



Subject: Financial Mathematics 1

Chapter: 5,6,7,8 (Unit 3 & 4)

Category: Assignment 2

Q1)

A young couple, Mr. & Mrs. Jones took a personal loan for an amount of INR 20,000 to refurbish their home. The loan is being repaid with payments of INR 427.90 made monthly in arrears for 5 years.

i) Calculate the Annual Percentage Rate (APR) of the couple's loan.

However, the couple have been struggling to repay the loan for last few months, since Mr. Jones lost his job. After exactly one year, the loan company offers to help the couple by restructuring their loan with new monthly payments of INR 274.49 made in arrears.

ii) Assuming the APR remains unchanged from the original loan; calculate the term of the new loan.

iii) Mr. & Mrs. Jones' initial reaction towards the restructured loan is positive, both in terms of the monthly repayment amount and the term of the loan. However, they are unsure if this may mean that they are paying too much as compared to the original loan. Calculate how much more interest in total the couple will pay on their restructured loan than on the original loan

Q2)

i) Define the Terms with reference to an investment Project.

a) Discounted payback period

b) Payback period

ii) Compare both the discounted payback period and the payback period with the net present value for determining whether to proceed with an investment project and explain why the latter is a superior measure.

iii) An investor is looking to invest in an infrastructure project. He has two choices available between two different cities. For each of the projects, the expected cashflows have been advised to the investor as being as follows.

For Project A, an initial expense of INR 170,000 has been estimated. This is followed by a further outgo of INR 20,000 at the end of first year and INR 10,000 at the end of second year. However, the project promises impressive incoming cash flows of INR 20,000 each at the end of year 1 and 2, and income of INR 200,000 at the end of year 3.

On the other hand, Project B demands an initial investment of INR 30,000 more than that of Project A, but no further ongoing expenses. In return, this project is expected to provide an income of INR 14,000 pa at the end of each of the first 6 years. This is followed by the return of the entire initial investment at the end of the 6th year.

a) Calculate the internal rate of return (correct to 1 decimal place) for each project.

b) Calculate the net present value of each project using a risk discount rate of 6% pa.

c) The investor is looking to explore the option of funding the projects through borrowing money from a bank. However, he is unsure about the impact this would have on the profitability of the projects. Calculate the maximum interest rate that could be charged by the bank, above which each project becomes unprofitable. Mention any other factors that the investor should consider when deciding between the projects

Q3)

On 1st November 1985, a man was in receipt of three annuities mentioned below:

Annuity 1: INR 200 p.a. payable annually on 1st February each year, the final payment being 1st February 2007

Annuity 2: INR 320 p.a. payable quarterly on 1st January, 1st April, 1st July and 1st October each year, the final payment being 1st January 2002

Annuity 3: INR 180 p.a. payable monthly on the first day of each month, the final payment being 1st August 2004

Immediately after receiving the monthly payment due on 1st November 1985, the man requested that these three annuities be combined into a single annuity payable half-yearly on 1st February and 1st August in each subsequent year, the final payment being 1st February 2007. The man's request was granted.

Find the amount of the revised annuity, given that it was calculated at an interest rate of 8% p.a. effective, all months are assumed to have equal length.

Q4)

Derive, from the first principles, the formula for $(I\ddot{a})_n$ for $i > 0$.

Q5)

During the period 1st January 2014 to 1st January 2015, the unit prices of two funds a “property fund” and “Equity Fund” are as mentioned below:

	Unit prices (2014)				Unit prices (2015)
	1 st January	1 st April	1 st July	1 st October	1 st January
Property Fund	12.4	13.1	14.8	15.8	16.4
Equity Fund	12.1	9.2	10.3	13.1	15.5

- i) Find the time-weighted rate of return for each of the fund for the year 2014
- ii) An investor bought units in the property fund on 1st January, 1st April, 1st July and 1st October 2014, and sold his units on 1st January 2015, find the yield on the complete transaction, if
 - a) He bought the same number of units on each date.
 - b) He invested the same sum of money on each date.
- iii) Repeat the calculation in ii(a) and ii(b) above, except that the investments are done in equity fund.

Q6)

A loan of INR 160,000 was issued to be repaid by a level annuity certain payable annually in arrears over a term of 10 years and calculated at an interest rate of 8% p.a. The terms of the loan provided that at any time the lender could alter the rate of interest, in which case the amount of the annual repayment would be revised appropriately.

- i) Find the initial amount of the annual repayment
- ii) Immediately after the fourth repayment, the annual rate of interest was increased to 10% p.a. Find the revised amount of the level annual repayment.
- iii) Immediately after the seventh repayment, the annual rate of interest was reduced to 9% p.a. There was no further change to the rate of interest. Find the final amount of the level annual repayment and the effective rate of interest paid by the borrower on the complete transaction.

Q7)

A Government owned company, ONGC has just purchased an oil rig for Rs 2.7 crores along with the drilling rights for three years in a section of Arabian Sea. The rig will be operated for three years at an annual cost of Rs 20,00,000 payable at the end of each year. At the end of third year the rig will be sold for Rs 10,00,000.

