INSTITUTE OF ACTUARIAL & QUANTITATIVE STUDIES

Subject: Financial Mathematics

Chapter:

Category: Assignment 2 Solutions

Ans 1:

i) The criterion to measure profitability of a capital project are

1. NPV- Net Present Value
2. IRR- Internal rate of Return
3. DPP- Discounted Payback Period

ii) If the investor is not short of capital the DPP is possibly not a good criterion to measure profitability of a project as it only indicates when a project would come into profitability and does not indicate how profitable the project is.

Ans 2:

Fund value after 25 years:

For the first 15 years, i(12) = 6%
Therefore, effective interest rate i = (1 + 6%/12)^12-1 = 6.168%
Effective interest rate half-yearly = 3.038%
Corresponding value for d =

For the next 10 years, j(2) = 6%
Therefore, effective interest rate j = (1 + 6%/2)^2-1 = 6.090%
Effective interest rate half-yearly = 3.000%

500 $\ddot{S}^{(3.038\%)}_{\overline{30|}}$ x$(1.03)^{20}$ +500x$\ddot{S}^{(3\%)}_{\overline{20|}}$
= 500 x 49.3215 x 1.8061 + 500 x 27.6765

= 58,378

Ans 3:

i) APR is the effective annual rate of interest….
… rounded to the nearest 1/10th of 1%

In the UK, lenders are required to provide the APR. As lenders provide the information on the interest rate in consistent format, the consumer is able to draw comparisons and make informed decisions.

ii) Prospective method involves finding the present value of future payments
Retrospective method involves calculating the accumulated value of the initial loan less the accumulated value of payments till date

A student takes out a student mortgage loan of Rs. 2,000,000 with a term of 15 years. The loan is repayable in monthly level instalments in arrears. Interest rate charged is 6% p.a. effective

iii) Let X be the monthly instalment

2000000 = 12X a15(12)

So, X = 16,706

Capital outstanding at the start of the seventh year is calculated prospectively as

12X a9 = 1,400,642

Capital outstanding at the end of the seventh year

12X a8 = 1,278,755

Capital repaid = 1400642 – 1278755 = 121,887

Interest component of the 85th instalment = 1278755 X (1.06^1/12 -1) = 6,224

iv) Loan outstanding at the end of 10 years = 12X a5 = 867,433

Monthly instalment = 150% x 16,706 = 25,059

Therefore the equation of value = 867,433 = 12 x 25,059 x an

Solving for n, we get 3.17, i.e. 13 years and 2 months

So loan reduces by 1 year and 10 months.

Total interest payments over term of loan as per original loan:
12 x 16,706 x 15 – 2000000 = 1,007,058

Total interest payments over term of loan as per revised schedule:
12 x 16,706 x 10 + 12 x 25,059 x (3 + 2/12) – 2000000 = 956,962

Interest saved = 1007058 – 956962 = 50,097

Ans 4:

i) The x-axis represents the discount rate and the y-axis represents amounts.

The point at which graph intersects x-axis represent the internal rate of return
The point at which graph intersects y-axis represents the sum of all cash flows

ii) Two key project appraisal metrics where borrowing is allowed:

1. Accumulated profit, taking into account conditions of the loan
2. Discounted payback period

Other factors to consider

Cash flows profile such as are the cash flow requirements consistent with other business needs, over what period will profits be produced.

Borrowing requirement: can the business raise the necessary cash at times required, what rate of interest will the business have to pay on borrowed funds, are the time limits or other restrictions imposed on borrowings

Resources: Are the other resources required for the project available, does the business have necessary staff, technical expertise, equipment

Risk: financial risks of the project, appropriateness of risk discount rate, can suppliers be relied on, can timelines be met

Investment conditions: What is the economic climate, are interest rates expected to rise or fall?

Indirect benefits: Are there any additional benefits associated with the project, will it provide value in future.

