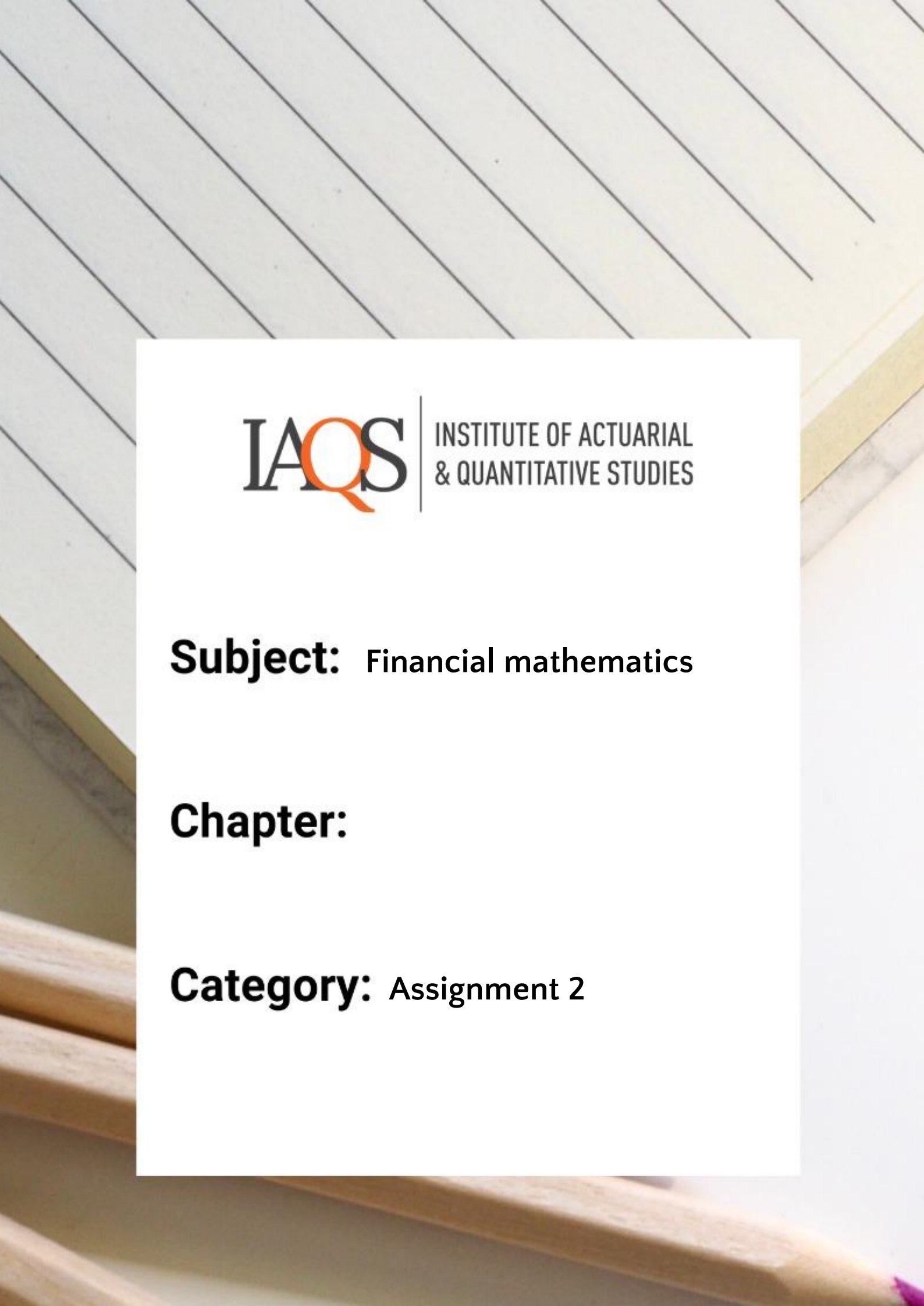

INSTITUTE OF ACTUARIAL & QUANTITATIVE STUDIES

Subject: Financial mathematics

Chapter:

Category: Assignment 2

1. i) Name the criterion to measure the profitability of a capital project.
ii) Explain why the discounted payback period is a poorer decision criterion than net present value assuming that the investor is not short of capital.

Answer:

i) The criterion to measure profitability of a capital project are

1. NPV- Net Present Value
2. IRR- Internal rate of Return
3. DPP- Discounted Payback Period

ii) If the investor is not short of capital the DPP is possibly not a good criterion to measure profitability of a project as it only indicates when a project would come into profitability and does not indicate how profitable the project is.

