



Subject: Financial mathematics

Chapter: Unit 4

Category: Practice question

Chapter 7

1. CT1 April 2010 Q8

A loan is repayable by annual instalments paid in arrear for 20 years. The first instalment is £4,650 and each subsequent instalment is £150 greater than the previous instalment. Calculate the following, using an interest rate of 9% per annum effective:

- (i) the amount of the original loan
- (ii) the capital repayment in the tenth instalment
- (iii) the interest element in the last instalment
- (iv) the total interest paid over the whole 20 years

[Ans: (i) 51,714.08, (ii) £1,943.84, (iii) £619.27, (iv) £69,785.92]

2. CT1 April 2011 Q7

A loan of £60,000 was granted on 1 July 1998.

The loan is repayable by an annuity payable quarterly in arrear for 20 years. The amount of the quarterly repayment increases by £100 after every four years. The repayments were calculated using a rate of interest of 8% per annum convertible quarterly.

- (i) Show that the initial quarterly repayment is £1,370.41.
- (ii) Calculate the amount of capital repaid that was included in the payment made on 1 January 1999.
- (iii) Calculate the amount of capital outstanding after the quarterly repayment due on 1 July 2011 has been made.

[Ans: (i) £1,370.41 per quarter, (ii) 173.82, (iii) £36,619]

3. CT1 April 2012 Q3

A company has borrowed £500,000 from a bank. The loan is to be repaid by level instalments, payable annually in arrear for ten years from the date the loan is made. The annual instalments are calculated at an effective rate of interest of 9% per annum.

(i) Calculate:

- (a) the amount of the level annual instalments.
- (b) the total amount of interest which will be paid over the ten-year term.

At the beginning of the eighth year, immediately after the seventh instalment has been made, the company asks for the loan to be rescheduled over a further four years from that date. The bank agrees to do this on condition that the rate of interest is increased to an effective rate of 12% per annum for the term of the rescheduled instalments and that repayments are made quarterly in arrear.

- (ii) (a) Calculate the amount of the new quarterly instalment.
- (b) Calculate the interest content of the second quarterly instalment of the rescheduled loan repayments.

[Ans: (i) a) 77,910.04, b) 279,100, (ii) a) £15,549.16, b) 5383.41]

4. CT1 September 2012 Q6

A loan is to be repaid by an increasing annuity. The first repayment will be £200 and the repayments will increase by £100 per annum. Repayments will be made annually in arrear for ten years. The repayments are calculated using a rate of interest of 6% per annum effective.

(i) Calculate the amount of the loan

(ii)(a) Calculate the interest component of the seventh repayment.

(b) Calculate the capital component of the seventh repayment.

(iii) Immediately after the seventh repayment, the borrower asks to have the original term of the loan extended to fifteen years and wishes to repay the outstanding loan using level annual repayments. The lender agrees but changes the interest rate at the time of the alteration to 8% per annum effective.

Calculate the revised annual repayment.

[Ans: (i) £4,432.25, (ii) a) £196.00, b) £604.00, (iii) £463.34]

5. CT1 April 2013 Q10

A loan is repayable by annual instalments in arrear for 20 years. The initial instalment is £5,000, with each subsequent instalment decreasing by £200. The effective rate of interest over the period of the loan is 4% per annum.

(i) Calculate the amount of the original loan.

(ii) Calculate the capital repayment in the 12th instalment.

After the 12th instalment is paid, the borrower and lender agree to a restructuring of the debt. The £200 reduction per year will no longer continue. Instead, future instalments will remain at the level of the 12th instalment and the remaining term of the debt will be shortened. The final payment will then be a reduced amount which will clear the debt.

(iii) (a) Calculate the remaining term of the revised loan.

(b) Calculate the amount of the final reduced payment.

(c) Calculate the total interest paid during the term of the loan.

[Ans – (i) - £45,638.56, (ii) - £2,189.66, (iii) a – 6 years, b - £764.11, c - £15,925.55]

6. CT1 September 2013 Q9

A bank makes a loan to be repaid by instalments paid annually in arrear. The first instalment is

£400, the second is £380 with the payments reducing by £20 per annum until the end of the 15th year, after which there are no further repayments. The rate of interest charged is 4% per annum effective.

- (i) Calculate the amount of the loan.
- (ii) Calculate the capital and interest components of the first payment.

At the beginning of the ninth year, the borrower can no longer make the scheduled repayments. The bank agrees to reduce the capital by 50 per cent of the loan outstanding after the eighth repayment. The bank requires that the remaining capital is repaid by a 10-year annuity paid annually in arrear, increasing by £2 per annum. The bank changes the rate of interest to 8% per annum effective.