Ans 5:

Option A

Workings only for reference:

Let the accumulated value of the investment be X and i_y =investment return for the year y
(working in '000s)

$E(X)=E[4 *(1+i_{2004})*(1+i_{2005})*(1+i_{2006}) +4*(1+i_{2005})*(1+i_{2006})+4*(1+i_{2006})+105+4]$

$= 4*(1.055*1.06*1.045+1.06*1.045+1.045)+109$

=122.29 per 1000

Ans 6:

i) Loan outstanding on 1 September 2008 = present value of annuity payments

$=1000*(v^{10/12}+1.05v^{14/12}+1.05^{2}*v^{18/12}+......+1.05^{14}v^{66/12})$

$=1000*\{1-[v^{4/12}(1.05)]^{15}\}/\{1-v^{4/12}(1.05)\}*v^{10/12}$

=17,692

ii) Loan outstanding on 1 June 2009 = $17692*(1.06)^{10/12}$

= 18,572

Interest paid in first instalment = 18,572 – 17,692 = 880

Capital repayment = 1,000 – 880 = 120

iii) Capital outstanding after 6 repayments = PV of payment as at 1 March 2011

$= 1000 * (1.05)^{6} *(v^{4/12}+1.05*v^{8/12}+....+(1.05)^{8}*v^{36/12})$

= 13,341

Interest in 7th Payment=$13,341 * ((1.06)^{4/12}-1) = 262$

= 262

Capital in 7th payment = $1000 * (1.05)^{6} – 262$

= 1340 – 262 = 1078

Ans 7:

Option B

(No marks for working)

The present value of payment as at 01/01/2023 is $5000\ \ddot{a}_{\overline{48}|}^{(12)}$

So, the present value as at 01/01/2022 is $5000v\ \ddot{a}_{\overline{48}|}^{(12)} = 5000 * v * \frac{1-v^{(12).48}}{d^{(12)}}$

$$= 5000 * 0.917431 * \frac{0.29157}{.0072} = Rs.186{,}912$$

Ans 8:

i) $L = 50000v + 48000v^2 + 46000v^3 \dots . +12000v^{20}$
$= 52000\,(v + v^2 + v^3 + \dots + v^{20})$ - $2000\,(v + 2v^2 + 3v^3 + \dots + 20v^{20})$

$= 52000\, a_{\overline{20|}} - 2000(\,Ia)_{\overline{20|}}$
$52000 * 13.5903 - 2000 * 125.155 = Rs.\,456385.60$

ii) **Interest component in 12th instalment:**

Amount of 12th instalment= 50,000 – 2000*12 = 28000
Loan outstanding after 11th instalment

$L_{11} = 28000v + 26000v^2 + \dots + 12000v^9$
$= 30000\, a_{\overline{9|}} - 2000(Ia)_9$
$= 30000 * 7.4353 - 2000 * 35.2366 = Rs.\,152585.80$
Interest component in 12th instalment= Rs.152585.80*.04 = Rs.6103.43
Hence, Capital repaid in 12th instalment =28000-6103.43 = Rs. 21896.57

iii) **Remaining term of Loan:**
Loan o/s after 12th instalment = 152585.80-21896.57 = Rs.130689.23
Level annual instalment= Rs.28000

Remaining term is given by equation:
$130689.23 = 28000 * a_{\overline{n|}}$

$a_{\overline{n|}} \geq \frac{130689.23}{28000}$ i.e. $a_{\overline{n|}} \geq 4.6675$

$a_{\overline{5|}} = 4.4518 \;\&\; a_{\overline{6|}} = 5.2421 \qquad Hence\ n = 6$

iv) **Calculating Final reduced payment**

Let R be the final payment

$$130689.23 = 28000 * a_{\overline{5|}} + Rv^6$$

$$130689.23 = 28000 * 4.4518 + 0.79031R$$

$$R = \frac{130689.23 - 28000 * 4.4518}{0.79013} = Rs.\,7641.09$$

v) **Interest paid during term of loan**

Total amount paid by way of annual instalments = Rs. 615641.09

Loan amount= Rs.456385.60

Interest paid= 615641.09-456385.60 = Rs. 159255.49

Ans 9:

i) Monthly effective interest rate = $i^{(12)}/12 = (1+i)^{(1/12)} - 1$ = 0.5262%

Annuity function (in arrears) $a_{60|}^{@0.5262\%}$ = 51.33692

X = PV of installments on 1 March 2017 = 200,000 * $a_{60|}^{@0.5262\%}$

= INR 10,267,383

Loan amount as at March 2015 = $X/(1+6.5\%)^2$ = INR 9,052,334

Savings consumed = 15,000,000 – 9,052,334 = 5,947,666

ii) As on 1 March 2020, there are 24 monthly instalments due for the student loan

Annuity function (in arrears) $a_{24|}^{@0.5262\%}$ = 22.49

Outstanding loan amount as at 1 March 2020 = 2,00,000 x 22.49 = INR 4,498,198

New loan amount = 14,498,198

This is to be paid in 10 annual instalments in arrears at 5% p.a.