2. Ram invests into a 25-year savings plan by paying Rs. 500 every half year in advance. The interest rate is 6% per annum convertible monthly for the first 15 years and 6% per annum convertible half yearly for the final 10 years. Determine the fund value at the end of policy term.

Answer:

Fund value after 25 years:

For the first 15 years, i(12) = 6%
Therefore, effective interest rate i = (1 + 6%/12)^12-1 = 6.168%
Effective interest rate half-yearly = 3.038%
Corresponding value for d =

For the next 10 years, j(2) = 6%
Therefore, effective interest rate j = (1 + 6%/2)^2-1 = 6.090%
Effective interest rate half-yearly = 3.000%

$500\,\ddot{s}_{\overline{30}|}^{(3.038\%)} \times (1.03)^{20} + 500 \times \ddot{s}_{\overline{20}|}^{(3\%)}$
= 500 x 49.3215 x 1.8061 + 500 x 27.6765

= 58,378

3. i) Define Annual Percentage Rate (APR). What is the purpose of the APR?
ii) Describe the prospective and retrospective methods of determining the loan outstanding.

ASSIGNMENT 2

A student takes out a student mortgage loan of Rs. 20,00,000 with a term of 15 years. The loan is repayable in monthly level instalments in arrears. Interest rate charged is 6% p.a. effective.

iii) For the loan above, calculate the capital repaid in the seventh year and the interest component of the 85th loan instalment.

iv) 10 years after taking out this loan, the student decides to repay back the loan by paying 150% of original loan instalment as the monthly instalment for the remainder of the term. Calculate the length of time by which the term of the loan will reduce if repayments are altered and determine the savings on interest payments.

Answer:

i) APR is the effective annual rate of interest....
... rounded to the nearest $1/10^{th}$ of 1%

In the UK, lenders are required to provide the APR. As lenders provide the information on the interest rate in consistent format, the consumer is able to draw comparisons and make informed decisions.

ii) Prospective method involves finding the present value of future payments
Retrospective method involves calculating the accumulated value of the initial loan less the accumulated value of payments till date

A student takes out a student mortgage loan of Rs. 2,000,000 with a term of 15 years. The loan is repayable in monthly level instalments in arrears. Interest rate charged is 6% p.a. effective

iii) Let X be the monthly instalment

2000000 = 12X a15(12)

So, X = 16,706

Capital outstanding at the start of the seventh year is calculated prospectively as

12X a9 = 1,400,642

Capital outstanding at the end of the seventh year

12X a8 = 1,278,755

Capital repaid = 1400642 – 1278755 = 121,887

Interest component of the 85th instalment = 1278755 X (1.06^1/12 -1) = 6,224

iv) Loan outstanding at the end of 10 years = 12X a5 = 867,433

Monthly instalment = 150% x 16,706 = 25,059

Therefore the equation of value = 867,433 = 12 x 25,059 x an

Solving for n, we get 3.17, i.e. 13 years and 2 months

So loan reduces by 1 year and 10 months.

Total interest payments over term of loan as per original loan:
12 x 16,706 x 15 – 2000000 = 1,007,058

Total interest payments over term of loan as per revised schedule:
12 x 16,706 x 10 + 12 x 25,059 x (3 + 2/12) – 2000000 = 956,962

Interest saved = 1007058 – 956962 = 50,097

4. i) A student develops a line graph plotting the net present value of a stream cash flows. The x-axis represents the discount rate and the y-axis represents amounts. What do the points at which this graph intersects with the x-axis and y-axis represent?
ii) Consider a project where external borrowing is involved. List the two key project appraisal metrics which should be considered. Further, provide a list of other factors for the company to consider when deciding between projects.

Answer:

i) The x-axis represents the discount rate and the y-axis represents amounts.

The point at which graph intersects x-axis represent the internal rate of return
The point at which graph intersects y-axis represents the sum of all cash flows

ASSIGNMENT 2

ii) Two key project appraisal metrics where borrowing is allowed:

1. Accumulated profit, taking into account conditions of the loan
2. Discounted payback period

Other factors to consider

Cash flows profile such as are the cash flow requirements consistent with other business needs, over what period will profits be produced.

Borrowing requirement: can the business raise the necessary cash at times required, what rate of interest will the business have to pay on borrowed funds, are the time limits or other restrictions imposed on borrowings

Resources: Are the other resources required for the project available, does the business have necessary staff, technical expertise, equipment

Risk: financial risks of the project, appropriateness of risk discount rate, can suppliers be relied on, can timelines be met

Investment conditions: What is the economic climate, are interest rates expected to rise or fall?

