INSTITUTE OF ACTUARIAL & QUANTITATIVE STUDIES

Subject: Financial mathematics

Chapter:

Category: Assignment 1 Solutions

Ans 1 :

<u>Solution 7:</u> Option B

Workings only for reference:

Return earned = 98/96.5-1 = 1.55%

We note that: (1+4%)^(144/365) = 1.55%. Therefore, Option B.

144 days

Ans 2:

i) Option B

Workings for reference (no marks to be awarded for showing steps as this is an MCQ)

Number of days the bill is held by the first purchaser = t

$103\ x\ (1.05)^t = 104$

$t = log 1.009709/log 1.05 = 0.198$

$Converted\ to\ days, this\ is\ 0.198\ x\ 365 = 72.2809\ or\ 72\ days$

ii) Option B

Workings for reference (no marks to be awarded for showing steps as this is an MCQ)

Number of days the bill was held by second investor = 91 – 72 = 19 days

$$104\ x\ (1 + \frac{19}{365} i) = 105$$

Solving this, we get i = 0.1875 = 19% return

Ans 3

i) At the outset the investor has a negative cashflow [½]

In return the investor receives a series of regular interest payments linked to an index [½] reflecting the effects of inflation [½]. A final capital repayment [½], which is also linked to an index [½] and may be subjected to a lag.

ii) An endowment assurance is a contract that provides a survival benefit at the end of the term, but it also provides a lump sum benefit on death before the end of the term. [1]

The benefits are provided in return for a series of regular premiums (or a single premium). [½]

The sum assured payable on death or survival need not be the same, although generally they often are. [½] The term of the contract could be fixed or up to certain age. [½]

iii) An investor may lend an amount of INR 1 at time 0 in return for a repayment of (1+i) at time 1. In this case, i is the interest paid at the end of the year and hence considered to be the effective rate of interest per unit time. [1]

In case an investor lends and amount of (1-d) at time 0 in return for a payment of 1 at time 1. In this case d is called the effective rate of discount per unit time. [½]

The accumulation factor for 4% pa simple interest rate over 5 years is:

$A(5) = 1+5*0.04 = 1.2$

[½]

The accumulation factor for effective interest rate over 5 years is:

$A(5) = (1+i)^{\wedge 5}$

[½]

If the two rates are equivalent, then they result in the same accumulation factors:

$(1+i)^{\wedge 5} = 1.2$

⇨ 0.03714 [½]

Ans 4

i)

a) $i^{(4)} = 6\%$ p.a.

Let i be the effective monthly rate of interest

$(1+i)^{\wedge 12} = (1+ i^{(4)}/4)^{\wedge 4}$

i = 0.498%

b) $i^{(6)} = 10\%$ p.a.

Let i be the effective monthly rate of interest

$(1+i)^{\wedge 12} = (1+ i^{(6)}/6)^{\wedge 6}$

i = 0.830%

ii)

given that

$$\delta(t) = \begin{cases} .05+.005\,t & 0<=t<5 \\ .08t - .01 & 5<=t<10 \\ .10 & 10<=t \end{cases}$$

The present value at time 2 of a payment of 5000 at time 15 years will be

$$PV = 5000 * e^{-\int_2^5 (.05+.005t)dt} * e^{-\int_5^{10} (.08t-.01)dt} * e^{-\int_{10}^{15} (.10)dt}$$

$$= 5000 * e^{-[.05t + \frac{.005t^2}{2}]_2^5} * e^{-[\frac{.08t^2}{2} - .01t]_5^{10}} * e^{-[.10t]_{10}^{15}}$$

$$= 5000 * e^{-[.05(5-2) + \frac{.005}{2}(25-4)]} * e^{-[.04(100-25) - .01(10-5)]} * e^{-[.10(15-10)]}$$

$$= 5000 * e^{-[.2025]} * e^{-[2.95]} * e^{-[.5]}$$

$$= 129.63$$

Hence the present value is Rs. 129.63

Ans 5:

ii) $\left(1 - \frac{91}{365} \times 0.07\right) = \left(1 + \frac{i^{(2)}}{2}\right)^{\frac{-91}{182.5}}$

$$\Rightarrow i^{(2)} = (0.982548^{\frac{-182.5}{91}} - 1) \times 2 = 0.03594 \times 2$$

$$\Rightarrow i^{(2)} = 0.07188 = 7.19\% \text{ p.a. convertible half-yearly}$$

Ans 6.