$a_{10|}^{@5\%}$ = 7.72

Annual instalment = 14,498,198 / 7.72 = 1,877,583

iii) Annuity function in advance $a(due)_{12|}^{@5\%}$ = 9.31

Loan amount = INR 10,000,000

Instalment = INR 10,000,000/9.31 = 1,074,528

Total loan amount as on 31 March 2020 = 14,498,198

Choice A = 10 instalments of INR 1,877,583

So rupee value of interest payment = 1,877,583 x10 – 14,498,198

= 4,277,632

Choice B = 24 instalments of INR 2,00,000 and 12 instalments of INR 10,74,528

So rupee value of interest payment = 2,00,000 * 24 + 10,74,528 * 12 – 1,44,98,198

= 1,76,94,333 – 1,44,98,198

= 31,96,134

Choice B has lower cumulative interest – so Pallavi will choose Option B

Ans 10:

Answer : c)

$$79.56 = v(Ia)_{\overline{n|}} + \frac{nv^{n+1}}{i}$$

$$= v\left[\frac{\ddot{a}_{\overline{n|}} - nv^n}{i}\right] + \frac{nv^{n+1}}{i}$$

$$= v\frac{\ddot{a}_{\overline{n|}}}{i} - \frac{nv^{n+1}}{i} + \frac{nv^{n+1}}{i}$$

$$= v\frac{\ddot{a}_{\overline{n|}}}{i}$$

$$= \frac{a_{\overline{n|}}}{i}$$

$$= \frac{1-v^n}{i^2}$$

$$= \frac{1-v^n}{(0.105)^2}$$

$$= \frac{1-v^n}{.011025}$$

$$\Rightarrow 1 - v^n = .871145$$

$$\Rightarrow v^n = .12285$$

$$\Rightarrow 1.105^{-n} = .12285$$

$$\Rightarrow n = -\frac{\ln(0.12285)}{\ln(1.105)}$$

$$\Rightarrow n = 21$$

Ans 11:

i) $i^{(2)} = 8\%$ p.a.

Let i be the effective annual rate of interest

$(1+i) = (1+ i^{(2)}/2)$^2

i= 8.16% p.a.

Let i_{12} and i_4 be the effective monthly and quarterly rate of interest

$i_{12} = (1+i)^{\wedge(1/12)}-1 = 0.66\%$

$i_4 = (1+i)^{\wedge(1/4)}-1 = 1.98\%$

Outgo:

- At outset = Rs 15,00,000
- Maintenance cost:
  Payable for 9 years
  Maintenance cost per quarter = 5%* 15,00,000/4 = 18,750

  PV of maintenance cost = $v^{@i}$ * 18750* $\ddot{a}_{\overline{36|}}{}^{@\ i_4}$

Total outgo =

15,00,000 + $v^{@i}$ * 18750* $\ddot{a}_{36\rceil}$$^{@}$ i_4

=15,00,000 + 4,51,956

=19,51,956

[1

Income:

- Buyback price:

  Depreciation (D) = 15,00,000*2%*10 = 3,00,000
  Buyback price at the end of 10^{th} year = 15,00,000-3,00,000 = 12,00,000

  [0.5

  PV of buyback price at outset = 12,00,000 * v^{10} @i
  = 5,47,664

  [1

- Rental income:

  Monthly rent = 2,40,000/12 = 20,000
  PV of rental income = 20,000 * $a_{120\rceil}$$^{@}$ i_{12}= 16,57,813

  Total income = 5,47,664 + 16,57,813 = 22,05,477

NPV = Income - Outgo

= 2,53,521

ii) NPV with Second option

Outgo:
PV of lease rent = (180000 / 12) * $a_{120\rceil}$$^{@}$ i_{12}
= 12,43,359

[1]

Investment amount = 15,00,000

Income:
PV of income = 15,00,000*$(1.10)^{\wedge 10}$ / $(1.0816)^{\wedge 10}$
= 17,75,625

[1]

NPV = Income – Outgo – Investment Income
= 17,75,625 - 12,43,359 – 15,00,000 = -9,67,734

[0.5]

The first option is more profitable as its NPV is more as compared to first option.