Indirect benefits: Are there any additional benefits associated with the project, will it provide value in future.

5. Mr. Khurana purchases INR100,000 nominal of a bond on 1 January 2022 which is redeemable at 105 in four years' time and pays coupons of 4% per annum at the end of each year. He wishes to invest the coupon payments in deposit until the bond is redeemed.
Calculate the mean value of the total accumulated investment on 31 December 2025 if the annual effective rate of interest has an expected value of 5.5% in 2022 and 2023, 6% in 2024 and 4.5% in 2025 assuming that the rate of interest at which the coupon payments can be invested is a random variable and the rate of interest in any one year is independent of that in any other year.
Calculate the accumulated value (workings are not required to be provided).

Answer:

Option A

Workings only for reference:

Let the accumulated value of the investment be X and i_y =investment return for the year y (working in '000s)

$E(X)=E[4*(1+i_{2004})*(1+i_{2005})*(1+i_{2006})+4*(1+i_{2005})*(1+i_{2006})+4*(1+i_{2006})+105+4]$

$= 4*(1.055*1.06*1.045+1.06*1.045+1.045)+109$

=122.29 per 1000

ASSIGNMENT 2

6. A loan was taken out on 1 September 2008 and was repayable by the following increasing annuity. The first repayment of INR1,000 was made on 1 July 2009. Thereafter, payments were made on 1 November, 1 March and 1 July each year until 1 March 2014, inclusive. Each payment was 5% greater than its predecessor. The effective rate of interest throughout the period was 6% per annum.

i) Calculate the initial loan amount.

ii) Calculate the capital repaid in first instalment.

iii) Calculate both the capital component and the interest component of the seventh repayment.

Answer:

i) Loan outstanding on 1 September 2008 = present value of annuity payments

$=1000*(v^{10/12}+1.05v^{14/12}+1.05^2*v^{18/12}+.....+1.05^{14}v^{66/12})$

$=1000*\{1-[v^{4/12}(1.05)]^{15}\}/\{1-v^{4/12}(1.05)\}*v^{10/12}$

$=17,692$

ii) Loan outstanding on 1 June 2009 = $17692*(1.06)^{10/12}$

$= 18,572$

Interest paid in first instalment = 18,572 – 17,692 = 880

Capital repayment = 1,000 – 880 = 120

iii) Capital outstanding after 6 repayments = PV of payment as at 1 March 2011

$= 1000 * (1.05)^6 *(v^{4/12}+1.05*v^{8/12}+....+(1.05)^8 *v^{36/12})$

$= 13,341$

Interest in 7th Payment=$13,341 * ((1.06)^{4/12}-1) = 262$

$= 262$

Capital in 7th payment = $1000 * (1.05)^6 – 262$

$= 1340 – 262 = 1078$

7. Allen wishes to plan for a pocket allowance of Rs.5,000 per month for his daughter during her four years degree course starting from 1st January 2023. He is thinking of purchasing a deferred annuity where deferment period is one year and pays Rs.5,000 at the beginning of each month during the period from 1st January 2023 to 31st December 2026.

Calculate the amount Allen requires to purchase the annuity as at 1st January 2022. Assume effective rate of interest 9% pa.

Answer:

ASSIGNMENT 2

Option B

(No marks for working)

The present value of payment as at 01/01/2023 is $5000\ \ddot{a}_{\overline{48}|}^{(12)}$

So, the present value as at 01/01/2022 is $5000v\ \ddot{a}_{\overline{48}|}^{(12)} = 5000 * v * \frac{1-v^{(12).48}}{d^{(12)}}$

$$= 5000 * 0.917431 * \frac{0.29157}{.0072} = Rs.186{,}912$$

8. A loan is repayable by annual installments in arrears for 20 years. The initial installment is Rs 50,000 and it decreases by Rs.2,000 each year subsequently. The effective rate of interest over the term of loan is 4% per annum.

i) Calculate the original loan amount.

ii) Calculate the capital repayment in the 12th installment.