- (iii) Calculate the first repayment under the revised loan.

[Ans – (i) - £3,052.65, (ii) - £277.894, (iii) - £74.16]

7. CT1 April 2014 Q10

A loan of £20,000 is repayable by an annuity payable annually in arrear for 25 years. The annual repayment is calculated at an effective interest rate of 8% per annum and increases by £50 each year.

- (i) Calculate the amount of the first payment.
- (ii) Calculate the capital outstanding after the first three payments have been made.
- (iii) Explain your answer to part (ii).

(iv) Calculate the total amount of interest paid over the term of the loan.

[Ans – (i) - £1,462.31, (ii) - £ 20,293.01, (iv) - £31557.66]

8. CT1 April 2015 Q11

On 1 January 2016, a student plans to take out a five-year bank loan for £30,000 that will be repayable by instalments at the end of each month. Under this repayment schedule, the instalment at the end of January 2016 will be X , the instalment at the end of February 2016 will be $2X$ and so on, until the final instalment at the end of December 2020 will be $60X$. The bank charges a rate of interest of 15% per annum convertible monthly.

(i) Prove that

$$(Ia)_n = \frac{\ddot{a}_n - nv^n}{i}$$

(ii) Show that $X = £26.62$.

The student is concerned that she will not be able to afford the later repayments and so she suggests a revised repayment schedule. The student would borrow £30,000 on 1 January 2016 as before. She would now repay the loan by 60 level monthly instalments of $36X = £958.32$ but the first repayment would not be made until the end of January 2019 and hence the final instalment is paid at the end of December 2023.

(iii) Calculate the APR on the revised loan schedule and hence determine whether you believe the bank should accept the student's suggestion.

(iv) Explain the difference in the total repayments made under the two arrangements.

[Ans – (iii) – 12.8%]

9. CT1 April 2016 Q5

A loan is to be repaid by a series of instalments payable annually in arrear for 15 years. The first instalment is £1,200 and payments increase thereafter by £250 per annum. Repayments are calculated using a rate of interest of 6% per annum effective.

Determine:

- (i) the amount of the loan.
- (ii) the capital outstanding immediately after the 9th instalment has been made.
- (iii) the capital and interest components of the final instalment.

[Ans – (i) - £26,043.29, (ii) - £19829.54, (iii) capital - £4,433.96, interest - £266.04]

10. CT1 September 2016 Q3

Describe the characteristics of a repayment loan (or repayment mortgage).

11. CT1 April 2017 Q2

A bank offers two repayment alternatives for a loan that is to be repaid over sixteen years:

Option 1: the borrower pays £7,800 per annum quarterly in arrear.

Option 2: the borrower makes payments at an annual rate of £8,200 every second year in arrear.

Determine which option would provide the better deal for the borrower at a rate of interest of 5% per annum effective.

[Ans – option 1]

12. CT1 September 2017 Q8

A loan is to be repaid by an increasing annuity. The first payment will be £100 and the payments will increase by £50 per annum. Payments will be made annually in arrear for ten years. The repayments are calculated using a rate of interest of 5% per annum effective.

- (i) Calculate the amount of the loan.
- (ii) Calculate:
 - (a) the interest component of the sixth instalment.
 - (b) the capital component of the sixth instalment.

Immediately after the sixth instalment, the borrower asks to repay the remaining loan using level annual instalments. The lender agrees, but changes the interest rate at the time of the alteration to 6% per annum effective.

- (iii) Calculate the revised instalment.

[Ans – (i) - £2,354.78, (ii) a- £96.36, b - £253.64, (iii) - £482.96]

13. CT1 April 2018 Q11

An n -year decreasing annuity is payable annually in arrear where the payment at the end of the first year is n , the payment at the end of the second year is $(n - 1)$, and so on until the final payment at the end of year n is 1.

- (i) Show that the present value of this annuity is $\frac{n-a_n}{i}$

A loan is to be repaid over 25 years by means of annual instalments payable in arrear. The amount of the first instalment is £8,000 and each subsequent instalment reduces by £200. The effective rate of interest charged by the lender is 5.5% per annum.

- (ii) Calculate the initial amount of the loan.
- (iii) Determine the interest and capital components of the 10th instalment.
- (iv) Calculate the total amount of interest payable over the term of the loan.

