



Subject: Financial mathematics

Chapter:

Category: Assignment 1

Q1.

How many days does one need to hold a 364 days Government Bond redeemable at INR 100 if he buys at INR 96.5 and sells at INR 98.0 after achieving a return of 4% per annum effective.

Q2.

A 91-day treasury bill was purchased for INR 103 at the time of issue and later sold to another investor for INR 104 who held the bill to maturity. The bill was redeemable at INR 105. The rate of return received by the initial purchaser was 5% p.a. effective.

- i) Calculate the length of time for which the initial purchaser held the bill.
- ii) Calculate the annual rate of return achieved by the second investor.

Q3.

Describe the following:

- i) Describe the cashflows for an investor who purchases an index-linked bond.
- ii) Describe main features of an endowment assurance contract.
- iii) Describe the effective rates of interest and discount. Calculate annual effective rate of interest that is equivalent to a simple interest rate of 4% over 5 years.

Q4.

- i) Calculate the effective monthly rate of interest corresponding to:
 - a) Nominal rate of interest of 6% p.a. convertible quarterly.
 - b) Nominal rate of interest of 10% p.a. convertible six times a year.

- ii) The rate of interest at time t is given by :

$$\delta(t) = \begin{cases} .05 + .005t & 0 \leq t < 5 \\ .08t - .01 & 5 \leq t < 10 \\ 0.10 & 10 \leq t \end{cases}$$

Calculate the present value at time 2 of a payment of 5000 at time 15 years.

Q5.

A Government issues a 91 day treasury bill at a simple rate of discount 7% per annum. Calculate the rate of return per annum convertible half yearly received by an investor who purchases the Bill and holds it to maturity.

Q6.

i) For a rate of interest of 7% per annum, convertible monthly, calculate:

- a) The equivalent rate of interest per annum convertible half yearly, and
- b) The equivalent rate of discount per annum convertible monthly

ii) The force of interest p.a. changes from δ_0 at time 0 to δ_m at time $t = m$ years and will thereafter remain constant at δ_m . Calculate the value of accumulation when $m = 16$, $n = 39$, $\delta_0 = \log_e(1.04)$ and $\delta_m = \log_e(1.03)$.

Q7. A new super market has earned a simple rate of interest of 8% p.a. over the last calendar year based on the following cash flows:

- i) Net investment income earned from above cash-flows over the year is Rs. 20,00,000. Assuming that all cash flows occur at the middle of the year, calculate the value of X.
- ii) Also calculate the effective yield of above cashflows.

Q8.

The force of interest at time t (where t is measured in years) is given by:

$$\delta(t) = 0.07 \text{ for } 0 \leq t < 4$$

$$\delta(t) = 0.06 \text{ for } 4 \leq t < 8$$

$$\delta(t) = 0.05 \text{ for } 8 \leq t < 20$$

- i) Derive expressions for $v(t)$, the present value of 1 due at time t .
- ii) Calculate the accumulated value at time 15 of an investment of INR 5,000 made at time 3.
- iii) What constant force of interest would produce the same accumulation as in (ii) for an investment of INR 5,000 over a period of 12 years?
- iv) Calculate the effective annual rate of interest that would have the same effect as the varying force of interest given above, over a period of 20 years.
- v) Calculate the present value at time 0 of an annuity of INR 1,000 per annum payable annually in advance for 10 years

Q9.

i) The rate of discount per annum convertible quarterly is 6%.

Calculate:

a) The equivalent rate of interest per annum convertible half yearly.

b) The equivalent rate of discount per annum convertible monthly.

ii) On 15th April, 2005 Amit borrowed Rs 2,00,000 to be repaid one year later by single payment of Rs 2,20,000. Amit repaid the loan early on 17th July, 2005.

a) Find the sum paid by Amit to terminate the contract assuming that the interest is reduced proportionately for early settlement.

b) Calculate APR on the completed transaction

Q10.

i) While valuing future payments an investor uses the formulae

$$v(t) = \alpha (\alpha + 1) / ((\alpha + t)(\alpha + t + 1)) \quad t \geq 0$$

where, $v(t)$ is the discounting factor at time t and α is a positive constant

Show that the force of interest (δ) at time t will be

$$\delta(t) = (2t + 2\alpha + 1) / ((\alpha + t)(\alpha + t + 1))$$

ii) For a bank deposit over a given year, the force of interest per annum was 15% at the start of year, 10% at middle of the year, and 8% at the end of year. The force of interest per annum follows a quadratic function given by

$$\delta(t) = a + bt + ct^2$$

where, a , b , c are constants and t is the time in years.

Find the accumulated amount at the end of year of a deposit of Rs 1,50,000 at the start of year.

Q11.

i) Describe the difference between following types of asset:

a) Debenture stocks and

b) Unsecured loan stocks.

ii) List reasons why investing in property may not be preferred alternative to investing in Government securities.

iii) a) How does Government issue a loan by tender?

b) What are the main advantages and disadvantages to the Government and to investors of issuing bonds at a set price as opposed to by tender?

c) Give the name of three securities, which have uncertain income and give reason for such uncertainty.

Q12.

If an investment fund offers to increase INR 20,000 to INR 26,000 in 17 months, calculate the following:

- i) The nominal rate of interest per annum convertible quarterly
- ii) The nominal rate of discount per annum convertible half yearly
- iii) Simple rate of interest per annum
- iv) Comment on the results.

Q13.

i) Explain the relationship $d = iw$ by general reasoning where d is the effective annual rate of discount and i is the effective annual rate of interest.

ii) Given $i = 0.08$,

Calculate

- a. $d^{(12)}$
- b. $i^{(365)}$
- c. δ
- d. $I^{(1/2)}$

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Q14.

$\delta(t)$, the force of interest per annum at time t years is defined as:-

$$\delta(t) = \begin{cases} 0.08 & \text{for } 0 \leq t \leq 5 \\ 0.06 & \text{for } 5 \leq t \leq 10 \\ 0.04 & \text{for } t \geq 10 \end{cases}$$

Using $\delta(t)$ Derive the expression for $v(t)$, the present value of 1 due at time t .