After the 12th installment has been paid, the borrower decides to repay the loan with level annual installments which equal the 12th installment in amount. There will not be any further reduction in installment amount leading to a reduced term of the loan. Final installment will also be reduced to the level to clear the debt.

iii) Calculate the remaining term of the loan.

iv) Calculate the final reduced payment made to clear the debt.

v) Calculate the total interest paid by the borrower during the term of this loan

Answer:

i) $L = 50000v + 48000v^2 + 46000v^3 \ldots. + 12000v^{20}$

$= 52000\ (v + v^2 + v^3 + \cdots + v^{20})$ - $2000\ (v + 2v^2 + 3v^3 + \cdots + 20v^{20})$

$= 52000\ a_{\overline{20}|} - 2000(Ia)_{\overline{20}|}$

$52000 * 13.5903 - 2000 * 125.155 = Rs.456385.60$

ii) **Interest component in 12th instalment:**

Amount of 12th instalment= 50,000 – 2000*12 = 28000

Loan outstanding after 11th instalment

$L_{11} = 28000v + 26000v^2 + \cdots + 12000v^9$

$= 30000\ a_{\overline{9}|} - 2000(Ia)_9$

$= 30000 * 7.4353 - 2000 * 35.2366 = Rs.152585.80$

Interest component in 12th instalment= Rs.152585.80*.04 = Rs.6103.43

Hence, Capital repaid in 12th instalment =28000-6103.43 = Rs. 21896.57

iii) **Remaining term of Loan:**

Loan o/s after 12th instalment = 152585.80-21896.57 = Rs.130689.23

Level annual instalment= Rs.28000

Remaining term is given by equation:

$$130689.23 = 28000 * a_{\overline{n}|}$$

$$a_{\overline{n}|} \geq \frac{130689.23}{28000} \text{ i.e. } a_{\overline{n}|} \geq 4.6675$$

$$a_{\overline{5}|} = 4.4518 \;\&\; a_{\overline{6}|} = 5.2421 \qquad Hence\ n = 6$$

iv) **Calculating Final reduced payment**

Let R be the final payment

$$130689.23 = 28000 * a_{\overline{5}|} + Rv^6$$

$$130689.23 = 28000 * 4.4518 + 0.79031R$$

$$R = \frac{130689.23 - 28000 * 4.4518}{0.79013} = Rs.7641.09$$

v) **Interest paid during term of loan**

Total amount paid by way of annual instalments = Rs. 615641.09

Loan amount= Rs.456385.60

Interest paid= 615641.09-456385.60 = Rs. 159255.49

9. Pallavi is a student studying in the USA. She funded her two years masters' degree through personal savings and a student loan. The total fee for her education was INR 15,000,000. On 1st March 2015, the student loan was made available to her by ABC Bank at an interest rate of 6.5% p.a. compounded annually. The loan was to be repaid through monthly instalments of INR 200,000 paid in arrears over a period of 5 years, with the first payment being due on 31 March 2017.

i) Calculate the amount of personal savings put in by Pallavi in March 2015 to fund her education.

On 1st March 2020, Pallavi decided to take out a home loan for INR 10,000,000. ABC bank offered a facility to combine the home loan with her outstanding student loan and also offered better terms. The total loan was to be repaid over a period of 10 years, through annual instalments paid in arrears at an interest rate of 5% p.a. compounded annually.

ii) Calculate the annual instalments that Pallavi would pay in future.

The other option for Pallavi is to keep the two loans separate. In this case the student loan will continue of the earlier agreed terms. On the other hand, the home loan will need to be repaid over a period of 12 years, through annual instalments paid in advance at an interest rate of 5% p.a. compounded annually. Pallavi has decided that she will choose the option, which results in a lower cumulative interest amount payable.

ASSIGNMENT 2

iii) Calculate the cumulative interest amount under both available options and identify the option that Pallavi should choose.

Answer:

i) Monthly effective interest rate = $i^{(12)}/12 = (1+i)^{(1/12)} - 1 = 0.5262\%$

Annuity function (in arrears) $a_{601}^{@0.5262\%} = 51.33692$

X = PV of installments on 1 March 2017 = 200,000 * $a_{601}^{@0.5262\%}$
= INR 10,267,383

Loan amount as at March 2015 = $X/(1+6.5\%)^2$ = INR 9,052,334

Savings consumed = 15,000,000 – 9,052,334 = 5,947,666

ii) As on 1 March 2020, there are 24 monthly instalments due for the student loan

Annuity function (in arrears) $a_{241}^{@0.5262\%} = 22.49$

Outstanding loan amount as at 1 March 2020 = 2,00,000 x 22.49 = INR 4,498,198

New loan amount = 14,498,198

This is to be paid in 10 annual instalments in arrears at 5% p.a.

