INSTITUTE OF ACTUARIAL & QUANTITATIVE STUDIES

Subject: Financial Mathematics

Chapter: 1,2,3,4 (Unit 1&2)

Category: Assignment 1 Solutions

Q 1)

(i)

$$e^{-\frac{\delta}{4}} = 1 - \frac{0.08}{4}$$

$$\delta = 0.080811$$

(ii)

$$(1+i)^{-1} = (1-\frac{0.08}{4})^4 = 0.92237$$

$$i = 0.084166$$

(iii)

$$(1-\frac{d^{(12)}}{12})^{12} = (1-\frac{0.08}{4})^4 = 0.92337$$
$$d^{(12)} = 0.080539$$

Q 2)

(i) Present value of 1 payable at the end of 10 year

=v^10

= Exp[- ($_0$?^{10}(0.004t + 0.0002t^2)dt)]

=Exp [-(0.004 t^2 /2 + 0.0002 t^3 /3)$_0$^{10}]

=Exp [-0.2 - 0.06667] =Exp[-26667]

= 0.765926

Present Value of 1000 payable at the end of 10 year is

= 765.926

(ii) Annual effective rate of interest i is such that

(1+i)^(-10) = 0.765926

(1+i)^(10) = 1.305609

I = 2.70%

Q 3)

(i) d is the interest payable at the beginning of the year on a loan of 1unit repayable at the end of the year. The corresponding interest payable at the end of year on 1unit of loan is i. Thus, d must equal the present value at the beginning of the year of the payment i payable at the end of the year ie d= iv .

(2)

(ii) Let $d^{(2)}$ be the nominal rate of discount per annum convertible half yearly

(a) 1-d = (1-0.0225)^4 = 0.912992

d = 0.087007

Hence $(1 - d^{(2)}/2)^2 = 0.912992$

$d^{(2)}$ = 2*(1- 0.912992^(1/2))

= 8.8988%

(2)

(b) 1-d = (1-2*0.05) = 0.9

d = 0.1

Hence $(1- d^{(2)}/2)^4 = 0.9$

$d^{(2)}$ = 2*(1-0.9^(1/4))

= 5.199%

(iii) Let d be the simple annual discount rate. Then

(1-d*91/365) = (1+i)^(-91/365) where i =0.05

(1-d*91/365) = 0.98791

d*91/365 = 0.01206

d = 4.8493%

Q 4)

i) d is the interest payable at the beginning of the year on a loan of 1unit repayable at the end of the year. The corresponding interest payable at the end of year on 1unit of loan is i.

Thus, d must equal the present value at the beginning of the year of the payment i payable at the end of the year ie $d = iv$. [2]

ii) Given i = 0.08

a. $d^{(12)}$

$v = 1 - d = \left[1 - d^{(12)}/12\right]^{12}$

$d^{(12)} = 12 \times \left[1 - v^{(1/12)}\right]$

$d^{(12)} = 12 \times \left[1 - 1.08^{\left(\frac{-1}{12}\right)}\right]$

= 0.076714776

INSTITUTE OF ACTUARIAL & QUANTITATIVE STUDIES

b. $i^{(365)}$

$(1+i) = \left[1 + i^{(365)}/365\right]^{365}$

$i^{(365)} = 365 \times \left[1.08^{\left(\frac{1}{365}\right)} - 1\right]$

= 0.076969155

c. δ

$\delta = \log(1 + i)$

$= \log(1.08)$

= 0.076961041

d. $i^{(1/2)}$

$i^{(1/2)} = 0.5 \times \left[1.08^{(2)} - 1\right]$

= 0.0832

Q 5)

i)

a) $1/(1+ i^{(2)} /2)^2 = (1- 0.06 /4)^4$

$(1+ i^{(2)} /2)^2 = 1.0623193$

$i^{(2)} = 6.1377\ \%$ p.a. convertible half-yearly

b) $(1- d^{(12)} /12)^{12} = (1- 0.06 /4)^4$

$(1- d^{(12)} /12)^{12} = 0.941337$

$d^{(12)} = 6.0303\%$ convertible monthly

ii) a) Original term of the loan = 365 day

No deferment – settlement date is actual date of repayment17th July 2005

Outstanding term of the loan = 272 days.