[Ans – (ii) - £82,372.95, (iii) capital - £3,366.17, interest - £2,833.83, (iv) - £57,627.05]

14. CM1A April 2019 Q12

A loan of £80,000 was taken out on 1 January 2016. The loan was to be repaid over 10 years in level instalments payable monthly in arrears.

- (i) Calculate the level monthly instalment using an effective rate of interest of 8% per annum.
- (ii) Calculate the amount of the loan outstanding on 1 November 2018 immediately after payment of the instalment then due.

On 1 November 2018, immediately after payment of the instalment, the borrower asked that the monthly instalment be reduced to £900 and the remaining term extended as required to clear the outstanding loan amount. The final payment would be equal to the outstanding loan at the time, if less than £900.

The lender agreed to this change, subject to the following conditions:

- the interest rate applied in future is increased to 9% per annum convertible monthly; and
- an administration fee of £250 is added to the loan outstanding at 1 November 2018.

- (iii) (a) Determine the new date on which the loan will be repaid.
- (b) Calculate the final instalment paid.

[Ans – (i) – 958.86, (ii) – 63180.76, (iii) – a - 101 months, b – 581.97]

15. CM1A September 2020 Q8

A loan is to be repaid by a series of instalments made annually in arrears. The first instalment is \$200 per annum and thereafter instalments increase by \$15 each year. The instalments are paid for 16 years and are calculated using an effective rate of interest of 5% per annum.

- (i) Calculate the amount of the loan.
- (ii) Construct a loan schedule that shows the capital and interest elements included in, and the amount of loan outstanding after, each of the 5th and 6th instalments.
- (iii) Calculate the capital and interest elements of the final instalment.

[Ans – (i) - \$3219.96, (iii) capital - 404.76, interest – 20.24]

16. CM1A April 2021 Q8

On 1 January 2022, a student plans to take out a 10-year bank loan for \$15,000. Under the repayment schedule, instalments will be paid monthly in arrears until the end of the term. The first instalment, at the end of January 2022, will be X , and the second instalment, at the end of February 2022, will be $2X$, and so on, until the instalment at the end of December 2026, which will be $60X$. The remaining instalments from the end of January 2027 will also be $60X$.

The bank charges a rate of interest of 12% p.a. effective.

- (i) Write down an equation of value to calculate X .
- (ii) Calculate the value of X using the equation of value in part (i).
- (iii) Write down an equation to calculate the loan outstanding, after the instalment paid at the end of December 2026, using the retrospective method.

- (iv) Calculate the loan outstanding after the instalment at the end of December 2026 has been paid, using the equation in part (iii).
- (v) Comment on your answer to part (iv).
- (vi) Write down an equation to calculate the total interest paid during 2027.
- (vii) Calculate the total interest paid during 2027 using the equation in part (vi).

The bank also offers the 10-year loan with the same interest rate but where the monthly instalments remain level throughout the term.

- (viii) Comment on whether the total interest paid by the student under this revised offer would be greater or less than that paid under the original repayment schedule. You should not perform any further calculations.

[Ans – (ii) – \$5.33, (iv) – \$14580, (vii) – \$1543]

17. CM1A September 2022 Q11

A loan of £300,000 was taken out on 1 January 2015. The loan is repaid by an annuity payable quarterly in arrears for 12 years. The amount of the quarterly instalment increases by £400 after every 4 years. The instalments were calculated based on a nominal rate of interest of 6% p.a. convertible quarterly.

- i. Assuming the equivalence principle applies, state an equation of value which can be used to calculate the initial quarterly instalment, R .
- ii. Using your answer to part (i), calculate R , showing all working.

- iii. Calculate, showing all working, the amount of capital repaid in the first quarterly instalment made on 31 March 2015.
- iv. Calculate, using an appropriate annuity function, the amount of loan outstanding immediately after the quarterly instalment due on 30 September 2022 has been made. You must show all working.

On 1 October 2022, the borrower asked to change the loan contract such that the quarterly payments would be level for the remainder of the term. The lender agreed, provided that the interest rate was increased to 7.5% p.a. effective and that the original term of the loan is unchanged.

- v. Calculate, using an appropriate annuity function, the new quarterly level instalment. You must show all your working.

[Ans – (ii) - £8475.42, (iii) - £3975.42, (iv) – 137880.71, (v) - £9506.55]

18. CM1 September 2023 Q11

A loan is repaid, over a term of 20 years, by monthly instalments paid in arrears. During the first year, the annual rate of payment is £5,000. This rate then reduces by £300 at the start of each subsequent year of the term, up to and including the start of the 10th year, after which the rate remains unchanged.