$a_{101}^{@5\%} = 7.72$

Annual instalment = 14,498,198 / 7.72 = 1,877,583

iii) Annuity function in advance $a(due)_{121}^{@5\%} = 9.31$

Loan amount = INR 10,000,000

Instalment = INR 10,000,000/9.31 = 1,074,528

Total loan amount as on 31 March 2020 = 14,498,198

Choice A = 10 instalments of INR 1,877,583

So rupee value of interest payment = 1,877,583 x10 – 14,498,198
= 4,277,632

Choice B = 24 instalments of INR 2,00,000 and 12 instalments of INR 10,74,528

So rupee value of interest payment = 2,00,000 * 24 + 10,74,528 * 12 – 1,44,98,198

= 1,76,94,333 – 1,44,98,198

= 31,96,134

Choice B has lower cumulative interest – so Pallavi will choose Option B

10. A perpetuity costs 79.56 and makes end-of-year payments. The perpetuity pays 1 at the end of year 2, 2 at the end of year 3, ..., n at the end of year (n+1). After year (n+1), the payments remain constant at n. The annual effective interest rate is 10.5%. Find n

Answer:

ASSIGNMENT 2

Answer : c)

$79.56 = v(Ia)_{n|} + \frac{nv^{n+1}}{i}$

$$= v\left[\frac{\ddot{a}_{n|} - nv^n}{i}\right] + \frac{nv^{n+1}}{i}$$

$$= v\frac{\ddot{a}_{n|}}{i} - \frac{nv^{n+1}}{i} + \frac{nv^{n+1}}{i}$$

$$= v\frac{\ddot{a}_{n|}}{i}$$

$$= \frac{\ddot{a}_{n|}}{i}$$

$$= \frac{1-v^n}{i^2}$$

$$= \frac{1-v^n}{(0.105)^2}$$

$$= \frac{1-v^n}{.011025}$$

$\Rightarrow 1 - v^n = .871145$

$\Rightarrow v^n = .12285$

$\Rightarrow 1.105^{-n} = .12285$

$\Rightarrow n = -\frac{\ln(0.12285)}{\ln(1.105)}$

$\Rightarrow n = 21$

11. An Insurance Company plans to explore replacing its existing leased IT system with a new policy administration system to gain better control on administration and enhance the operational efficiency. The cost of procuring the system has been estimated at INR 15,00,000 and its usual working lifetime is assumed to be 10 years. The vendor supplying the new system will buy it back at the end of 10 years at the price assuming depreciation of 2% p.a. at simple rate.

The annual maintenance cost incurred by the insurance company will be 5% p.a. of the procurement cost payable quarterly starting from the beginning of the second year till its usual working lifetime.

The Company will further leverage the system to one of its clients & receive a rent of INR 2,40,000/- per annum payable monthly in arrears starting from first month.

i) Calculate NPV of the project assuming rate of interest of 8% p.a. convertible half yearly.

The Company instead of purchasing the system decides to continue with the lease arrangement and decides to invest the procurement cost in a 10-year zero-coupon bond giving a return of 10% p.a. compounding yearly. In addition, the Company pays the lessor an annual amount of INR 1,80,000/- payable monthly at the end of each month for using the leased asset for 10 years.

ii) Calculate NPV of the leased arrangement assuming rate of interest of 8% p.a. convertible half yearly and comment which of the arrangement will be profitable for the Company

Answer:

ASSIGNMENT 2

i) $i^{(2)} = 8\%$ p.a.

Let i be the effective annual rate of interest

$(1+i) = (1+ i^{(2)}/2)^{\wedge 2}$

i= 8.16% p.a.

Let i_{12} and i_4 be the effective monthly and quarterly rate of interest

$i_{12} = (1+i)^{\wedge(1/12)}-1 = 0.66\%$

$i_4 = (1+i)^{\wedge(1/4)}-1 = 1.98\%$

Outgo:

- At outset = Rs 15,00,000
- Maintenance cost:
  Payable for 9 years
  Maintenance cost per quarter = 5%* 15,00,000/4 = 18,750

  PV of maintenance cost = $v^{@i} * 18750* \ddot{a}_{361}^{@ i_4}$

Total outgo =

$15,00,000 + v^{@i} * 18750* \ddot{a}_{361}^{@ i_4}$

=15,00,000 + 4,51,956

=19,51,956

[1

Income:

- Buyback price:

  Depreciation (D) = 15,00,000*2%*10 = 3,00,000

  Buyback price at the end of 10^{th} year = 15,00,000-3,00,000 = 12,00,000

  [0.5

  PV of buyback price at outset = $12,00,000 * v^{10}$ @i

  = 5,47,664

  [1

- Rental income:

