Lecture 1

IAQS | INSTITUTE OF ACTUARIAL & QUANTITATIVE STUDIES

Class: FY MSc

Subject : Financial Mathematics

Subject Code: PPSAS102

Chapter: Unit 1 – Chapter 1

Chapter Name: Financial products and Its Cash flows

Today's Agenda

1. Cash Flow Models
    1. Cash Flow Process
    2. Certainty

2. Financial Products
    1. Zero Coupon Bond
    2. Fixed Interest Security
    3. Index linked security
    4. Cash on deposit
    5. Equity
    6. Annuity Certain
    7. Interest only loan
    8. Repayment Loan (or mortgage)

1 Cash flow Models

- Cash flow Process

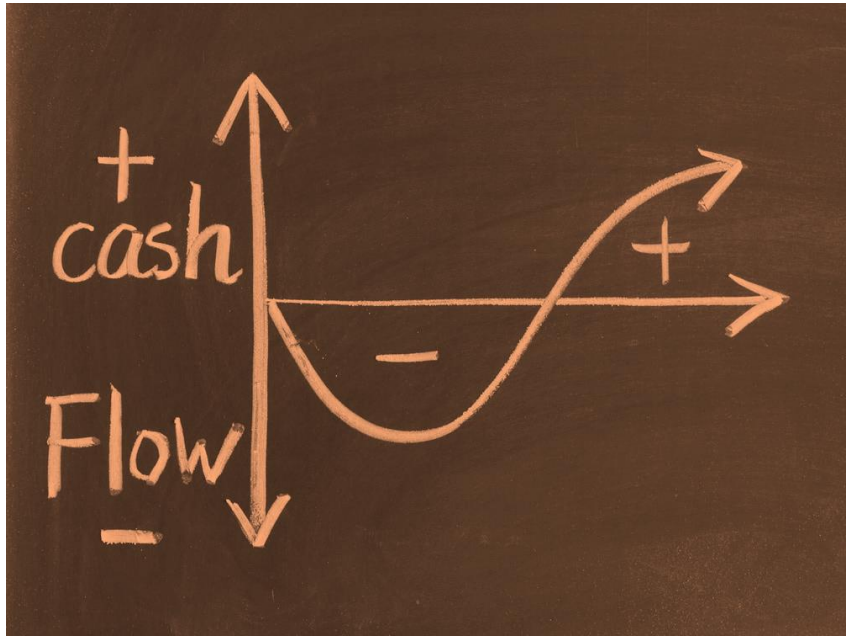
- Examples
  - Zero – Coupon bond
  - Fixed interest security
  - Index linked security
  - Cash on deposit
  - Equity
  - Annuity Certain
  - Interest only loan
  - Repayment Loan (or mortgage)

1.1 Cash flow Process

Analysis of cash flow involves the mathematical projection of the payments arising from a financial transaction.

- Payments received – income (Positive cash flow)
- Payments paid – outgo (negative cash flow)
- Net cash flow (income less outgo)

Practical Work – Management of cash flows



1.1 Cash flow Process

Things to be considered:

- Money – Paid or Received
- Timing – Known or Uncertain
- Amount - Known or Uncertain

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Case Study

1. Operating a Bridge or a Road

1. Life Insurance Company

1.2 Great Certainty = Certain?

What do you think about the certainty of cash flows?

Whether amount is certain, time is certain or both uncertain or something else?

1.2 Great Certainty = Certain?

An Employee, gets paid on the last Friday of every month will be certain or almost certain to receive a payment on the last Friday of this month?

And Why?

2.1 Zero Coupon Bond

ZCB is a contract to provide a specified lump sum at some specified future date.

For the Investor:
-ve cash flow at point of investment
Single known +ve cash flow on the specified future date

Certainty of +ve cash flow:
Amount – Known
Timing – Known
Payment – Not certain (i.e. Risk of Default)

When Issued by governments of developed countries: Low Risk
When Issued by companies: High Risk

It is like a Loan from investor to the issuer.
No interest payments before redemption.