The instalments were calculated using a nominal rate of interest of 9% p.a. convertible monthly for the first 10 years and 10.5% p.a. convertible monthly for the remainder of the term.

- (i) Calculate, using appropriate annuity factors and showing all working, the amount of the loan.
- (ii) Calculate, using appropriate annuity factors and showing all working, the capital and interest components of the 6th monthly instalment.

[Ans: (i) £31,246.42, (ii) Int = £227.41, Capital = £189.26]

19. CM1 September 2024 Q7

A 30-year loan of \$500,000 is repaid by quarterly instalments in arrears. The instalments are level over each year and increase by \$100 at the start of each following year. The effective rate of interest applicable to the loan is 7% p.a. compounded quarterly.

- (i) Calculate, showing all working and using appropriate annuity factors, the initial quarterly instalment.
- (ii) Calculate, showing all working and using appropriate annuity factors, the capital and interest elements of the third instalment.
- (iii) Calculate, showing all working and using appropriate annuity factors, the capital outstanding before the payment of the penultimate (i.e. the second to last) quarterly instalment.
- (iv)
 - (a) Write down a formula for the total interest paid over the term of the loan.
 - (b) Calculate the total interest paid over the term of the loan using your formula from part (iv)(a).

[Ans: (i) \$9,032, (ii) int = \$8,740, capital = \$292, (iii) \$23,253,

(iv)a) $4 \times X \times 30 + 4 \times 100 \times [1 + 2 + 3 + \dots + 29] - 500,000$ where $X = 9,032.37$, (b) \$757,884]

Chapter 8

1. CT1 April 2010 Q9

A company is undertaking a new project. The project requires an investment of £5m at the outset, followed by £3m three months later.

It is expected that the investment will provide income over a 15 year period starting from the beginning of the third year. Net income from the project will be received continuously at a rate of £1.7m per annum. At the end of this 15 year period there will be no further income from the investment.

Calculate at an effective rate of interest of 10% per annum:

(i) the net present value of the project

(ii) the discounted payback period

A bank has offered to loan the funds required to the company at an effective rate of interest of 10% per annum. Funds will be drawn from the bank when required and the loan can be repaid at any time. Once the loan is paid off, the company can earn interest on funds from the venture at an effective rate of interest of 7% per annum.

(iii) Calculate the accumulated profit at the end of the 17 years.

[Ans: (i) £3.283m, (ii) 10.1, (iii) £14.95m]

2. CT1 April 2010 Q10

A pension fund's assets were invested with two fund managers.

On 1 January 2007 Manager A was given £120,000 and Manager B was given £100,000. A further £10,000 was invested with each manager on 1 January 2008 and again on 1 January 2009.

The values of the funds were:

	31 December 2007	31 December 2008	31 December 2009
Manager A	£130,000	£135,000	£180,000
Manager B	£140,000	£145,000	£150,000

- (i) Calculate the time-weighted rates of return earned by Manager A and Manager B over the period 1 January 2007 to 31 December 2009.
- (ii) Show that the money-weighted rate of return earned by Manager A over the period 1 January 2007 to 31 December 2009 is approximately 9.4% per annum.
- (iii) Explain, without performing further calculations, whether the money-weighted rate of return earned by Manager B over the period 1 January 2007 to 31 December 2009 was higher than, lower than or equal to that earned by Manager A.
- (iv) Discuss the relative performance of the two fund managers.

[Ans: (i) A- 9.05%, B- 9.41%]

3. CT1 September 2010 Q6

On 1 January 2001 the government of a particular country bought 200 million shares in a particular bank for a total price of £2,000 million. The shares paid no dividends for three years. On 30 June 2004 the shares paid dividends of 10 pence per share. On 31 December 2004, they paid dividends of 20 pence per share. Each year, until the end of 2009, the dividend payable every 30 June rose by 10% per annum compound and the dividend payable every 31 December rose by 10% per annum compound. On 1 January 2010, the shares were sold for their market price of £3,500 million.

- (i) Calculate the net present value on 1 January 2001 of the government's investment in the bank at a rate of interest of 8% per annum effective.

(ii) Calculate the accumulated profit from the government's investment in the bank on the date the shares are sold using a rate of interest of 8% per annum effective.

[Ans: (i) £31.66m, (ii) £63.30m]

4. CT1 September 2010 Q10

On 1 April 2003 a company issued securities that paid no interest and that were to be redeemed for £70 after five years. The issue price of the securities was £64. The securities were traded in the market and the market prices at various different dates are shown in the table below.