ASSIGNMENT 2

Monthly rent = 2,40,000/12 = 20,000

PV of rental income = 20,000 * $a_{120}^{@\ i_{12}}$ = 16,57,813

Total income = 5,47,664 + 16,57,813 = 22,05,477

NPV = Income - Outgo

= 2,53,521

ii) NPV with Second option

Outgo:

PV of lease rent = (180000 / 12) * $a_{120}^{@\ i_{12}}$

= 12,43,359

[1]

Investment amount = 15,00,000

Income:

PV of income = 15,00,000*(1.10)^10 / (1.0816)^10

= 17,75,625

[1]

NPV = Income – Outgo – Investment Income

= 17,75,625 - 12,43,359 – 15,00,000 = -9,67,734

[0.5]

The first option is more profitable as its NPV is more as compared to first option.

12. An investor has an amount of INR 10,000 to invest in two investment options. She has to put atleast 25% of the capital in each of the options. Calculate the maximum accumulated value of the investments after a period of 5 years.

Under each of the options, the investment made at time 0 accumulates at:

Option A: Compound discount rate of 7% per annum convertible half-yearly for 2 years followed by nominal rate of interest of 8% per annum convertible quarterly for the balance period.

Option B: Force of interest $\delta(t) = 0.05+0.003t$ for $0 \le t \le 5$.

Answer:

ASSIGNMENT 2

iv) Accumulated value of Rs. 1 invested at time 0 after 5 years for each of the options is:

Option A:

Accumulated Value = $1 * 1 / [(1 - 0.07/2)^4]^* (1 + 0.08/4)^{12}$

$= 1 * (1 - 0.07/2)^{-4} * (1 + 0.08/4)^{12}$
$= 1 * 1.1532 * 1.2682$
$= 1.46248 \sim 1.4625$ ---------- (1)

Option B:

$$A(0,5) = \exp \int_0^5 [0.05 + 0.003t]\, dt$$

$$= \exp \left[0.05t + 0.003 \frac{t^2}{2}\right]_0^5$$

$$= \exp \left[0.05 * (5) + 0.003 * \frac{(5)^2}{2}\right]$$

$$= \exp [0.25 + 0.0375]$$

$$= \exp [0.2875]$$

$$= 1.333091$$

Accumulated Value = $1 * 1.333091 \sim 1.3331$ ---------- (2)

From (1) and (2) above, the maximum accumulated value of the investments after a period of 5 years is under Option A. Therefore, to maximise the return, the investor should invest 75% in Option A and 25% in Option B.

Hence, the accumulated amount of Rs. 10,000 after 5 years is :

$1000 [0.75 * 1.4625 + 0.25 * 1.3331]$

= Rs. 14301.5 [4]

13. A builder is constructing a shopping mall which will have 42 shops. The total cost of construction is expected to be INR 105 crores out of which INR 10 crores will be incurred at the start of the project followed by INR 9.5 crores at the end of the first month. Thereafter every month end the expenses will reduce by INR 0.5 crores as compared to that incurred in the preceding month till it becomes NIL. It is expected that the rental income from each of the shops will be INR 5 lakhs per month payable in advance starting from 21st month. Maintenance cost of the mall is expected to be INR 10 lakhs per month payable in advance starting from 25th month.

i) Explain what is meant by a) Discounted Payback Period (DPP) of a project; b) Internal Rate of Return.
ii) Calculate the present value of cost of construction at an effective rate of interest of 9% p.a. effective.
iii) Calculate the DPP for the project at an effective rate of interest of 9% p.a. effective.

ASSIGNMENT 2

iv) After 15 years from the start of construction, the builder decides to sell the shopping mall. Calculate the price at which the mall should be sold to get an IRR of 9% p.a. effective.

Answer:

i)

a) Discounted Payback Period of a project: An investment project usually requires expenditure (or investment) in the early years before returns (or profits) emerge in the later years. So during the early years the 'project account' will be overdrawn and interest will be incurred at the borrowing rate.

The discounted payback period is the time from the beginning of the project until the 'project account' balance first becomes positive

Or

Discounted payback period ends at the first point in time such that the net present value (or accumulated value) of project cashflows up to that time is positive.

b) The internal rate of return for an investment project is the effective rate of interest that equates the present value of income and outgo, i.e. it makes the net present value of the cashflows equal to zero.

