INSTITUTE OF ACTUARIAL & QUANTITATIVE STUDIES

Class: FY MSc

Subject: Financial Mathematics

Chapter: Unit 3 - Chapter 2

Chapter Name: Equation of Value

Today's Agenda

0. Introduction

1. Equation of Value

2. Uses
   1. Unknown rate of Interest
   2. Unknown Time or Number of Payments

0 Introduction

Till Now

- ❖ We studied various examples and exercises involving accumulated values and present values ;
- ❖ Understood equivalent rates of interest and discount ;
- ❖ Learnt about annuity and types of annuity.

Further

- ❖ We will see the value these add when combined in the practical world.
- ❖ Any transaction or project that we invest in includes exchange of money and these components are used to help make better decisions accounting for time value of money.

1 Equation of Value

As a consequence of time value of money , two or more amounts of money payable at different points in time cannot be compared until all the amounts are accumulated or discounted to a common date . This date is called the comparison date and the equation which accumulates or discounts each payment to the comparison date is called the **Equation Of Value.**

What is an equation of value?

An *equation of value* equates the present value of money received to the present value of money paid out:

"PV income = PV outgo"

or equivalently:

"PV income – PV outgo = 0"

1 Example

❖ From the prospective of an investor in real-estate project,

INFLOW	OUTFLOW
Periodic rent on the property	Money to buy land
Sale proceeds from the property	Periodic wage to workers
	Expenses of raw materials
	Periodic salary to heads

❖ In a transaction of purchase of dividend paying stock,

INFLOW	OUTFLOW
Periodic dividend payments	Purchase price of the stock
Sale proceeds stock (if sold)	

2 Uses

Used to find the unknown quantity:

1. Amount to be invested (PV)
2. Annuity amount
3. Interest rate
4. Term/ Tenure

2.1 Unknown rate of Interest

We understand these concepts better through an example.

Question

At what rate of interest, convertible quarterly, is \$16000 the present value of \$1000 paid at the end of every quarter for five years?
Hint: Make the equation of value for the transaction and then find the one unknown.

Solution

Let $j = i^{(4)}/4$, so that the equation of value becomes

$$1000a_{\overline{20}|j} = 16{,}000$$

or

$$a_{\overline{20}|j} = 16.$$

This problem is ideally set up to use a financial calculator. We set

$$\begin{aligned} \text{N} &= 20 \\ \text{PV} &= 16 \\ \text{PMT} &= -1 \end{aligned}$$

and compute I obtaining

$$\text{I} = 2.2262.$$

Thus, we have

$$j = .022262$$

so that

$$i^{(4)} = 4(.022262) = .08905.$$

Question

Find the rate at which $\ddot{s}_{\overline{2|}} = 2.5$

Solution

$$\ddot{s}_{\overline{2}|} = (1+i)^2 + (1+i) = 2.5.$$

Thus, we have a quadratic which simplifies to $i^2 + 3i - .5 = 0$ and applying the quadratic formula

$$i = \frac{-3 \pm \sqrt{(3)^2 + (4)(.5)}}{2}$$

$$= \frac{-3 \pm \sqrt{11}}{2}.$$

Only the positive root is reasonable, so that

$$i = \frac{-3 + \sqrt{11}}{2} = .1583, \text{ or } 15.83\%.$$

Question

Gustavo Larson has saved $20,000. On 1 January, he purchases a perpetuity that makes end-of-year payments. The perpetuity price is based on an annual effective interest rate of 5%.

What are the annual payments of Gustavo's perpetuity?

Solution

Solution The value on January 1 of a perpetuity with annual end-of-year payments of Q is $Q(\frac{1}{.05}) = 20Q$. Setting this value equal to \$20,000, we find $Q = \$1,000$.

2.2 Unknown Time or Number of Payments

Question

Smith wishes to accumulate 1000 by means of semiannual contributions earning interest at a nominal rate of $i^{(2)} = 0.08$. The regular deposits will be of 50 each.

Find the number of regular deposits required and the additional fractional deposit,

i) If the fractional deposit is made at the time of last regular deposit.

ii) If the fractional deposit is made six month after the last deposit.

Solution

We solve the relationship $1000 = 50 \cdot s_{\overline{n}|.04}$ for n. Writing this equation as

$$1000 = 50 \times \frac{(1.04)^n - 1}{.04}$$

results in a value of $n = \frac{\ln(1.8)}{\ln(1.04)} = 14.9866$. Thus 14 deposits of the full amount of 50 are required. The accumulated amount on deposit at the time of, and including, the 14^{th} deposit is $50s_{\overline{14}|.04} = 914.60$. If the additional fractional deposit is made at the time of the 14^{th} regular deposit, then it must be $1000 - 914.60 = 85.40$, which is actually larger than the regular semiannual deposit. If the account is allowed to accumulate another half-year, then the accumulated amount in the account six months after the 14^{th} deposit, is $50\ddot{s}_{\overline{14}|.04} = 50(1.04)s_{\overline{14}|.04}$ $= 951.18$. In this case an additional fractional deposit (also called a *balloon payment*) of amount $1000 - 951.18 = 48.82$ is required to bring the amount on deposit to a total of 1000.