<i>Date</i>	<i>Market price of securities (£)</i>
1 April 2003	64
1 April 2004	65
1 April 2005	60
1 April 2006	65
1 April 2007	68
1 April 2008	70

(i) Explain why the price of the securities might have fallen between 1 April 2004 and 1 April 2005.

Two investors bought the securities at various dates. Investor X bought 100 securities on 1 April 2003 and 1,000 securities on 1 April 2005. Investor Y bought 100 securities every year on 1 April from 2003 to 2007 inclusive. Both investors held the securities until maturity.

(ii) Construct a table showing the nominal amount of the securities held and the market value of the holdings for X and Y on 1 April each year, just before any purchases of securities.

(iii) (a) Calculate the effective money weighted rate of return per annum for X for the

period from 1 April 2003 to 1 April 2008.

(b) Calculate the effective time weighted rate of return per annum for X for the period from 1 April 2003 to 1 April 2008.

(iv) (a) Determine whether the effective money weighted rate of return for Y is lower or higher than that for X for the period from 1 April 2003 to 1 April 2008.

(b) Determine the effective time weighted rate of return per annum for Y for the period from 1 April 2003 to 1 April 2008.

(v) Discuss the relationship between the different rates of return that have been calculated.

Date	Market price of securities (£)	X		Y	
		No of securities held before purchases	Market value of holdings before purchases (£)	No of securities held before purchases	Market value of holdings before purchases (£)
1 April 2003	64	–	–	–	–
1 April 2004	65	100	6,500	100	6,500
1 April 2005	60	100	6,000	200	12,000
1 April 2006	65	1,100	71,500	300	19,500
1 April 2007	68	1,100	74,800	400	27,200
[Ans: (ii) 1 April 2008	70	1,100	77,000	500	35,000

(iii) a) 4.73%, b) 1.808%, (iv) a) MWRR is less of Y, b) 1.808%]

5. CT1 April 2011 Q6

The value of the assets held by a pension fund on 1 January 2010 was £10 million. On 30 April 2010, the value of the assets had fallen to £8.5 million. On 1 May 2010, the fund received a contribution payment of £7.5 million and paid out £2 million in benefits. On 31 December 2010, the value of the fund was £17.1 million.

- (i) Calculate the annual effective money-weighted rate of return (MWRR) for 2010.
- (ii) Calculate the annual effective time-weighted rate of return (TWRR) for 2010.
- (iii) Explain why the MWRR is higher than the TWRR for 2010.

The fund manager's bonus for 2010 is based on the return achieved by the fund over the year.

- (iv) State, with reasons, which of the two rates of return calculated above would be more appropriate for this purpose.

[Ans: (i) 11.8%p.a., (ii) 3.821%p.a.]

6. CT1 April 2011 Q9

A company is considering investing in a project. The project requires an initial investment of three payments, each of £105,000. The first is due at the start of the project, the second six months later, and the third payment is due one year after the start of the project.

After 15 years, it is assumed that a major refurbishment of the infrastructure will be required, costing £200,000.

The project is expected to provide a continuous income stream as follows:

- £20,000 in the second year
- £23,000 in the third year
- £26,000 in the fourth year
- £29,000 in the fifth year

Thereafter the continuous income stream is expected to increase by 3% per annum (compound) at the start of each year. The income stream is expected to cease at the end of the 30th year from the start of the project.

(i) Show that the net present value of the project at a rate of interest of 8% per annum effective is £4,000 (to the nearest £1,000).

(ii) Calculate the discounted payback period for the project, assuming a rate of interest of 8% per annum effective.

[Ans: (ii) 29.31]

7. CT1 September 2011 Q4

A pension fund makes the following investments (£m):

<i>1 January 2009</i>	<i>1 July 2009</i>	<i>1 January 2010</i>
1.5	6.0	4.0

The rates of return earned on money invested in the fund were as follows:

<i>1 January 2009 to 30 June 2009</i>	<i>1 July 2009 to 31 December 2009</i>	<i>1 January 2010 to 31 December 2010</i>
1%	2%	5%

Assume that 1 January to 30 June and 1 July to 31 December are precise half-year periods.

(i) Calculate the time-weighted rate of return per annum effective over the two years from 1 January 2009 to 31 December 2010.

(ii) Calculate the money-weighted rate of return per annum effective over the two years from 1 January 2009 to 31 December 2010.