2.1 Zero Coupon Bond - Example

For example, an investor who purchases a 3-year ZCB bond at a discount for $920 will receive $1,000. The $80 return, plus coupon payments received on the bond, is the investor's earnings or return for holding the bond.

Time	0	3
Cash Flow	- $920	+ $1000

2.1 Zero Coupon Bond

ZCB Value

Zero Coupon Bond

$$\text{Zero-Coupon Bond Value} = \frac{\text{Maturity Value}}{(1+i)^{\text{Number of Years}}}$$

Question

1.The principal amount of a bond that is repaid at the end of the loan term is called the bond's:

A) Coupon
B) Face value.
C) Maturity
D) Yield to maturity.
E) Coupon rate.

Answer: B

2.1 Zero Coupon Bond

Watch Video – Zero Coupon Bond

https://www.youtube.com/watch?v=_f_eCQLNFQ0

2.2 Fixed Interest Security

This is form of a loan.

Holder of the bond – receives a lump sum of specified amount at some specified future date together with series of regular level interest payment until the redemption of the lump sum.

Regular level interest payments are called Coupons

2.2 Fixed Interest Security

For investor:
-ve cash flow at point of investment
Single known +ve cash flow on the specified future date
Smaller known +ve cash flows on a regular set of specified future dates.

Certainty of +ve cash flow:
Amount – Known
Timing – Known
Payment –Not certain (i.e. Risk of Default)

Issued by governments of developed countries: Low Risk
Issued by companies: High Risk

2.2 Fixed Interest Security

Example of a Fixed Interest Security – Bond

2.3 Index Linked Bond

An index-linked bond is a bond in which payment of interest income on the principal is related to a specific price index, usually the Consumer Price Index (CPI).

Index-linked bonds are issued by governments to help mitigate the impact of inflation, paying a real yield plus accrued inflation. Due to inflation the purchasing power diminishes. Hence investors might be interested in coupons and final redemption amount to be linked with index that reflects the effects of inflation.

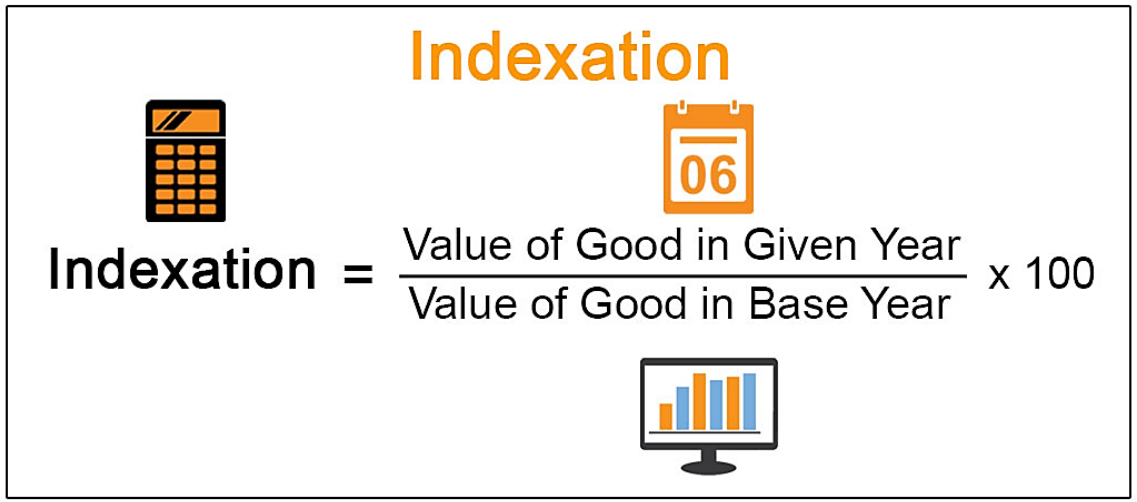
For investor:
-ve cashflow at point of investment
Single unknown +ve cashflow on the specified future date
Smaller unknown +ve cash flows on a regular set of specified future dates