At most one oil strike can occur in any year and the probabilities of striking oil in first, second and third years are 0.25, 0.2 and 0.1 respectively. These probabilities are independent of each other. The company assumes that there will be a continuous output from each strike at the rate of 50,000 barrels per year for ten years commencing at the end of the year of the strike.

i) Assuming a constant oil price of Rs 200 per barrel, find the effective annual rate of interest at which the present value of the company's outlays equals the expected present value of its income from the project.

ii) If a successful oil strike occurs only in the third year and oil price following that remains constant at Rs 170 per barrel, calculate the Net present value (NPV) of the project at 10% p.a. effective rate of return.

Also comment whether it is profitable to invest in the project.

Q8)

The ABC trust runs the investment fund which had following fund value at the various dates:

Date	Fund Value (in Rs crores)
1 st January, 2011	30.50
30 th September, 2011	33.00
31 st December, 2011	38.50
1 st July, 2012	41.05
31 st December, 2012	45.00
30 th June, 2013	45.60
31 st December, 2013	47.00

Over the three years period the investment fund has the following inflows and outflows:

Date	In-flows (in Rs crores)	Out-flows (in Rs crores)
1 st October, 2011	6.00	-
30 th June, 2012	4.50	-
1 st July, 2013	-	2.50

- Calculate the annual effective TWRR & MWRR earned in the fund from 1/1/2011 – 31/12/2013
- Calculate the linked rate of return from 1/1/2011 – 31/12/2013
- Why TWRR is better measure of investment performance than MWRR

Q9)

Mr. Sam wishes to purchase a house and has decided to take the loan from a bank to finance the purchase. The bank agrees to finance the purchase with the following terms:

- 80% of the house cost will be financed by the bank and the remaining cost will be borne by Mr. Sam.
- The loan is repayable by an increasing annuity, payable annually in arrears for 15 years. The repayment at the end of first year is Rs. 2,00,000 and subsequent repayments increase by Rs. 20,000 every year.

The repayments were calculated using a rate of interest of 12% per annum effective.

- i) Calculate the amount of loan granted by the bank and the total cost of the house.
- ii) Construct the schedule of repayment for years nine and ten (i.e. immediately after the 8th repayment installment), showing the outstanding loan at the beginning of the year, the repayment installment, interest portion and capital repayment portion in each year.

Immediately after the tenth installment of repayment, Mr. Sam wishes to transfer its balance loan to another bank which offered him the loan at the effective rate of 10% per annum with regular annual payments of fixed amount.

The new bank charges the upfront processing fees of 1% of loan outstanding. The other terms of the loan are unchanged.

Calculate the revised annual payments for Mr. Sam.

Based on your calculations, comment whether it is advisable for Mr. Sam to transfer its loan to a new bank.

Q10)

Given below is the information concerning a pension fund:
(Figures in crores)

Date	Fund Value	Cash flow
1/1/2015	460	
31/3/2015	500	
1/4/2015		40
31/12/2015		-50
1/1/2016	550	
1/7/2016	600	
31/12/2016	X	

- i) If the TWRR earned on the fund during the period from 1/1/2015 to 31/12/2016 is 20% p.a. effectively, calculate X the value of fund on 31/12/2016
- ii) Calculate annual effective MWRR
- iii) Calculate annual effective linked rate of return using subintervals of one year.
- iv) State when the linked internal rate of return will be identical to the TWRR
- v) Which is a better measure of fund performance TWRR or MWRR? Why?

Q11)

A businessman has decided to purchase a leasehold property for INR 800,000 with a further payment of INR 50,000 for repairs in one year's time. The income associated with letting the property will be INR 100,000 per annum payable continuously for 20 years commencing in two years' time.

- i) Explain what is a discounted payback period (DPP) for a project
- ii) a) Given that the venture will be financed by bank loans on the basis of an effective annual rate of 7% and that the loans may be repaid continuously, find the discounted payback period (DPP) for the project.
- b) After the loan has been repaid the businessman will deposit all the available income in an account which will earn interest of 6% p.a. Find the accumulated amount in the account in 22 years' time.
- iii) Assuming that the bank loan is repaid partially at the end of each complete year, interest is paid annually in arrear and the businessman earns interest on his rental income at an annual effective rate of 6%, find the DPP and the accumulated amount in the businessman's account after 22 years.

Q12)

A loan of INR 988,000 was granted on 10th July 2015. The loan is repayable by a level annuity payable monthly in arrears (on the 10th of each month) for 25 years and is calculated on the basis of interest rate of 7% p.a. effective.

Determine the following:-

- i) The monthly repayment
- ii) The loan outstanding immediately after the repayment on 10th March 2029
- iii) The capital to be repaid on 10th October 2026
- iv) The capital to be repaid and the amount of interest to be paid in monthly installments due between 10th April 2033 and 10th March 2034 (both dates inclusive).



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