Lecture 4

Unit 2 :

INSTITUTE OF ACTUARIAL
& QUANTITATIVE STUDIES

Class: FY MSc

Subject : Financial Mathematics

Subject Code: PPSAS102

Chapter: Unit 2 Chapter 2

Chapter Name: Discounting & Accumulating

Today's Agenda

1. Present value of cash flows
   1. Discrete cash flow
   2. Continuous payable cash flow
   3. Present value of General cash flow

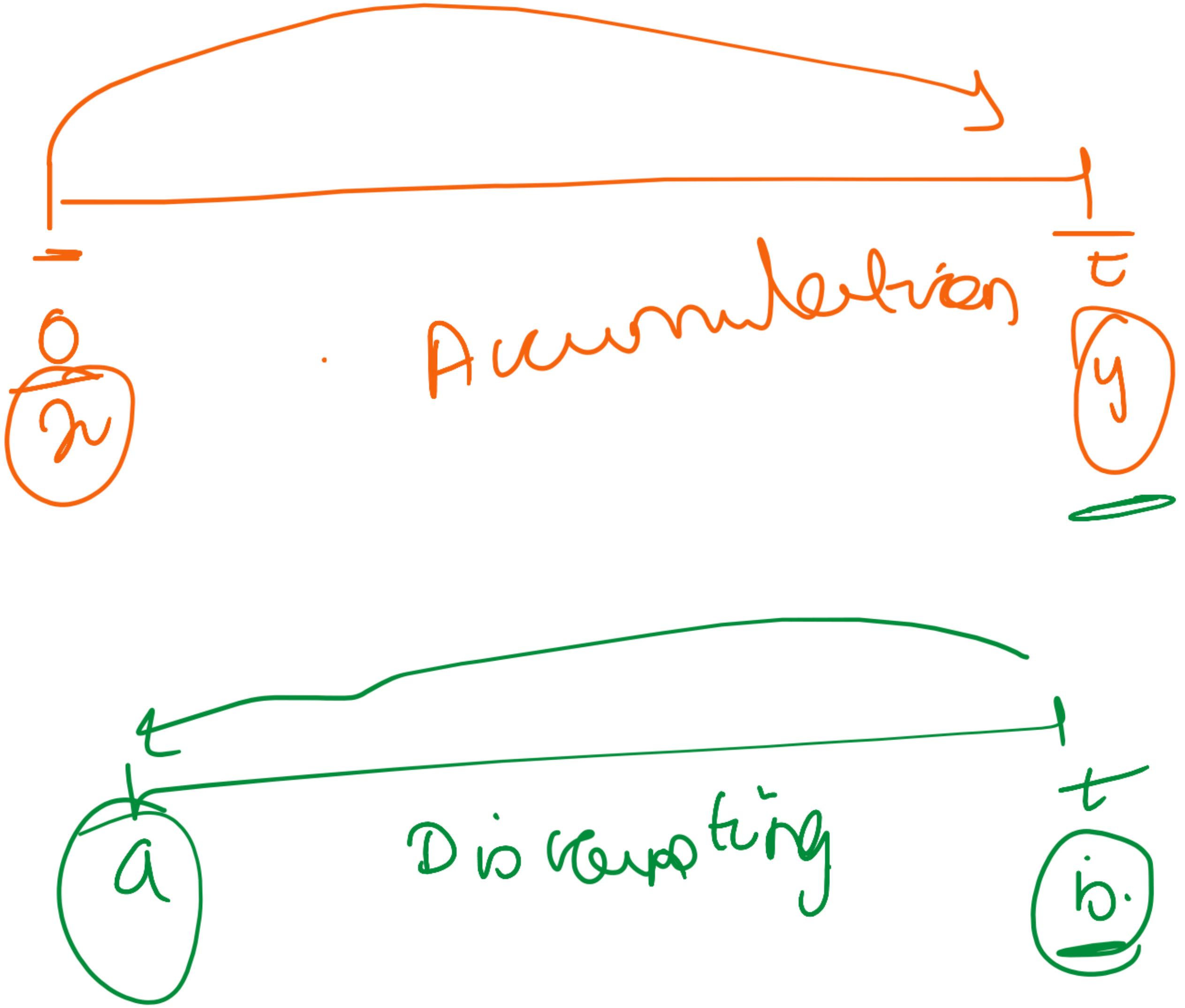
2. Changes in interest rates

1. Interest income

Unit 2

1. Acc. & disc of CFs -

2. Annuities



tle

$y > x$

$a < b$

Discounting & Accumulating

Changes in CFs.

multiple CFs.