[Ans: (i) 4.005%, (ii) 0.04629 or 4.63%]

8. CT1 September 2011 Q10

A country's football association is considering whether to bid to host the World Cup in 2026. Several countries aspiring to host the World Cup will be making bids. Regardless of whether the bid is successful, the association will incur various costs. For two years, starting on 1 January 2012, the association will incur costs at a rate of £2m per annum, assumed to be paid continuously, to prepare the bid.

If the football association is successful, the following costs will be incurred from 1 January 2016 until 31 December 2025:

- One stadium will be built each year for ten years. The first stadium will be built in 2016 and is expected to cost £200m; the stadium built in 2017 is expected to cost £210m; and so on, with the cost of each stadium rising by 5% each year. The costs of building each stadium are assumed to be incurred halfway through the relevant year.
- Administration costs at a rate of £100m per annum will be incurred, payable monthly in advance from 1 January 2025 until 31 December 2026.
- Revenues from television, ticket receipts, advertising and so on are expected to be £3,300m and are assumed to be received continuously throughout 2026.

(i) Explain why the payback period is not a good indicator of whether this project is worthwhile.

The football association decides to judge whether to go ahead with the bid by calculating the net present value of the costs and revenues from a successful bid on 1 January 2012 at a rate of interest of 4% per annum effective.

(ii) Determine whether the association should make the bid.

The football association is discussing how it might factor into its calculations the fact that it is not certain to win the right to host the World Cup because other countries are also bidding.

(iii) Explain how you might adjust the above calculations if the probability of winning the right to host the World Cup is 0.1 and whether this adjustment would make it more likely or less likely that the bid will go ahead.

[Ans: (ii) NPV = -£1.617m, not make a bid]

9. CT1 April 2012 Q2

The value of the assets held by an investment fund on 1 January 2011 was £2.3 million.

On 30 April 2011, the value of the assets had risen to £2.9 million and, on 1 May 2011, there was a net cash inflow to the fund of £1.5 million. On 31 December 2011, the value of the assets was £4.2 million.

- (i) Calculate the annual effective time-weighted rate of return (TWRR) for 2011.
- (ii) Calculate, to the nearer 0.1%, the annual effective money-weighted rate of return (MWRR) for 2011.
- (iii) Explain why the TWRR is significantly higher than the MWRR for 2011.

[Ans: (i) 0.204 or 20.4%, (ii) 12.2%,]

10. CT1 April 2012 Q5

An investor is considering two projects, Project A and Project B. Project A involves the investment of £1,309,500 in a retail outlet. Rent is received quarterly in arrear for 25 years, at an initial rate of £100,000 per annum. It is assumed that the rent will increase at a rate of 5% per annum compound, but with increases taking place every five years. Maintenance and other expenses are incurred quarterly in arrear, at a rate of £12,000 per annum. The retail outlet reverts to its original owner after 25 years for no payment.

Project B involves the purchase of an office building for £1,000,000. The rent is to be received quarterly in advance at an initial rate of £85,000 per annum. It is assumed that the rent will increase to £90,000 per annum after 20 years. There are no maintenance or other expenses. After 25 years the property reverts to its original owner for no payment.

- (i) Show that the internal rate of return for project A is 9% per annum effective.
- (ii) Calculate the annual effective internal rate of return for Project B. Show your working.

(iii) Discuss the extent to which the answers to parts (i) and (ii) above will influence the investor's decision over which project to choose.

[Ans: (i) 9%, (ii) 7.5%,]

11. CT1 September 2012 Q3

An investment fund is valued at £120m on 1 January 2010 and at £140m on 1 January 2011. Immediately after the valuation on 1 January 2011, £200m is paid into the fund. On 1 July 2012, the value of the fund is £600m.

(i) Calculate the annual effective time-weighted rate of return over the two-and-a half year period.

(ii) Explain why the money-weighted rate of return would be higher than the time weighted rate of return.

[Ans: (i) 33.49%]

12. CT1 September 2012 Q10

Two investment projects are being considered.

(i) Explain why comparing the two discounted payback periods or comparing the two payback periods are not generally appropriate ways to choose between two investment projects.

The two projects each involve an initial investment of £3m. The incoming cash flows from the two projects are as follows:

Project A

In the first year, Project A generates cash flows of £0.5m. In the second year it will

generate cash flows of £0.55m. The cash flows generated by the project will continue to increase by 10% per annum until the end of the sixth year and will then cease. Assume that all cash flows are received in the middle of the year.

Project B

Project B generates cash flows of £0.64m per annum for six years. Assume that all cash flows are received continuously throughout the year.