Certainty of +ve cashflow:
Amount – Unknown &
Timing – Known
Payment –Not certain (i.e. Risk of Default)

2.3 Index Linked Bond

Cash flows are said to be known in **real** terms.
Real terms means taking into account inflation, whereas nominal means ignoring inflation

Calculation:



2.3 Index Linked Bond

Index linked security – Purchased on 1/1/2015
Coupons – 600 each year and Redemption payment – 11,000
(if there had been no inflation)

Inflation Index

Date	1/1/2015	1/1/2016	1/1/2017	1/1/2018
Index	100	105	108	113

Calculate the payments actually received by the investor.

2.4 Cash on Deposit

Eg: Bank Account (Call deposit)

For investor:
Investor chooses when to invest and disinvest and receive interest during the period of investment
Interest rate is subject to regular change.

Certainty of +ve/-ve cashflow:
Amount – Unknown
Timing – Unknown

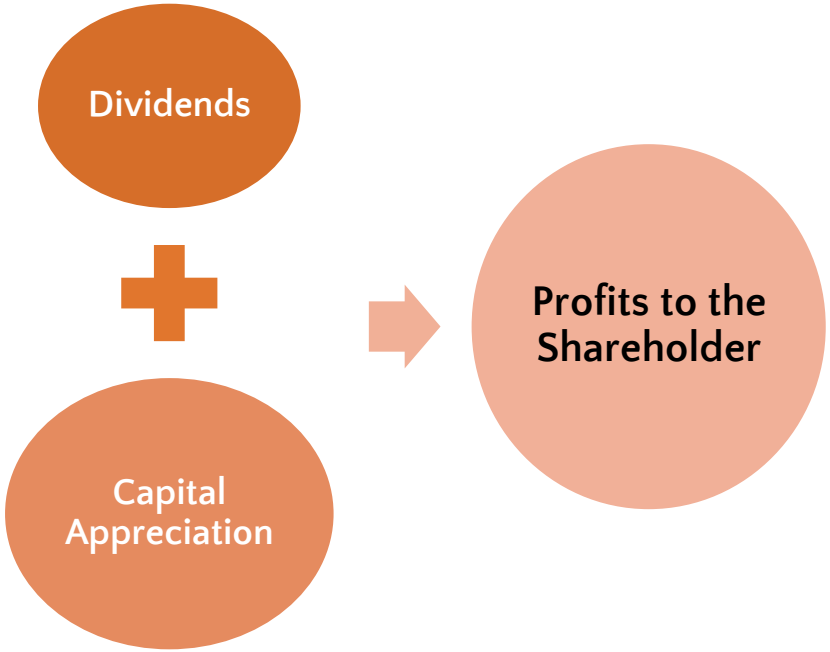
Another type of deposit is term deposit.

2.5 Equity/ Shares/ Stock

What do you know about Equity and Shares?

A share indicates a unit of ownership of the particular company. If you are a shareholder of a company, it implies that you as an investor, hold a percentage of ownership of the issuing company. As a shareholder you stand to benefit in the event of the company's profits, and also bear the disadvantages of the company's losses.

Small company – few individuals or institutions
Large organisation – many thousands of shareholders



2.5 Equity/ Shares/ Stock

Dividend rates are variable (since dependent on company's profit). Relationship between dividends and profits is not a simple one.

Buyback is another way of distributing profits

Equity can be held in perpetuity (hence, dividends too can continue indefinitely)

Risk of failing – Equity holders entitled to any assets which remain after creditors are paid

<u>For investor:</u>
-ve cashflow at point of investment
Amount – Unknown
Timing – Known (in some way)
Payment – Not certain (possibility of future +ve cashflow < Intial invested amount)

2.6 Annuity Certain

An annuity certain provides a series of regular payments in return for a single lump sum paid at the outset.