If all the payments for the project were transacted through a bank account that earned interest at the same rate as the internal rate of return, the net proceeds at the end of the project (i.e. the accumulated profit) would be zero. A higher internal rate of return indicates a more "profitable" project.

ii) Working in terms of monthly units of time, effective monthly interest rate =

$i(12)/12 = 0.086488/12$

$= 0.007207333$

The project plan is as below:

Cashflow item	**Timing**	**Amount (in crores)**
Cost of construction	Every month starting from Time 0	$-[10 - t(0.5)]$ where $t = 1, 2, \ldots 19$ (in months)
Maintenance Cost of 24 months	Every month, starting Immediately after completion	- 0.10
Rental income	At the start of each month beginning with 21st month	+ 2.1

Let C be the present value of cost of construction.

Therefore, $C = 10 + (10 - 0.5)\, v + [10 - (0.5)\, 2]\, v^2 + \ldots\ldots + [10 - (0.5)\, 19]\, v^{19}$

$= 10\, \ddot{a}_{\overline{20}|} - 0.5\, (Ia)_{\overline{19}|}$

$= 10\, \frac{1 - v^{20}}{d} - 0.5\, \frac{(\ddot{a}_{\overline{19}|} - 19\, v^{19})}{i} = 186.97044 - 86.57$

$= 100.3910$

Hence, the present value of cost of construction is Rs. 100.3910 crores.

$v^{19} = 0.872452$
$v^{20} = 0.866208$
$i = 0.007207$
$d = 0.007156$
$\ddot{a}_{\overline{20}|} = 18.697044$
$\ddot{a}_{\overline{19}|} = 17.824592$

iii) Let *t* be the DPP in months

$\Rightarrow \quad - C + 2.1 * \ddot{a}_{\overline{t-20}|}\, v^{20} - 0.1 * \ddot{a}_{\overline{t-24}|}\, v^{24} \geq 0$

$= - 100.3910 + 2.1 * \ddot{a}_{\overline{4}|}\, v^{20} + 2.1 * \ddot{a}_{\overline{t-24}|}\, v^{24} - 0.1 * \ddot{a}_{\overline{t-24}|}\, v^{24}$

$= - 100.3910 + 2.1 * 3.957272 * 0.866214 + 2 * \ddot{a}_{\overline{t-24}|}\, v^{24}$

$= - 100.3910 + 7.198473 + 2 * 0.841686 * \ddot{a}_{\overline{t-24}|}$

$= - 93.192527 + 1.683372 * \ddot{a}_{\overline{t-24}|}$

$= - 93.192527 + 1.683372 * \frac{(1 - v^{t-24})}{d}$

$= - 93.192527 + 1.683372 * \frac{(1 - v^{t-24})}{0.007155}$

$= - 93.192527 + 235.2721174 * (1 - v^{t-24})$

$93.192527 = 235.2721174 * (1 - v^{t-24})$

$93.192527 = 235.2721174 - 235.2721174 * v^{t-24}$

$93.192527 - 235.2721174 = 235.2721174 * v^{t-24}$

$142.07959 = 235.2721174 * v^{t-24}$

$v^{t-24} = \frac{142.07959}{235.272117}$

$v^{t-24} = 0.603895$

$\frac{v^t}{v^{24}} = 0.603895$

$\frac{v^t}{0.841686} = 0.603895$

$v^t = 0.508290$

$\frac{1}{(1+i)^t} = 0.508290$

$\frac{1}{(1.007207)^t} = 0.508290$

At t = 100, LHS = 0.48767
t = 90, LHS = 0.523979

Using interpolation t = 95 months

iv) Let P be the price at which Mall is sold after 15 years i.e. 180 months

Using the calculations in 5iii) above with t =180 months, the equation of value becomes:

$142.07959 = 235.2721\ v^{(180-24)} + P\ v^{180}$

$P\ v^{180} = 142.07959 - 235.2721\ v^{156}$ @0.007207333

$v^{156}=1.0072^{-156} = 0.3265$

$v^{180}=1.0072^{-180}= 0.2749$

$0.2749P = 65.2633$

P = Rs. 237.41 crores

14.