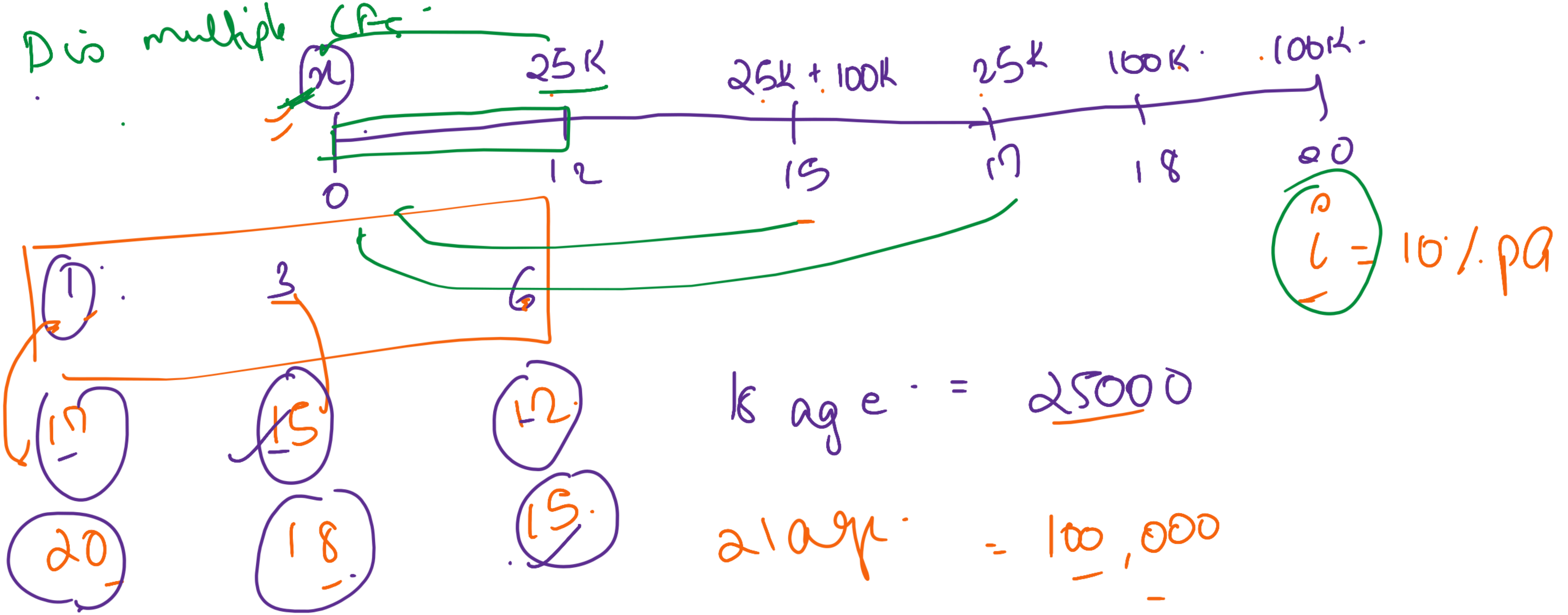
cont. CFs

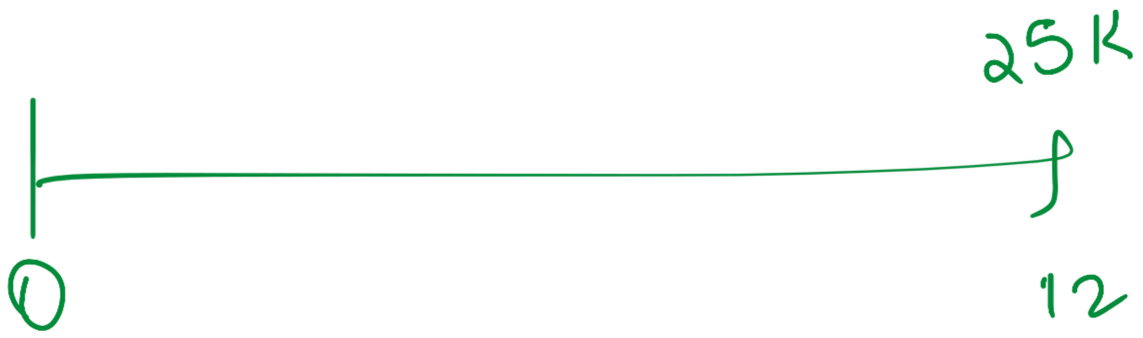
$$\int_{t_1}^{t_2} \rho(t)\cdot v(t)\,dt$$

Changes in IR.

Variable IRs

The parents of three children aged 1,3 and 6 wish to set up a trust fund that will pay INR 25000 to each child upon attainment of age 18 and INR 100,000 to each child upon attainment of age 21. If the trust fund will earn effective annual interest rate of 10%, what amount must the parents now invest in the trust fund





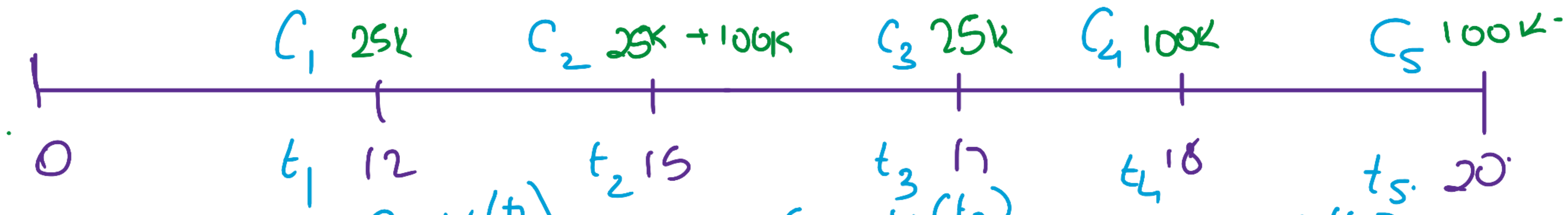
$i = 10\%\ p$

$$PV = \frac{25000}{(1+10\%)^{12}}$$

75686.13513

$$AF = (1+i)^n$$

$$DF = \frac{1}{AF} = \frac{1}{(1+i)^n}$$



$$\text{Total PV} = \underbrace{25000\, v(12)}_{C_1\, v(t_1)} + \underbrace{125{,}000\, v(15)}_{C_2\, v(t_2)} + \underbrace{25000\, v(17)}_{C_3\, v(t_3)} + \dots$$

$$100000\, v(18) + 100000\, v(20)$$

$$v(t) = e^{-\delta t}$$

$$v(t) = \frac{1}{(1+i)^t} \qquad \frac{1}{(1+i)} = v \qquad \boxed{v(t) = v^t}$$

$$\text{Total PV} = \underbrace{25000\, v^{12}}_{C_1 v^{t_1}} + \underbrace{125000\, v^{15}}_{C_2 v^{t_2}} + \underbrace{25000\, v^{17}}_{C_3 v^{t_3}} + \underbrace{100000\, v^{18}}_{C_4 v^{t_4}}$$

