INSTITUTE OF ACTUARIAL & QUANTITATIVE STUDIES

Subject: Financial mathematics

Chapter:

Category: Assignment 1 Solution

Answer 1 :

Solution 7: Option B

Workings only for reference:

Return earned = 98/96.5-1 = 1.55%

We note that: (1+4%)^(144/365) = 1.55%. Therefore, Option B.

144 days

Answer 2:

i) Option B

Workings for reference (no marks to be awarded for showing steps as this is an MCQ)

Number of days the bill is held by the first purchaser = t

$103\ x\ (1.05)^t\ =\ 104$

$t\ =\ log1.009709/log1.05\ =\ 0.198$

$Converted\ to\ days, this\ is\ 0.198\ x\ 365\ =\ 72.2809\ or\ 72\ days$

ii) Option B

Workings for reference (no marks to be awarded for showing steps as this is an MCQ)

Number of days the bill was held by second investor = 91 – 72 = 19 days

$104\ x\ (1\ +\ \frac{19}{365}\ i)\ =\ 105$

Solving this, we get i = 0.1875 = 19% return

Answer 3

i) At the outset the investor has a negative cashflow [½]
In return the investor receives a series of regular interest payments linked to an index [½] reflecting the effects of inflation [½]. A final capital repayment [½], which is also linked to an index [½] and may be subjected to a lag.

ii) An endowment assurance is a contract that provides a survival benefit at the end of the term, but it also provides a lump sum benefit on death before the end of the term. [1]
The benefits are provided in return for a series of regular premiums (or a single premium). [½]

1. In this case, i is the interest paid at the end of the year and hence considered to be the effective rate of interest per unit time. [1]

In case an investor lends and amount of (1-d) at time 0 in return for a payment of 1 at time 1. In this case d is called the effective rate of discount per unit time. [½]

The accumulation factor for 4% pa simple interest rate over 5 years is:

$A(5) = 1+5*0.04=1.2$

[½]

The accumulation factor for effective interest rate over 5 years is:

$A(5) = (1+i)^{5}$

[½]

If the two rates are equivalent, then they result in the same accumulation factors:

$(1+i)^{5} = 1.2$

$\Rightarrow$ 0.03714 [½]

Answer 4

i)

a) $i^{(4)} = 6\%$ p.a.

Let i be the effective monthly rate of interest

$(1+i)^{12} = (1+ i^{(4)}/4)^{4}$

$i = 0.498\%$

b) $i^{(6)} = 10\%$ p.a.

Let i be the effective monthly rate of interest

$(1+i)^{12} = (1+ i^{(6)}/6)^{6}$

$i = 0.830\%$

$$\delta(t) = \begin{cases} .05+.005\,t & 0<=t<5 \\ .08t - .01 & 5<=t<10 \\ .10 & 10<=t \end{cases}$$

The present value at time 2 of a payment of 5000 at time 15 years will be

PV= $5000 * e^{-\int_2^5 (.05+.005t)dt} * e^{-\int_5^{10} (.08t-.01)dt} * e^{-\int_{10}^{15} (.10)dt}$

$= 5000 * e^{-[.05t+\frac{.005t^2}{2}]_2^5} * e^{-[\frac{.08t^2}{2} - .01t]_5^{10}} * e^{-[.10t]_{10}^{15}}$

$= 5000 * e^{-[.05(5-2)+\frac{.005}{2}(25-4)]} * e^{-[.04(100-25) - .01(10-5)]} * e^{-[.10(15-10)]}$

$= 5000 * e^{-[.2025]} * e^{-[2.95]} * e^{-[.5]}$

= 129.63

Hence the present value is Rs. 129.63

Answer 5:

ii) $\left(1 - \frac{91}{365} \times 0.07\right) = \left(1 + \frac{i^{(2)}}{2}\right)^{\frac{-91}{182.5}}$

$\Rightarrow i^{(2)} = (0.982548^{\frac{-182.5}{91}} - 1) \times 2 = 0.03594 \times 2$

$\Rightarrow i^{(2)} = 0.07188 = \mathbf{7.19\%}$ **p.a. convertible half-yearly**

Answer 6.