Term of the annuity – Known
Frequency of payments – Known
Payment amount – Level or Vary in line with some Index (eg: Inflation Index)

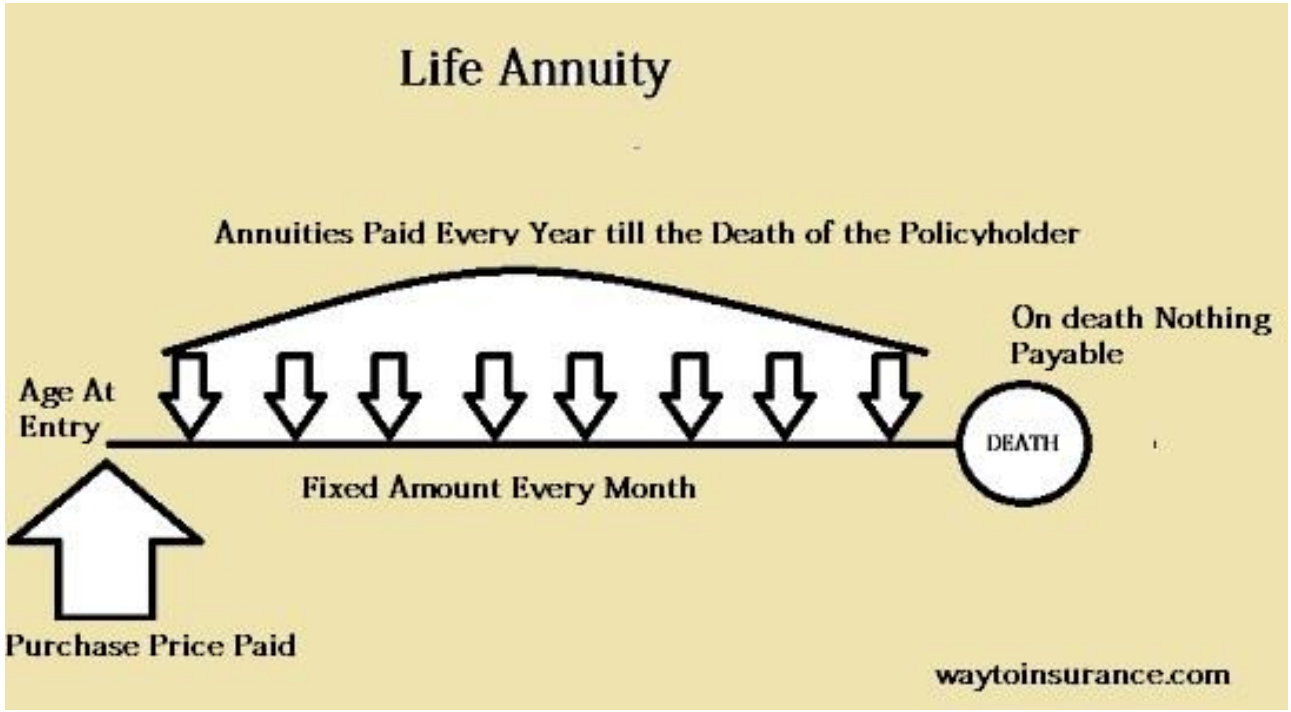
<u>For investor:</u>
Initial –ve cashflow
Series of smaller +ve cashflows

<u>Certainty of +ve cashflow:</u>
Amount – Known/Unknown
Timing – Known

Cash flows are similar to fixed interest security (but no redemption payment).