$$+ \underbrace{100000\, v^{20}}_{C_5 \cdot v^{t_5}}$$

$$= 75686.135$$

$$\sum_{i=1}^{5} C_i\, v(t_i) = \sum_{i=1}^{5} C_i\, v^{t_i}$$

$i = 10\%$

$$v = \frac{1}{1+i} = \frac{1}{(1.)} \rightarrow A$$

$$25000 \cdot A^{12} + 125000 \cdot A^{15}$$

1.1 Present Value of Cash flows

Discrete Cash flow

The present value of the sums $c_{t_1}, c_{t_2}, \ldots, c_{t_n}$ due at times $t_1, t_2, \ldots, t_n$ (where $0 \le t_1 < t_2 < \ldots < t_n$) is:

$$\boldsymbol{c_{t_1} v(t_1) + c_{t_2} v(t_2) + \ldots + c_{t_n} v(t_n) = \sum_{j=1}^{n} c_{t_j} v(t_j)}$$

annuity

$$\sum_{j=1}^{n}$$

If the number of payments is infinite the present value is defined to be:

$$\boldsymbol{\sum_{j=1}^{\infty} c_{t_j} v(t_j)}$$

provided that this series converges. It usually will in practical problems.

$\sum c_{t_i} \cdot v(t_i)$ $\quad n = \infty$

1.1 Example

Under its current rent agreement, a company is obliged to make annual payment of \$7500 for the building its occupies. Payments are due on 1 January 2006, 1 January 2007, 1 January 2008. If a company wishes to cover these payments by investing a single sum in its bank account that pays 7.5% pa compound. eff ann.

What sum must be invested on 1 January 2005 ?

1-Jan05	Jan06	Jan07	Jan08
	7500	7500	7500

$$v = \frac{1}{1+7.5\%}$$

$i = 7.5\%$ pa eff.

$$7500v^1 + 7500v^2 + 7500v^3$$

$$7500\left(v^1 + v^2 + v^3\right).$$

19503.94305.

Continuous CF

- Theoretical concept, not practical
- weekly/daily $\approx$ cont.

20 years:
= week $\approx$ cont

1 month
1 week $\not\approx$ cont

Rate of payment / payment stream $\rightarrow \rho(t)$

$\Downarrow$

Cont. CF

=

$\rho(t) =$
$25000 + 2t$
$25000 + 2(1)$
$25000 + 2(2)$

$M(t) =$ Total payment made/received b/w time 0 and time t.

$\rho(t)$

individual CF

$M(t)$ and $\rho(t)$

$$M(t) = \int_{0/t_1}^{t/t_2} \rho(t)\,dt$$

$$\rho(t) = M'(t)$$

$\int$

Qu.

$\rho(0.1)\ \cdot V(0.1)$

$0 \quad 0.1 \quad 0.2 \quad 0.3 \quad 0.4 \quad 0.5 \quad \dots \quad T$

$\rho(t)$

$$\boxed{PV = \int_0^T \rho(t) \cdot V(t)\, dt}$$

δ

$e^{-\delta n} \Big/ e^{-\int_{t_1}^{t_2} \delta(t)\, dt}$

n

1.2 Continuous payable Cash flows

The present value of the entire cash flow is obtained by integration as

$\delta(t)$

$$\int_0^T v(t)\rho(t)\,dt$$

If T is infinite, we obtain by similar argument:

$$\int_0^\infty v(t)\rho(t)\,dt$$

1.3 Present Value of General Cash flows

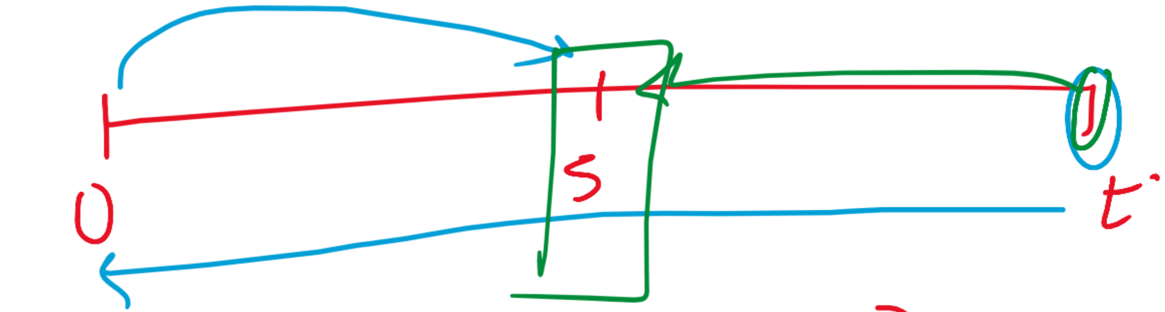
By combining the results of discrete and continuous cash flows, we obtain formula as:

$$\sum_{j=1}^{\infty} \boldsymbol{c_{t_j} v(t_j)} + \int_0^{\infty} \boldsymbol{v(t)\rho(t)dt}$$

This is the formula for the present value of a general cash flow.

Assuming a constant rate of interest, the formula simplifies to

$$\sum_{j=1}^{\infty} c_{t_j} v^t + \int_0^{\infty} v^t \rho(t) dt$$



$$v(t) = v(s) \cdot v(s,t)$$

$$v(s,t) = \frac{v(t)}{v(s)}$$

$$\boxed{v(s,t) = v(t) \cdot A(s)}$$

$$A(s) = \frac{1}{v(s)}$$

1.3 Example

A life office starts issuing a new type of 10-year saving policy to young investor who pay weekly premiums of $10. Assuming that the life office sells 10,000 policies evenly over each year & that no policyholder stop paying premiums.

What will the rate of premium income be fore the office during the first few years ?

1.3 Solution

Solution

After t years the office will have sold $10{,}000t$ policies.

So the weekly premium income will be: $10{,}000t \times £10 = £100{,}000t$

Since there are 52.18 $(365.25/7)$ weeks in a year, this corresponds to an annual rate of income of:

$$52.18 \times £100{,}000t = £5{,}218{,}000t$$

Question

A company expects to receive for the next 5 years a continuous cash flow of $350 pa. it also expects to have to pay out $600 at the end of the first year and $400 at the end of the third year. Calculate the net present value of these cash flows if v(t) = 1 – t/100 for $0 \leq t \leq 5$.

2

Changes in Interest rates

Example

Calculate the present values as at 1 January 2005 of the following payments:

- A single payment of £2000 payable on 1 July 2009.
- A single payment of £5000 payable on 31 December 2016.

Assume effective rate of interest of 8% per annum until 31 December 2011 and 6% per annum thereafter.

2 Solution

Solution

(i) Here, the interest rate is constant throughout the relevant period, so the present value is just:

$$2,000v^{4\frac{1}{2}@8\%} = 2,000 \times 0.70728 = £1,415$$

(ii) Here, we need to break the calculation up at 31 December 2011 when the interest rate changes:

$$5,000v^{7@8\%} \times v^{5@6\%} = 5,000 \times 0.58349 \times 0.74726 = £2,180$$

3

Interest Income

If we invest an amount of capital C , then the present value of the proceeds we receive from this investment should equal our original amount of capital.

Example

An investor deposits £2,000 in a bank account and receives income at the end of each of the next three years. The rate of interest is 4% *pa* effective. The investor withdraws the capital after three years.

At the end of each year the investor receives $0.04 \times 2,000 = £80$.

The present value of the interest received is:

$$80(v + v^2 + v^3) = £222.01$$

The present value of the capital received after three years is:

$$2,000v^3 = £1,777.99$$

The present value of the capital plus the present value of the interest equals the initial investment, *ie*:

$$1,777.99 + 222.01 = £2,000$$