Ans 12:

iv) Accumulated value of Rs. 1 invested at time 0 after 5 years for each of the options is:

Option A:

Accumulated Value = $1 * 1 / [(1 - 0.07/2)^4] * (1 + 0.08/4)^{12}$

$= 1 * (1 - 0.07/2)^{-4} * (1 + 0.08/4)^{12}$
$= 1 * 1.1532 * 1.2682$
$= 1.46248 \sim 1.4625$ ----------------------- (1)

Option B:

$$A(0,5) = \exp \int_0^5 [0.05 + 0.003t]\, dt$$

$$= \exp \left[0.05t + 0.003 \frac{t^2}{2}\right]_0^5$$

$$= \exp \left[0.05 * (5) + 0.003 * \frac{(5)^2}{2}\right]$$

$$= \exp [0.25 + 0.0375]$$

$$= \exp [0.2875]$$

$$= 1.333091$$

Accumulated Value = $1 * 1.333091 \sim 1.3331$ ----------------------- (2)

From (1) and (2) above, the maximum accumulated value of the investments after a period of 5 years is under Option A. Therefore, to maximise the return, the investor should invest 75% in Option A and 25% in Option B.

Hence, the accumulated amount of Rs. 10,000 after 5 years is :

1000 [0.75 * 1.4625 + 0.25 * 1.3331]

= Rs. 14301.5 [4]

Ans 13:

i)

a) Discounted Payback Period of a project: An investment project usually requires expenditure (or investment) in the early years before returns (or profits) emerge in the later years. So during the early years the 'project account' will be overdrawn and interest will be incurred at the borrowing rate.

The discounted payback period is the time from the beginning of the project until the 'project account' balance first becomes positive

Or

Discounted payback period ends at the first point in time such that the net present value (or accumulated value) of project cashflows up to that time is positive.

b) The internal rate of return for an investment project is the effective rate of interest that equates the present value of income and outgo, i.e. it makes the net present value of the cashflows equal to zero.

If all the payments for the project were transacted through a bank account that earned interest at the same rate as the internal rate of return, the net proceeds at the end of the project (i.e. the accumulated profit) would be zero. A higher internal rate of return indicates a more "profitable" project.

ii) Working in terms of monthly units of time, effective monthly interest rate =

$i^{(12)}/12 = 0.086488/12$

$= 0.007207333$

The project plan is as below:

Cashflow item	Timing	Amount (in crores)
Cost of construction	Every month starting from Time 0	- [10 – t (0.5)] where t= 1, 2, ... 19 (in months)
Maintenance Cost of 24 months	Every month, starting Immediately after completion	- 0.10
Rental income	At the start of each month beginning with 21st month	+ 2.1

Let C be the present value of cost of construction.

Therefore, C = 10 + (10 – 0.5) v + [10 – (0.5) 2] v^2 + ... + [10 – (0.5) 19] v^{19}

$= 10\, \ddot{a}_{\overline{20}|} - 0.5\, (Ia)_{\overline{19}|}$

$= 10\, \frac{1 - v^{20}}{d} - 0.5\, \frac{(\ddot{a}_{\overline{19}|} - 19\, v^{19})}{i} = 186.97044 - 86.57$

= 100.3910

Hence, the present value of cost of construction is Rs. 100.3910 crores.

v^{19} = 0.872452
v^{20} = 0.866208
i = 0.007207
d = 0.007156
$\ddot{a}_{\overline{20}|}$ = 18.697044
$\ddot{a}_{\overline{19}|}$ = 17.824592

iii) Let *t* be the DPP in months

$=> \; - C + 2.1 * \ddot{a}_{\overline{t-20}|}\, v^{20} - 0.1 * \ddot{a}_{\overline{t-24}|}\, v^{24} \geq 0$

$= - 100.3910 + 2.1 * \ddot{a}_{\overline{4}|}\, v^{20} + 2.1 * \ddot{a}_{\overline{t-24}|}\, v^{24} - 0.1 * \ddot{a}_{\overline{t-24}|}\, v^{24}$

$= - 100.3910 + 2.1 * 3.957272 * 0.866214 + 2 * \ddot{a}_{\overline{t-24}|}\, v^{24}$

$= - 100.3910 + 7.198473 + 2 * 0.841686 * \ddot{a}_{\overline{t-24}|}$

$= - 93.192527 + 1.683372 * \ddot{a}_{\overline{t-24}|}$

$= - 93.192527 + 1.683372 * \frac{(1-v^{t-24})}{d}$

$= - 93.192527 + 1.683372 * \frac{(1-v^{t-24})}{0.007155}$

$= - 93.192527 + 235.2721174 * (1 - v^{t-24})$

$93.192527 = 235.2721174 * (1 - v^{t-24})$

$93.192527 = 235.2721174 - 235.2721174 * v^{t-24}$

$93.192527 - 235.2721174 = 235.2721174 * v^{t-24}$

$142.07959 = 235.2721174 * v^{t-24}$

$v^{t-24} = \frac{142.07959}{235.272117}$

$v^{t-24} = 0.603895$

$\frac{v^t}{v^{24}} = 0.603895$

$\frac{v^t}{0.841686} = 0.603895$

$v^t = 0.508290$

$\frac{1}{(1+i)^t} = 0.508290$

$\frac{1}{(1.007207)^t} = 0.508290$

At t = 100, LHS = 0.48767
t = 90, LHS = 0.523979

Using interpolation t = 95 months

iv) Let P be the price at which Mall is sold after 15 years i.e. 180 months

Using the calculations in 5iii) above with t =180 months, the equation of value becomes:

