



**Subject:** Financial mathematics

**Chapter:**

**Category:** Assignment 2

1. i) Name the criterion to measure the profitability of a capital project.
- ii) Explain why the discounted payback period is a poorer decision criterion than net present value assuming that the investor is not short of capital.

2. Ram invests into a 25-year savings plan by paying Rs. 500 every half year in advance. The interest rate is 6% per annum convertible monthly for the first 15 years and 6% per annum convertible half yearly for the final 10 years. Determine the fund value at the end of policy term.

3. i) Define Annual Percentage Rate (APR). What is the purpose of the APR?
- ii) Describe the prospective and retrospective methods of determining the loan outstanding.

A student takes out a student mortgage loan of Rs. 20,00,000 with a term of 15 years. The loan is repayable in monthly level instalments in arrears. Interest rate charged is 6% p.a. effective.

- iii) For the loan above, calculate the capital repaid in the seventh year and the interest component of the 85th loan instalment.
- iv) 10 years after taking out this loan, the student decides to repay back the loan by paying 150% of original loan instalment as the monthly instalment for the remainder of the term. Calculate the length of time by which the term of the loan will reduce if repayments are altered and determine the savings on interest payments.

4. i) A student develops a line graph plotting the net present value of a stream cash flows. The x-axis represents the discount rate and the y-axis represents amounts. What do the points at which this graph intersects with the x-axis and y-axis represent?
- ii) Consider a project where external borrowing is involved. List the two key project appraisal metrics which should be considered. Further, provide a list of other factors for the company to consider when deciding between projects.

5. Mr. Khurana purchases INR100,000 nominal of a bond on 1 January 2022 which is redeemable at 105 in four years' time and pays coupons of 4% per annum at the end of each year. He wishes to invest the coupon payments in deposit until the bond is redeemed.

Calculate the mean value of the total accumulated investment on 31 December 2025 if the annual effective rate of interest has an expected value of 5.5% in 2022 and 2023, 6% in 2024 and 4.5% in 2025 assuming that the rate of interest at which the coupon payments can be invested is a random variable and the rate of interest in any one year is independent of that in any other year.

Calculate the accumulated value (workings are not required to be provided).

6. A loan was taken out on 1 September 2008 and was repayable by the following increasing annuity. The first repayment of INR1,000 was made on 1 July 2009. Thereafter, payments were made on 1

iii) Calculate both the capital component and the interest component of the seventh repayment.

7. Allen wishes to plan for a pocket allowance of Rs.5,000 per month for his daughter during her four years degree course starting from 1st January 2023. He is thinking of purchasing a deferred annuity where deferment period is one year and pays Rs.5,000 at the beginning of each month during the period from 1st January 2023 to 31st December 2026.

Calculate the amount Allen requires to purchase the annuity as at 1st January 2022. Assume effective rate of interest 9% pa.

8. A loan is repayable by annual installments in arrears for 20 years. The initial installment is Rs 50,000 and it decreases by Rs.2,000 each year subsequently. The effective rate of interest over the term of loan is 4% per annum.

i) Calculate the original loan amount.

ii) Calculate the capital repayment in the 12th installment.

After the 12th installment has been paid, the borrower decides to repay the loan with level annual installments which equal the 12th installment in amount. There will not be any further reduction in installment amount leading to a reduced term of the loan. Final installment will also be reduced to the level to clear the debt.

iii) Calculate the remaining term of the loan.

iv) Calculate the final reduced payment made to clear the debt.

v) Calculate the total interest paid by the borrower during the term of this loan

9. Pallavi is a student studying in the USA. She funded her two years masters' degree through personal savings and a student loan. The total fee for her education was INR 15,000,000. On 1st March 2015, the student loan was made available to her by ABC Bank at an interest rate of 6.5% p.a. compounded annually. The loan was to be repaid through monthly instalments of INR 200,000 paid in arrears over a period of 5 years, with the first payment being due on 31 March 2017.

i) Calculate the amount of personal savings put in by Pallavi in March 2015 to fund her education.

On 1st March 2020, Pallavi decided to take out a home loan for INR 10,000,000. ABC bank offered a facility to combine the home loan with her outstanding student loan and also offered better terms. The total loan was to be repaid over a period of 10 years, through annual instalments paid in arrears at an interest rate of 5% p.a. compounded annually.

ii) Calculate the annual instalments that Pallavi would pay in future.

The other option for Pallavi is to keep the two loans separate. In this case the student loan will continue of the earlier agreed terms. On the other hand, the home loan will need to be repaid over a period of 12 years, through annual instalments paid in advance at an interest rate of 5% p.a. compounded annually. Pallavi has decided that she will choose the option, which results in a lower cumulative interest amount payable.