$(1+\frac{i^{(2)}}{2})$ = $(1+\frac{0.07}{12})^6$

$i^{(2)}$ = 7.103%

b) $(1-\frac{d^{(12)}}{12})^{12}$ = $(1+\frac{i^{\wedge}((12)}{12})^{-12}$

$(1-\frac{d^{12}}{12})$ = $(1+\frac{0.07}{12})^{-1}$

$d^{(12)}$ = 6.959%

ii) δ(t) = $\delta_0 + \frac{t}{m}(\delta_m - \delta_0)$ for 0 <= t <= m

δ(t)= δ_m for t>m

Required accumulated value

exp $(\int_0^n \delta(t)dt$

= Exp { $\int_0^m [\delta_0 + \frac{t}{m}(\delta_m - \delta_0)]$ dt + $\delta_m\ (n-m)\}$

=exp { $[\delta_0 t + \frac{t^2}{2m}(\delta_m - \delta_0)]_{t=0}^{t=m}$ +$\delta_m\ (n-m)\}$

= exp $[\delta_0 m + \frac{m}{2}(\delta_m - \delta_0) + \delta_m\ (n-m)]$

= exp $[\ \frac{m}{2}(\delta_0 - \delta_m) + \delta_m\ n\ ]$

= $[\exp(\delta_0 - \delta_m)]^{m/2}\ [exp(\delta_m)]^n$

= $(\frac{1.04}{1.03})^8\ (1.03)^{39}$ = 3.4215

Answer 7 :

ASSIGNMENT 1

$X = 29{,}50{,}000$

Assets at the end of the year

= 2,50,00,000+29,50,000 +20,00,000-22,00,000-7,50,000

=2,70,00,000

ii) Hence effective yield is

2,70,00,000 =2,50,00,000 * (1+i) +(29,50,000- 22,00,000-7,50,000)*$(1+i)^{0.5}$

2,70,00,000 =2,50,00,000 * (1+i) +0*$(1+i)^{0.5}$

(1+i) = 1.08 => i = 8%

Answer 8:

i) $v(t) = \exp(-\int_0^t \delta(s)\,ds)$

$v(t) = \exp(-\int_0^t 0.07\,ds) = \exp(-0.07t)$ for $0 \le t < 4$

$v(t) = \exp(-\int_0^4 0.07\,ds - \int_4^t 0.06\,ds) = \exp(-0.04 - 0.06t)$ for $4 \le t < 8$

$v(t) = \exp(-\int_0^4 0.07\,ds - \int_4^8 0.06\,ds - \int_8^t 0.05\,ds) = \exp(-0.08 - 0.04 - 0.05t)$

$v(t) = \exp(-0.12 - 0.05t)$ for $8 \le t$

ii) Accumulated value = 5000* v(3)/v(15)

= 5000* exp(-0.21)/ exp(-0.12-0.75)

= 5000 * exp(0.66)

= INR 9673.96

iii) Let $\delta(t)$ be the constant force of interest

5000* exp(12 δ) = 5000* exp(0.66)

12 δ = 0.66

δ = 0.055

So i= 0.0576

v) Present value

1000*(1+v(1)+v(2) +……….v(9))

1000* (1+e(-0.07)+ e(-0.14)+ e(-0.21)+ e(-0.28)+ e(-0.34)+ e(-0.40)+ e(-0.46)+ e(-0.52)+ e(-0.57))

= INR 7,541.54

Answer 9

i)

a) $1/(1+ i^{(2)} /2)^2 = (1- 0.06 /4)^4$

$(1+ i^{(2)} /2)^2 = 1.0623193$

$i^{(2)} =$ 6.1377 % p.a. convertible half-yearly

b) $(1- d^{(12)} /12)^{12} = (1- 0.06 /4)^4$

$(1- d^{(12)} /12)^{12} = 0.941337$

$d^{(12)} =$ 6.0303% convertible monthly

ii) a) Original term of the loan = 365 day

No deferment – settlement date is actual date of repayment17th July 2005

Outstanding term of the loan = 272 days.

The rebate allowed = (272/365) x 20000 = 14904.1096

Sum paid to settle the loan = (220000 – 14904.1096) = 205095.89

b) The loan was repaid after 93 days, the APR obtained is:

$200000\,(1+i)^{93/365}$ = 205095.89

⇨ i = 10.3%

Answer 10:

ii) Measure time in years from the start of the given year

Given $\delta(0) = 0.15$ and let

$\delta(t) = 0.15 + bt + ct^2$ (A)

Putting t= ½ in the equation (A)

$0.10 = 0.15 + ½ b + 1/4c$

Putting t= 1 in the equation (A)

$0.08 = 0.15 + b + c$

Solving the above two equations, b= -0.13 and c= 0.06

Putting the values of b and c in equation A

$\delta(t) = 0.15 - 0.13t + 0.06t^2$

This implies that $\int_0^1 \delta(t)\,dt = 0.105$

The accumulated amount at the end of year is:

1,50,000x exp(0.105) = 1,50,000x 1.1107= Rs1,66,606.59