$142.07959 = 235.2721\ v^{(180-24)} + P\ v^{180}$

$P\ v^{180} = 142.07959 - 235.2721\ v^{156}$ @0.007207333

$v^{156} = 1.0072^{-156} = 0.3265$

$v^{180} = 1.0072^{-180} = 0.2749$

0.2749P = 65.2633

P = Rs. 237.41 crores

Ans 14:

i) **TWRR**

$(1+i)^3 = 33/30.50 * (41.05-4.5)/(33+6) * 45.6/41.05 * 47/(45.6-2.50)$

$(1+i)^3 = 1.081967 * 0.937179 * 1.11084 * 1.090487$

$(1+i)^3 = 1.228313$

$(1+i) = 1.070951$

TWRR = 7.0951%

MWRR

$30.5*(1+i)^3 + 6*(1+i)^{2.25} + 4.5*(1+i)^{1.5} - 2.5*(1+i)^{0.5} = 47$

At i = 7%, LHS = 46.74

At i = 7.5%, LHS = 47.37

By interpolation,

MWRR = 7.206%

ii) Linked rate of return

Year 2011 – equation of value

$30.5*(1+i) + 6*(1+i)^{0.25} = 38.50$

i= 6%, LHS = 38.42

i= 6.5%, LHS = 38.57

By interpolation, i = 6.266%

Year 2012 – equation of value

$38.50*(1+i) + 4.50*(1+i)^{0.50} = 45$

i= 4.5%, LHS = 44.83

i= 5%, LHS = 45.03

By interpolation

i = 4.92%

Year 2013 – equation of value

$45*(1+i) - 2.50*(1+i)^{0.5} = 47$

i= 10%, LHS = 46.88

i= 10.5%, LHS = 47.097

By interpolation

i = 10.276%

Linked rate of return

$(1+i)^3 = (1+.06266)*(1+.0492)*(1+.10276)$

$(1+i)^3 = 1.2295$

i = 7.13%

iii)

TWRR eliminates the effect of the cashflow amount and timing, and therefore gives a fairer basis for assessing the investment performance for the fund. MWRR is sensitive to the amount and timing of the net cashflows.

Ans 15:

i) Time weighted rate of return is given by:

$(1+i)^2 = \frac{500}{460} \times \frac{(550+50)}{(500+40)} \times \frac{x}{550}$

$(1+i)^2 = 1.44$

$\Rightarrow$ X = (1.44 x 550) /(1.086957 x 1.1111)

$\Rightarrow$X = 655.78

ii) Money weighted rate of return is given by the equation:

$460\,(1+i)^2 + 40(1+i)^{7/4} - 50(1+i) = 655.78$

For i = 20%, LHS = 657.4335

For i = 19%, LHS = 646.1394

So, MWRR = 19.85% p.a.

iii) The effective rate of return for the year 2015 is given by solving the equation of value:

$460\,(1+i) + 40\,(1+i)^{\left(\frac{3}{4}\right)} = 600$

⇨ i= 20.44%

The effective rate of return for the year 2016

$(1+i) = \frac{655.78}{550} = 1.1923$

So the linked rate of return is

$(1+i)^2 = 1.1923 * 1.2044$

Hence LIRR = 19.83%

iv) When the sub-intervals chosen for calculating the Linked internal rate of return coincide with the intervals between net cash flows being received then the Linked internal rate of return will be identical to the time weighted rate of return. [1]

v)

- ⇨ Time Weighted Rate of Return (TWRR) is a better measure of fund manager's performance
- ⇨ Money-weighted rate of return (MWRR) is sensitive to the amounts and timings of the net cash flows, but fund manager does not have any control on these.
- ⇨ Time-weighted rate of return (TWRR) is calculated using the "growth factors" reflecting the change in fund value between the times of consecutive cash flows. Thus TWRR is independent of amount and timing of cash-flows.
- ⇨ Hence Time-weighted rate of return (TWRR) is a better measure of fund manager's performance [2]