- (ii) (a) Calculate the payback period from Project B.
- (b) Calculate the discounted payback period from Project B at a rate of interest of 4% per annum effective.
- (iii) Show that there is at least one “cross-over point” for Projects A and B between 0% per annum effective and 4% per annum effective where the cross-over point is defined as the rate of interest at which the net present value of the two projects is equal.
- (iv) Calculate the duration of the incoming cash flows from Projects A and B at a rate of interest of 4% per annum effective.
- (v) Explain why the net present value of Project A appears to fall more rapidly than the net present value of Project B as the rate of interest increases.

[Ans: (ii) a) 4.6875 years, b) 5.1798 yrs, (iv) A 3.163 yrs, B 2.882 yrs,]

13. CT1 April 2013 Q1

The value of the assets held by an investment fund on 1 January 2012 was £1.3 million.
On 30 September 2012, the value of the assets was £1.9 million.
On 1 October 2012, there was a net cash outflow from the fund of £0.9 million.
On 31 December 2012, the value of the assets was £0.8 million.

- (i) Calculate the annual effective time-weighted rate of return (TWRR) for 2012.
- (ii) Calculate the annual effective money-weighted rate of return (MWRR) for 2012 to the nearest 1%.
- (iii) Explain why the MWRR is significantly higher than the TWRR.

[Ans: (i) 16.9%, (ii) 36% p.a.]

14. CT1 April 2013 Q8

A car manufacturer is to develop a new model to be produced from 1 January 2016 for six years until 31 December 2021. The development costs will be £19 million on 1 January 2014, £9 million on 1 July 2014 and £5 million on 1 January 2015.

It is assumed that 6,000 cars will be produced each year from 2016 onwards and that all will be sold.

The production cost per car will be £9,500 during 2016 and will increase by 4% each year with the first increase occurring in 2017. All production costs are assumed to be incurred at the beginning of each calendar year.

The sale price of each car will be £12,600 during 2016 and will also increase by 4% each year with the first increase occurring in 2017. All revenue from sales is assumed to be received at the end of each calendar year.

- (i) Calculate the discounted payback period at an effective rate of interest of 9% per annum.
- (ii) Without doing any further calculations, explain whether the discounted payback period would be greater than, equal to, or less than the period calculated in part (i) if the effective rate of interest were substantially less than 9% per annum.

[Ans: (i) 6 years]

15. CT1 September 2013 Q6

A pension fund is considering investing in a major infrastructure project. The fund has been asked to make an investment of £2m for a 1% share in revenues from building a road. No other costs will be incurred by the pension fund. The following revenues are expected to arise from the project:

In the first year, 40,000 vehicles a day will use the road, each paying a toll of £1.

In the second year, 50,000 vehicles a day will use the road, each paying a toll of £1.10.

In the third year, both the number of vehicles using the road and the level of tolls will rise by 1% from their level in the second year. They will both continue to rise by 1% per annum compound until the end of the 20th year.

At the end of the 20th year, it is assumed that the road has no value as it will have to be completely rebuilt.

You should assume that all revenue is received continuously throughout the year and that there are 365 days in all years.

Calculate the net present value of the investment in the road at a rate of interest of 8% per annum effective.

[Ans: £275,324]

16. CT1 April 2014 Q1

You are given the following information in respect of a pension fund:

<i>Calendar Year</i>	<i>Value of fund at 1 January</i>	<i>Value of fund at 30 June</i>	<i>Net cash flow received on 1 July</i>
2011	£870,000	£872,000	£26,000
2012	£914,000	£902,000	£27,000
2013	£953,000	£962,000	£33,000
2014	£990,000		

Calculate, to the nearest 0.1%, the annual effective money-weighted rate of return earned by the fund during the period from 1 January 2011 to 1 January 2014.

[Ans: 1.2% p.a.]

17. CT1 April 2014 Q8

An insurance company borrows £50 million at an effective interest rate of 9% per annum. The insurance company uses the money to invest in a capital project that pays £6 million per annum payable half-yearly in arrear for 20 years. The income from the project is used to repay the loan. Once the loan has been repaid, the insurance company can earn interest at an effective interest rate of 7% per annum.

- (i) Calculate the discounted payback period for this investment.
- (ii) Calculate the accumulated profit the insurance company will have made at the end of the term of the capital project.

[Ans: (i) – 15.5 years, (ii) - 32,153.20]

18. CT1 September 2014 Q10

A student is considering whether to attend university or enter a profession immediately upon leaving school. If he enters the profession immediately, his salary is expected to be as follows.