The rebate allowed = (272/365) x 20000 = 14904.1096

Sum paid to settle the loan = (220000 – 14904.1096) = 205095.89

Q 6)

i) Assume that the retail price of the toy is 100. The cash price is 70 or one can pay 75 in 6 months' time. Hence the effective rate of discount d is

70/75 = (1-d) ^ (0.5), Where d = 0.1289 or 12.89%

ii)

- $d^{(12)}$ = 12 x [1 – $(1-d)^{1/12}$] = 13.72% ,
- i = $\frac{d}{(1-d)}$ = 14.79% ,and $i^{(2)}$ = 2 x [$(1+i)^{(1/2)}$-1] = 14.28%
- (1+i) =e^{δ} hence δ =0.1380

Q 7)

i) 20,000 * $(1 + i^4 / 4)^{4x17/12}$ = 26,000

$(1 + i^4 / 4)$ = $(26,000/20,000)^{12/68}$

i^4 = (1.0474 – 1) * 4 = 18.96%

ii) 26,000 * $(1 – d^2 / 2)^{2x17/12}$ = 20,000

$(1 + d^2 / 2)$ = $(20,000/26,000)^{12/34}$

d^4 = (1 - .91156) * 2 = 17.69%

iii) 20,000 * (1 + i * 17/12) = 26,000

i = (26,000/20,000) * 12/17 – 1 = 21.18%

iv)

- If using simple interest, the rate of interest needs to be higher when compared to compound interest in order to achieve the same overall amount.
- Interest rate will get lower with higher frequency of compounding.

Q 8)

The relationship between i and i^p can be represented in a formula as follows:-

$(1 + i^p / p)^p = (1 + i)$

Also when $p \rightarrow \infty$; the nominal rate of interest rate will become convertible continuously and is known as force of interest.

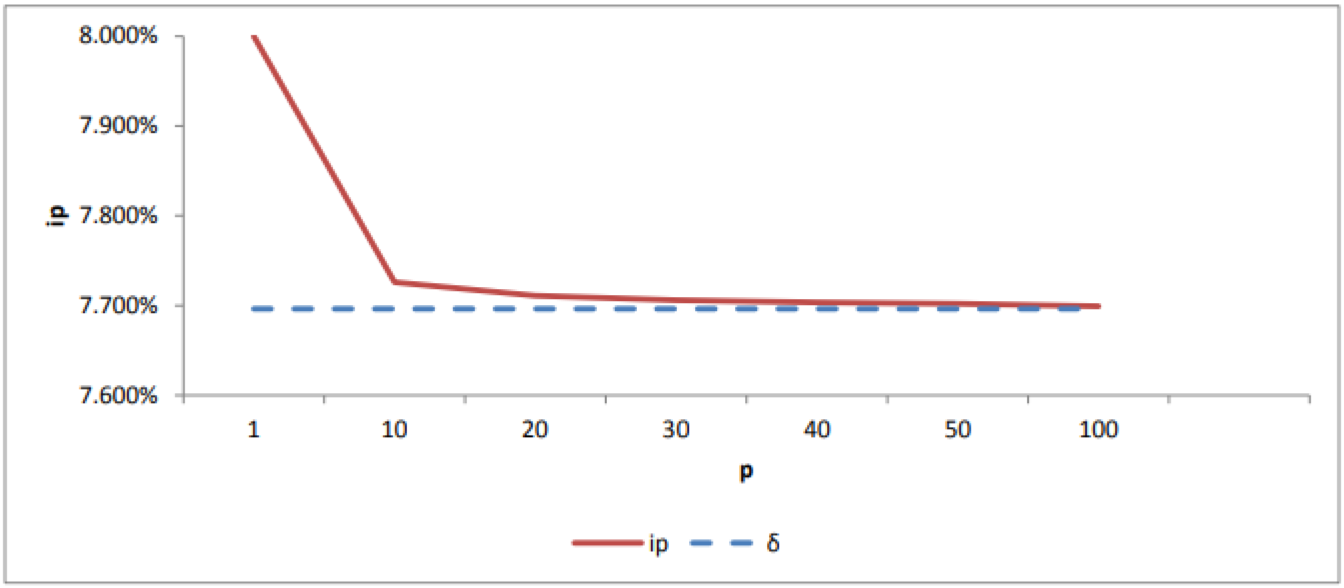
Calculating the force of interest:

$e^\delta = (1 + i) = 1.08$

$\delta = \ln(1.08) = 7.696\%$

The Force of interest will be the lowest value which will be attained by i^p

The graph depicting relationship between i^p and p will be:



Q 9)

i) **Force of Interest:-**
The force of interest can be defined as the nominal rate of interest per unit time at time t convertible momently. i.e., for each value of t there is a number $\delta(t)$ such that
$\lim_{h \to 0+} i_h(t) = \delta(t)$, where $\delta(t)$ is called the force of interest per unit time at time t.

