0.12 - 0.05t)$ for $8 \le t$

ii) Accumulated value = 5000* v(3)/v(15)

= 5000* exp(-0.21)/ exp(-0.12-0.75)

= 5000 * exp(0.66)

= INR 9673.96

iii) Let δ(t) be the constant force of interest

5000* exp(12 δ) = 5000* exp(0.66)

12 δ = 0.66

δ = 0.055

ASSIGNMENT 1

iv) $1/v(20) = 1/\exp(-0.12-1) = 3.06485 = (1+i)^{20}$

Where i is the effective annual rate

So i= 0.0576

v) Present value

1000*(1+v(1)+v(2) +……….v(9))

1000* (1+e(-0.07)+ e(-0.14)+ e(-0.21)+ e(-0.28)+ e(-0.34)+ e(-0.40)+ e(-0.46)+ e(-0.52)+ e(-0.57))

= INR 7,541.54

9. i) The rate of discount per annum convertible quarterly is 6%.
Calculate:
a) The equivalent rate of interest per annum convertible half yearly.
b) The equivalent rate of discount per annum convertible monthly.

ii) On 15th April, 2005 Amit borrowed Rs 2,00,000 to be repaid one year later by single payment of Rs 2,20,000. Amit repaid the loan early on 17th July, 2005.
a) Find the sum paid by Amit to terminate the contract assuming that the interest is reduced proportionately for early settlement.
b) Calculate APR on the completed transaction

Answer:

i)

a) $1/(1+ i^{(2)}/2)^2 = (1- 0.06/4)^4$

$(1+ i^{(2)}/2)^2 = 1.0623193$

$i^{(2)} = 6.1377\%$ p.a. convertible half-yearly

b) $(1- d^{(12)}/12)^{12} = (1- 0.06/4)^4$
$(1- d^{(12)}/12)^{12} = 0.941337$
$d^{(12)} = 6.0303\%$ convertible monthly

ii) a) Original term of the loan = 365 day
No deferment – settlement date is actual date of repayment17th July 2005

Outstanding term of the loan = 272 days.
The rebate allowed = (272/365) x 20000 = 14904.1096
Sum paid to settle the loan = (220000 – 14904.1096) = 205095.89

b) The loan was repaid after 93 days, the APR obtained is:

$200000\,(1+i)^{93/365} = 205095.89$
⇨ i = 10.3%

10. i) While valuing future payments an investor uses the formulae

ASSIGNMENT 1

$$v(t) = \alpha(\alpha+1)/((\alpha+t)(\alpha+t+1)) \quad t \geq 0$$

where, $v(t)$ is the discounting factor at time t and α is a positive constant

Show that the force of interest (δ) at time t will be

$$\delta(t) = (2t + 2\alpha + 1)/((\alpha+t)(\alpha+t+1))$$

ii) For a bank deposit over a given year, the force of interest per annum was 15% at the start of year, 10% at middle of the year, and 8% at the end of year. The force of interest per annum follows a quadratic function given by

$\delta(t) = a+bt+ct^2$

where, a, b, c are constants and t is the time in years.

Find the accumulated amount at the end of year of a deposit of Rs 1,50,000 at the start of year.

Answer:

i) $\int_0^t \delta(t)dt = -\log v(t)$

$\delta(t) = -(v'(t)/v(t))$

$\delta(t) = (2t + 2\alpha + 1)/((t+\alpha)(t+\alpha+1))$

ii) Measure time in years from the start of the given year

Given $\delta(0) = 0.15$ and let

$\delta(t) = 0.15 + bt + ct^2$ (A)

Putting t= ½ in the equation (A)

$0.10 = 0.15 + ½ b + 1/4c$

Putting t= 1 in the equation (A)

$0.08 = 0.15 + b + c$

Solving the above two equations, b= -0.13 and c= 0.06

Putting the values of b and c in equation A

$\delta(t) = 0.15 - 0.13t + 0.06t^2$

This implies that $\int_0^1 \delta(t)\,dt = 0.105$

The accumulated amount at the end of year is:

1,50,000x exp(0.105) = 1,50,000x 1.1107= Rs1,66,606.59