The ABC trust runs the investment fund which had following fund value at the various dates:

Date	Fund Value (in Rs crores)
1st January, 2011	30.50
30th September, 2011	33.00
31st December, 2011	38.50
1st July, 2012	41.05
31st December, 2012	45.00
30th June, 2013	45.60
31st December, 2013	47.00

Over the three years period the investment fund has the following inflows and outflows:

Date	In-flows (in Rs crores)	Out-flows (in Rs crores)
1st October, 2011	6.00	-
30th June, 2012	4.50	-
1st July, 2013	-	2.50

i) Calculate the annual effective TWRR & MWRR earned in the fund from 1/1/2011 – 31/12/2013
ii) Calculate the linked rate of return from 1/1/2011 – 31/12/2013
iii) Why TWRR is better measure of investment performance than MWRR

Answer:

i) **TWRR**

$(1+i)^3$ = 33/30.50 * (41.05-4.5)/(33+6) * 45.6/41.05 * 47/(45.6-2.50)

$(1+i)^3$ = 1.081967 * 0.937179 * 1.11084 * 1.090487

$(1+i)^3$ = 1.228313

(1+i) = 1.070951

TWRR = 7.0951%

MWRR

$30.5*(1+i)^3 + 6*(1+i)^{2.25} + 4.5*(1+i)^{1.5} - 2.5*(1+i)^{0.5} = 47$

At i = 7%, LHS = 46.74

At i = 7.5%, LHS = 47.37

By interpolation,

MWRR = 7.206%

ii) Linked rate of return

Year 2011 – equation of value

$30.5*(1+i) + 6*(1+i)^{0.25} = 38.50$

i= 6%, LHS = 38.42

i= 6.5%, LHS = 38.57

By interpolation, i = 6.266%

Year 2012 – equation of value

$38.50*(1+i) + 4.50*(1+i)^{0.50} = 45$

i= 4.5%, LHS = 44.83

i= 5%, LHS = 45.03

By interpolation

i = 4.92%

Year 2013 – equation of value

$45*(1+i) - 2.50*(1+i)^{\wedge 0.5} = 47$

i= 10%, LHS = 46.88

i= 10.5%, LHS = 47.097

By interpolation

i = 10.276%

Linked rate of return

$(1+i)^{\wedge 3} = (1+.06266)*(1+.0492)*(1+.10276)$

$(1+i)^{\wedge 3} = 1.2295$

i = 7.13%

iii)

TWRR eliminates the effect of the cashflow amount and timing, and therefore gives a fairer basis for assessing the investment performance for the fund. MWRR is sensitive to the amount and timing of the net cashflows.

15.

Given below is the information concerning a pension fund:
(Figures in crores)

Date	Fund Value	Cash flow
1/1/2015	460	
31/3/2015	500	
1/4/2015		40
31/12/2015		-50
1/1/2016	550	
1/7/2016	600	
31/12/2016	X	

i) If the TWRR earned on the fund during the period from 1/1/2015 to 31/12/2016 is 20% p.a. effectively, calculate X the value of fund on 31/12/2016

ii) Calculate annual effective MWRR

iii) Calculate annual effective linked rate of return using subintervals of one year.

iv) State when the linked internal rate of return will be identical to the TWRR

v) Which is a better measure of fund performance TWRR or MWRR? Why?

Answer:

i) Time weighted rate of return is given by:

$$(1+i)^2 = \frac{500}{460} \times \frac{(550+50)}{(500+40)} \times \frac{x}{550}$$

$$(1+i)^2 = 1.44$$

$$\Rightarrow X = (1.44 \times 550) / (1.086957 \times 1.1111)$$

$$\Rightarrow X = 655.78$$

ii) Money weighted rate of return is given by the equation:

$$460\,(1+i)^2 + 40(1+i)^{7/4} - 50(1+i) = 655.78$$

For i = 20%, LHS = 657.4335

For i = 19%, LHS = 646.1394

So, MWRR = 19.85% p.a.

iii) The effective rate of return for the year 2015 is given by solving the equation of value:

$460\,(1+i) + 40\,(1+i)^{\left(\frac{3}{4}\right)} = 600$

⇨ i= 20.44%

The effective rate of return for the year 2016

$(1+i) = \frac{655.78}{550} = 1.1923$

So the linked rate of return is

$(1+i)^2 = 1.1923 * 1.2044$

Hence LIRR = 19.83%

iv) When the sub-intervals chosen for calculating the Linked internal rate of return coincide with the intervals between net cash flows being received then the Linked internal rate of return will be identical to the time weighted rate of return. [1]

v)

- ⇨ Time Weighted Rate of Return (TWRR) is a better measure of fund manager's performance
- ⇨ Money-weighted rate of return (MWRR) is sensitive to the amounts and timings of the net cash flows, but fund manager does not have any control on these.
- ⇨ Time-weighted rate of return (TWRR) is calculated using the "growth factors" reflecting the change in fund value between the times of consecutive cash flows. Thus TWRR is independent of amount and timing of cash-flows.
- ⇨ Hence Time-weighted rate of return (TWRR) is a better measure of fund manager's performance [2]