We may also define $\delta(t)$ directly in terms of the accumulation factor as

$$\delta(t) = \lim_{h \to 0+} \left(\frac{A(t,t+h) - 1}{h} \right)$$

ii) Given $i^{(2)}=0.0775$

$i=(1+ i^{(2)}/2)^2-1 = (1+.0775/2)^2-1=1.079002-1=7.9002\%$

$\delta = \ln(1+i)=\ln(1.079)=7.6036\%$

$d^{(12)}= 12*(1-v^{(1/12)}) = 12*(1-(1/1.079)^{(1/12)}) = 7.5796\%$

$i^{(1/2)}=(1/2)*((1+i)^2-1) = 0.5*(1.079^2-1) = 8.2122\%$

Q 10)

i) v(t) = exp $\left[-\int_0^t \delta(s)ds\right]$

Hence

$$\int_0^t \delta(s)ds = \begin{cases} 0.08t & for\ 0 \le t \le 5 \\ 0.1 + 0.06\,t & for\ 5 \le t \le 10 \\ 0.3 + 0.04\,t & for\ t \ge 10 \end{cases}$$

$$v(t) = \begin{cases} \exp(-0.08t) & for\ 0 \le t \le 5 \\ \exp(-0.1 - 0.06\,t) & for\ 5 \le t \le 10 \\ \exp(-0.3 - 0.04\,t) & for\ t \ge 10 \end{cases}$$

Q 11)

a)

The amount initially lent was $50{,}000(1-\frac{9}{12}*0.18)=43{,}250$

b)

i.

$$(1+i)=e^{\delta}=1.0618$$
$$v=0.9418$$
$$d^{(12)}=12(1-v^{(1/12)})=5.98\%$$

ii.

$$v=1-d=\left(1-\frac{d^{(4)}}{4}\right)^4=0.9413$$

$$i=v^{-1}-1=6.24\%$$

$$i^{(12)}=12\left((1+i)^{\frac{1}{12}}-1\right)=6.07\%$$

c)

The order is determined based on when the interest is paid, so, for example d corresponds to interest paid immediately which requires a smaller payment amount. Similarly, i corresponds to interest paid at the end of time and δ corresponds to payment throughout

$$d<\delta<i^{(4)}<i$$

d)

With 1 unit of initial capital, *d is* the interest payable at the beginning of the year. Similarly, with the same capital, *i* is the interest payable at the end of the year. So *d* must be equal to the present value at the beginning of the year of the payment *i* payable at the end of the year. i.e. *d = iv*

Q 12)

a) $i^{(12)} = 6\%$, thus monthly effective i is given by

$$i = \frac{i^{(12)}}{12} = 0.5\%$$

Accumulation factor for two years (working in months) = $(1 + 0.5\%)^{24}$ = 1.1272

Accumulated amount after two years = 7945.63

Accumulation factor for next 2.5 years (working in quarters) = (1 + 10*1.5%) = 1.15

Accumulated amount after four and a half years = 9137.48

$$v = 1 - d = \left(1 - \frac{d^{(12)}}{12}\right)^{12} = 0.94162$$

$$i = v^{-1} - 1 = 6.2\%$$

Accumulation factor for next 1.5 years (working in years) = $(1 + 6.2\%)^{1.5}$ =1.0944

Accumulated amount after 6 years = 7,049*1.1272*1.15*1.0944 = 10,000

Q 13)

i. $2 = (1+i)^n$

$$n = \frac{\ln(2)}{\ln(1.1)} = 7.272 \text{ years}$$

ii. $d^{(12)} = 10\%$; $1+i = \left(1 - \frac{d^{(p)}}{p}\right)^{(-p)}$

$i = 10.563\%$ p.a

Thus, as in part (a), n = 6.902 years

CHAPTER – 1, 2, 3, 4

ASSIGNMENT 1 SOLUTION