2.6 Annuity Certain

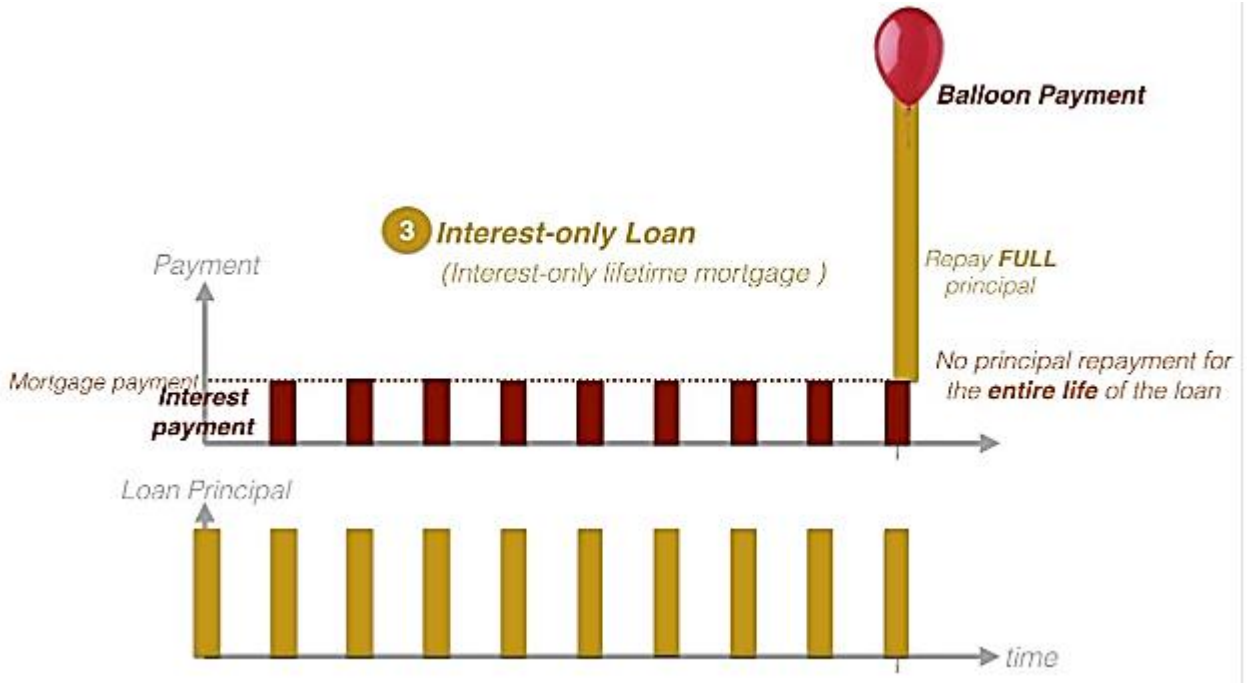
Example of annuity certain is A Life Annuity



2.7 An “Interest Only” Loan

This is a form of a loan. It is repayable by a series of interest payments followed by a return of the initial loan amount.

Note: The Capital amount that is borrowed is owed throughout the term of the loan



2.7 An "Interest Only" Loan

It is the reverse of the fixed interest security.
Provider of the loan is buying fixed interest security from the borrower.

For Provider:
Initial –ve cashflow
Series of smaller +ve cashflows
Larger known +ve cashflow at the end of the term

Certainty of +ve cashflow:
Amount – Unknown (Interest rate need not be fixed in advance)
Timing – Unknown (possibility of early repayment)

Question

1. Given the following information on an interest-only mortgage, calculate the monthly mortgage payment. Loan amount: $56000, Term: 15 years, Interest rate: 7.5%

A. $169.13
B. $350
C. $519.13
D. $4200

Answer: B

2.8 Repayment/ Mortgage Loan

It is a form of a loan. This loan is repayable by a series of payments (partial repayment of loan + interest).

Simple form:
Amount – known (interest rate is fixed)
Timing – Known
Can be complicated if prepayment and variable interest rate is considered.