i)

a) $(1+\frac{i^{(2)}}{2})^2 = (1+\frac{i^{\wedge}((12)}{12})^{12}$

$(1+\frac{i^{(2)}}{2}) = (1+\frac{0.07}{12})^6$

$i^{(2)} = 7.103\%$

b) $(1-\frac{d^{(12)}}{12})^{12} = (1+\frac{i^{\wedge}((12)}{12})^{-12}$

$(1-\frac{d^{12}}{12}) = (1+\frac{0.07}{12})^{-1}$

$d^{(12)} = 6.959\%$

ii) $\delta(t) = \delta_0 + \frac{t}{m}(\delta_m - \delta_0)$ for 0 <= t <= m

$\delta(t) = \delta_m$ for t>m

Required accumulated value

$\exp\left(\int_0^n \delta(t)dt\right.$

$= \text{Exp}\{\int_0^m[\delta_0 + \frac{t}{m}(\delta_m - \delta_0)]\,dt + \delta_m(n-m)\}$

$=\exp\{[\delta_0 t + \frac{t^2}{2m}(\delta_m - \delta_0)]_{t=0}^{t=m} + \delta_m(n-m)\}$

$= \exp[\delta_0 m + \frac{m}{2}(\delta_m - \delta_0) + \delta_m(n-m)]$

$= \exp[\frac{m}{2}(\delta_0 - \delta_m) + \delta_m n]$

$= [\exp(\delta_0 - \delta_m)]^{m/2}[exp(\delta_m)]^n$

$= (\frac{1.04}{1.03})^8 (1.03)^{39} = 3.4215$

Ans 7 :

i) 20,00,000 = 8% x[2,50,00,000 + 0.5 * (X- 22,00,000-7,50,000)]

= 18,82,000 +0.04 X

X = 29,50,000

Assets at the end of the year

= 2,50,00,000+29,50,000 +20,00,000-22,00,000-7,50,000

=2,70,00,000

ii) Hence effective yield is

2,70,00,000 =2,50,00,000 * (1+i) +(29,50,000- 22,00,000-7,50,000)*$(1+i)^{0.5}$

2,70,00,000 =2,50,00,000 * (1+i) +0*$(1+i)^{0.5}$

(1+i) = 1.08 => i = 8%

Ans 8:

i) $v(t) = \exp(- \int_0^t \delta(s)\, ds)$

$v(t) = \exp(- \int_0^t 0.07\, ds) = \exp(- 0.07t)$ for $0 \le t < 4$

$v(t) = \exp(- \int_0^4 0.07\, ds - \int_4^t 0.06\, ds) = \exp(- 0.04 - 0.06t)$ for $4 \le t < 8$

$v(t) = \exp(- \int_0^4 0.07\, ds - \int_4^8 0.06\, ds - \int_8^t 0.05\, ds) = \exp(-0.08 - 0.04 - 0.05t)$

$v(t) = \exp(-0.12 - 0.05t)$ for $8 \le t$

ii) Accumulated value = 5000* v(3)/v(15)

= 5000* exp(-0.21)/ exp(-0.12-0.75)

= 5000 * exp(0.66)

= INR 9673.96

iii) Let δ(t) be the constant force of interest

5000* exp(12 δ) = 5000* exp(0.66)
12 δ = 0.66

δ = 0.055

iv) $1/v(20) = 1/\exp(-0.12-1) = 3.06485 = (1+i)^{\wedge}20$

Where i is the effective annual rate

So i= 0.0576

v) Present value

1000*(1+v(1)+v(2) +……….v(9))

1000* (1+e(-0.07)+ e(-0.14)+ e(-0.21)+ e(-0.28)+ e(-0.34)+ e(-0.40)+ e(-0.46)+ e(-0.52)+ e(-0.57))

= INR 7,541.54

Ans 9

i)

a) $1/(1+ i^{(2)} /2)^2 = (1- 0.06 /4)^4$

$(1+ i^{(2)} /2)^2 = 1.0623193$

$i^{(2)} =$ 6.1377 % p.a. convertible half-yearly

b) $(1- d^{(12)} /12)^{12} = (1- 0.06 /4)^4$
$(1- d^{(12)} /12)^{12} = 0.941337$
$d^{(12)} = 6.0303\%$ convertible monthly

ii) a) Original term of the loan = 365 day
No deferment – settlement date is actual date of repayment17th July 2005

Outstanding term of the loan = 272 days.
The rebate allowed = (272/365) x 20000 = 14904.1096
Sum paid to settle the loan = (220000 – 14904.1096) = 205095.89

b) The loan was repaid after 93 days, the APR obtained is:

$200000\,(1+i)^{93/365}$ = 205095.89
⇨ i = 10.3%

Ans 10:

i) $\int_0^t \delta(t)dt = -\log v(t)$

$\delta(t) = -(v'(t)/v(t))$

$\delta(t) = (2t + 2\alpha + 1)/((t + \alpha)(t + \alpha + 1))$

ii) Measure time in years from the start of the given year

Given $\delta(0) = 0.15$ and let

$\delta(t) = 0.15 + bt + ct^2$ (A)

Putting t= ½ in the equation (A)