10. A perpetuity costs ₹9.56 and makes end-of-year payments. The perpetuity pays ₹1 at the end of year 2, ₹2 at the end of year 3, ..., ₹n at the end of year (n+1). After year (n+1), the payments remain constant at ₹n. The annual effective interest rate is 10.5%. Find n

11. An Insurance Company plans to explore replacing its existing leased IT system with a new policy administration system to gain better control on administration and enhance the operational efficiency. The cost of procuring the system has been estimated at INR 15,00,000 and its usual working lifetime is assumed to be 10 years. The vendor supplying the new system will buy it back at the end of 10 years at the price assuming depreciation of 2% p.a. at simple rate.

The annual maintenance cost incurred by the insurance company will be 5% p.a. of the procurement cost payable quarterly starting from the beginning of the second year till its usual working lifetime.

The Company will further leverage the system to one of its clients & receive a rent of INR 2,40,000/- per annum payable monthly in arrears starting from first month.

i) Calculate NPV of the project assuming rate of interest of 8% p.a. convertible half yearly.

The Company instead of purchasing the system decides to continue with the lease arrangement and decides to invest the procurement cost in a 10-year zero-coupon bond giving a return of 10% p.a. compounding yearly. In addition, the Company pays the lessor an annual amount of INR 1,80,000/- payable monthly at the end of each month for using the leased asset for 10 years.

ii) Calculate NPV of the leased arrangement assuming rate of interest of 8% p.a. convertible half yearly and comment which of the arrangement will be profitable for the Company

12. An investor has an amount of INR 10,000 to invest in two investment options. She has to put at least 25% of the capital in each of the options. Calculate the maximum accumulated value of the investments after a period of 5 years.

Under each of the options, the investment made at time 0 accumulates at:

Option A: Compound discount rate of 7% per annum convertible half-yearly for 2 years followed by nominal rate of interest of 8% per annum convertible quarterly for the balance period.

Option B: Force of interest  $\delta(t) = 0.05 + 0.003t$  for  $0 \leq t \leq 5$ .

13. A builder is constructing a shopping mall which will have 42 shops. The total cost of construction is expected to be INR 105 crores out of which INR 10 crores will be incurred at the start of the project followed by INR 9.5 crores at the end of the first month. Thereafter every month end the expenses will reduce by INR 0.5 crores as compared to that incurred in the preceding month till it becomes NIL. It is expected that the rental income from each of the shops will be INR 5 lakhs per month payable in advance starting from 21st month. Maintenance cost of the mall is expected to be INR 10 lakhs per month payable in advance starting from 25th month.

- Explain what is meant by a) Discounted Payback Period (DPP) of a project; b) Internal Rate of Return.
- Calculate the present value of cost of construction at an effective rate of interest of 9% p.a. effective.
- Calculate the DPP for the project at an effective rate of interest of 9% p.a. effective.

14.

The ABC trust runs the investment fund which had following fund value at the various dates:

| Date                             | Fund Value (in Rs crores) |
|----------------------------------|---------------------------|
| 1 <sup>st</sup> January, 2011    | 30.50                     |
| 30 <sup>th</sup> September, 2011 | 33.00                     |
| 31 <sup>st</sup> December, 2011  | 38.50                     |
| 1 <sup>st</sup> July, 2012       | 41.05                     |
| 31 <sup>st</sup> December, 2012  | 45.00                     |
| 30 <sup>th</sup> June, 2013      | 45.60                     |
| 31 <sup>st</sup> December, 2013  | 47.00                     |

Over the three years period the investment fund has the following inflows and outflows:

| Date                          | In-flows (in Rs crores) | Out-flows (in Rs crores) |
|-------------------------------|-------------------------|--------------------------|
| 1 <sup>st</sup> October, 2011 | 6.00                    | -                        |
| 30 <sup>th</sup> June, 2012   | 4.50                    | -                        |
| 1 <sup>st</sup> July, 2013    | -                       | 2.50                     |

- Calculate the annual effective TWRR & MWRR earned in the fund from 1/1/2011 – 31/12/2013
- Calculate the linked rate of return from 1/1/2011 – 31/12/2013
- Why TWRR is better measure of investment performance than MWRR

15.

Given below is the information concerning a pension fund:  
(Figures in crores)

| Date       | Fund Value | Cash flow |
|------------|------------|-----------|
| 1/1/2015   | 460        |           |
| 31/3/2015  | 500        |           |
| 1/4/2015   |            | 40        |
| 31/12/2015 |            | -50       |
| 1/1/2016   | 550        |           |
| 1/7/2016   | 600        |           |
| 31/12/2016 | X          |           |

- If the TWRR earned on the fund during the period from 1/1/2015 to 31/12/2016 is 20% p.a. effectively, calculate X the value of fund on 31/12/2016
- Calculate annual effective MWRR
- Calculate annual effective linked rate of return using subintervals of one year.



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