Cash flows similar to those of annuity certain

Breakdown of 'Interest' and 'capital' changes significantly over a period of time.
1st Repayment – Almost entirely of Interest (very small capital repayment)
Last Repayment – Very small interest content (almost entirely capital payment)

2.8 Repayment/ Mortgage Loan

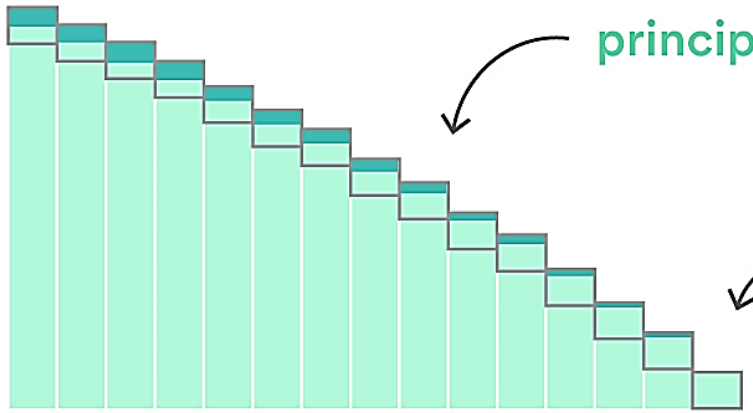
Structure of Payments

How do payments change over the life of a loan?

As the principal gets smaller with each payment, less interest accrues.

Later payments will include more principal.

Early payments include relatively more interest.



Your final payment

2.8 Repayment/ Mortgage Loan

Example

Consider John, who takes a $10,000 loan with a 10% annual interest over 10 annual payments. The loan repayment schedule would look as follows:

End of Year ...	Total Payment	Principal Payment	Interest Payment	Unpaid Balance
0				$10,000.00
1	$1,627.45	$627.45	$1,000.00	$9,372.55
2	$1,627.45	$690.20	$937.25	$8,682.35
3	$1,627.45	$759.22	$868.23	$7,923.13
4	$1,627.45	$835.14	$792.31	$7,087.99
5	$1,627.45	$918.66	$708.80	$6,169.33
6	$1,627.45	$1,010.52	$616.93	$5,158.81
7	$1,627.45	$1,111.57	$515.88	$4,047.24
8	$1,627.45	$1,222.73	$404.72	$2,824.51
9	$1,627.45	$1,345.00	$282.45	$1,479.50
10	$1,627.45	$1,479.50	$147.95	-$0.00
TOTAL:	$16,274.54	$10,000.00	$6,274.54	

Question

Nisha takes a Loan of Rs.100,000 at 8.5% p.a. interest (reducing balance rate) for one year (12 months).
Calculate the bifurcation of interest and capital repayment for each payment.

Solution

Year	Total Payr	Principal	Interest	Capital Outstanding
0	0	0	0	100000
1	8722	8013.666667	708.3333333	91986.33333
2	8722	8070.430139	651.5698611	83915.90319
3	8722	8127.595686	594.4043143	75788.30751
4	8722	8185.166155	536.8338449	67603.14135
5	8722	8243.144415	478.8555846	59359.99694
6	8722	8301.533355	420.466645	51058.46358
7	8722	8360.335883	361.664117	42698.1277
8	8722	8419.554929	302.4450712	34278.57277
9	8722	8479.193443	242.8065571	25799.37933
10	8722	8539.254396	182.7456036	17260.12493
11	8722	8599.740782	122.2592183	8660.38415
12	8722	8660.655612	61.34438773	-0.271461875

Recap

- Analysis of cash flow involves the mathematical projection of the payments arising from a financial transaction.
- Zero coupon bond is a contract to provide a specified lump sum at some specified future date.
- In Fixed Interest Security, holder of the security – receives a lump sum of specified amount at some specified future date together with series of regular level interest payment until the redemption of the lump sum.
- An index-linked bond is a bond in which payment of interest income on the principal is related to a specific price index.
- Cash on deposits include bank account deposits, term deposits etc.
- A share indicates a unit of ownership of the particular company. As a shareholder you stand to benefit in the event of the company's profits, and also bear the disadvantages of the company's losses.

Recap

- An annuity certain provides a series of regular payments in return for a single lump sum paid at the outset.
- An Interest only loan is repayable by a series of interest payments followed by a return of the initial loan amount.
- Mortgage is a form of a loan. This loan is repayable by a series of payments (partial repayment of loan + interest).