$0.10 = 0.15 + \frac{1}{2}b + 1/4c$

Putting t= 1 in the equation (A)

$0.08 = 0.15 + b + c$

Solving the above two equations, b= -0.13 and c= 0.06

Putting the values of b and c in equation A

$\delta(t) = 0.15 - 0.13t + 0.06t^2$

This implies that $\int_0^1 \delta(t)\,dt = 0.105$

The accumulated amount at the end of year is:

1,50,000x exp(0.105) = 1,50,000x 1.1107= Rs1,66,606.59

Ans 11.

i) a) Debentures are part of the loan capital of the company. The term "loan capital" usually refers to long term borrowings rather than short term. The issuing company provides some form of security to holders of the debenture. This is usually in the form of a floating charge against the assets of the company.

b) Unsecured loan stocks have no explicit assets backing them and holders rank alongside other unsecured creditors. Yields will be higher than on the debentures to reflect the higher risk of default.

ii) Reasons: • The return on property may be lower, especially in the short term. • There is less flexibility because of the large unit sizes. • It may be difficult and expensive to value the property. • There are large buying and selling expenses. • The maintenance costs associated with property investment are also high. • There may be periods when the property is void and no income is received. • Marketability is poor

iii) a) Government may raise money by floating a loan on stock exchange. The terms of the issue are set out by tender. The investors are invited to nominate the price that they are prepared to pay and the loan is issued to the highest bidders, subject to certain rules of allocation.

b) Advantages: The government will know the price that the investor will pay at outset so it will know the cost of borrowing money. It will be administratively less difficult by tender. Disadvantages: The investor may be willing to pay more for the bond than the set price and so the money could be borrowed more cheaply if the tender is used Insufficient investor may be prepared to pay the set price causing only part of the offer to be old and the Government may not meet it finance requirement

c) Securities with uncertain income include: 1) Equities – dividend vary according to the performance of the company issuing stocks and may be zero 2) Property – rent payments are subject to review. 3) Index-linked bonds – coupon payment and final redemption price may increase in proportion to the increase in the relevant index of inflation

Ans 12.

i) $20{,}000 * (1 + i^4 / 4)^{4x17/12} = 26{,}000$

$(1 + i^4 / 4) = (26{,}000/20{,}000)^{12/68}$

$i^4 = (1.0474 – 1) * 4 = 18.96\%$

ii) $26{,}000 * (1 – d^2 / 2)^{2x17/12} = 20{,}000$

$(1 + d^2 / 2) = (20{,}000/26{,}000)^{12/34}$

$d^4 = (1 - .91156) * 2 = 17.69\%$

iii) $20{,}000 * (1 + i * 17/12) = 26{,}000$

$i = (26{,}000/20{,}000) * 12/17 – 1 = 21.18\%$

iv)

- If using simple interest, the rate of interest needs to be higher when compared to the compound interest in order to achieve the same overall amount.
- Interest rate will get lower with higher frequency of compounding.

Ans 13.

i) *d* is the interest payable at the beginning of the year on a loan of 1 unit repayable at the end of the year. The corresponding interest payable at the end of year on 1unit of loan is i. Thus, d must equal the present value at the beginning of the year of the payment i payable at the end of the year i.e. $d= iv$.

ii) Given i = 0.08

a. $d^{(12)}$

$$v=1-d = \left[1 - d^{(12)}/12\right]^{12}$$
$$d^{(12)} = 12 \times \left[1 - v^{(1/12)}\right]$$
$$d^{(12)} = 12 \times \left[1 - 1.08^{\left(\frac{-1}{12}\right)}\right]$$
$$= 0.076714776$$

b. $i^{(365)}$

$$(1+i) = \left[1 + i^{(365)}/365\right]^{365}$$
$$i^{(365)} = 365 \times \left[1.08^{\left(\frac{1}{365}\right)} - 1\right]$$
$$= 0.076969155$$

c. δ

$$\delta = \log(1+i)$$
$$= \log(1.08)$$
$$= 0.076961041$$

d. $i^{(1/2)}$

$$i^{(1/2)} = 0.5 \times \left[1.08^{(2)} - 1\right]$$
$$= 0.0832$$

Ans 14

i) $v(t) = \exp\left[-\int_0^t \delta(s)ds\right]$

Hence

$$\int_0^t \delta(s)ds = \begin{cases} 0.08t & for\ 0 \le t \le 5 \\ 0.1 + 0.06\,t & for\ 5 \le t \le 10 \\ 0.3 + 0.04\,t & for\ t \ge 10 \end{cases}$$

$$v(t) = \begin{cases} \exp(-0.08t) & for\ 0 \le t \le 5 \\ \exp(-0.1 - 0.06\,t) & for\ 5 \le t \le 10 \\ \exp(-0.3 - 0.04\,t) & for\ t \ge 10 \end{cases}$$