Year 1: £15,000

Year 2: £18,000

Year 3: £20,000

In each subsequent year the expected salary would rise by 1% per annum compound. The salary is assumed to be received monthly in arrear for 40 years.

If he attends university, the fees and other costs will be £15,000 per annum for three years, paid annually in advance. After attending university, the student's potential earnings will rise. Immediately after leaving university, he expects to earn £22,000 in the first year, £25,000 in the second year and £28,000 in the third year. Thereafter, his salary is expected to rise each year by 1.5% per annum compound. The salary would be paid monthly in arrear for 37 years.

- (i) Calculate the present value of the student's earnings if he enters the profession immediately at a rate of interest of 7% per annum effective.
- (ii) Calculate the net present value of the decision to attend university at a rate of interest of 7% per annum effective and hence determine whether attending university would be a more attractive option.
- (iii) Explain why attending university would be relatively more attractive at lower interest rates.

The student wishes to consider the effect of taxation on earnings.

- (iv) Determine the rate of income tax above which the option of attending university would be less attractive financially than that of entering the profession immediately.

[Ans – (i) - £297,537.30, (ii) - £11,463.89, (iv) - 21.39%]

19. CT1 September 2014 Q4

A fund had a value of £2.0 million on 1 January 2013. On 1 May 2013, £2.5 million was invested. Immediately before this investment, the value of the fund was £2.1 million. At the close of business on 31 December 2013, the value of the fund was £4.2 million.

- (i) Calculate the annual effective time-weighted rate of return for 2013.
- (ii) Calculate the annual effective money-weighted rate of return for 2013.
- (iii) Comment on your answers to parts (i) and (ii).

[Ans – (i) – 4.130% p.a., (ii) -8.1% p.a.]

20. CT1 April 2015 Q9

A property development company has just purchased a retail outlet for \$4,000,000. A further \$900,000 will be spent refurbishing the outlet in six months' time.

An agreement has been made with a prospective tenant who will occupy the outlet beginning one year after the purchase date. The tenant will pay rent to the owner for five years and will then immediately purchase the outlet from the property development company for \$6,800,000. The initial rent will be \$360,000 per annum and this will be increased by the same percentage compound rate at the beginning of each successive year. The rental income is received quarterly in advance.

Calculate the compound percentage increase in the annual rent required to earn the company an internal rate of return of 12% per annum effective.

[Ans: 6.83%]

21. CT1 September 2015 Q9

A student has inherited £1m and is considering investing the money in two projects, A and B.

Project A requires the investment of the whole sum in properties that are to be let out to tenants. The details are:

- The student expects to receive an income from rents at an annual rate of £60,000 a year for four years after an initial period of one year in which no income will be received.
- Rents are expected to rise thereafter at the start of each year at a rate of 0.5% per annum.
- The income will be received monthly in advance.
- The project involves costs of £10,000 per annum in the first year, rising at a constant rate of 0.5% per annum.
- The costs will be incurred at the beginning of each year.

- f) At the end of 20 years, the student expects to be able to sell the properties for £2m after which there will be no further revenue or costs.

Project B involves the investment of the whole sum in an investment fund.

- g) The fund is expected to pay an income of £60,000 per annum annually in advance and return the whole invested sum at the end of 20 years.

- (i) (a) Calculate the payback period for project B.

- (b) Show, by general reasoning or otherwise, that the payback period from project A is longer than that from project B.

- (ii) (a) Define the discounted payback period.

- (b) Determine the discounted payback period from project B at a rate of interest of 1% per annum effective.

- (c) Show, by general reasoning or otherwise, that the discounted payback period from project A is longer than that from project B.

- (iii) Determine the internal rate of return from project B expressed as an annual effective return.

- (iv) Show that the internal rate of return from project A is higher than that from project B.

- (v) Discuss which project is the better project given your answers to parts (i)–(iv) above.

[Ans – (i) a – 16 years, (ii) b – 18 years, (iii) – 6.383%]

22. CT1 April 2016 Q12 (part ii onwards)

A company is considering the purchase of a gold mine which has recently ceased production.

The company forecasts that:

- the cost of re-opening the mine will be \$900,000, which will be incurred continuously over the first twelve months.
- additional costs are expected to be constant throughout the term of the project at \$200,000 per annum, excluding the first year. These are also incurred continuously.
- after the first twelve months, the rate of revenue will grow continuously and linearly from zero per annum to \$3,600,000 per annum at a constant rate of \$300,000 per annum.
- when the rate of revenue reaches \$3,600,000 per annum it will then decline continuously and linearly at a constant rate of \$150,000 per annum until it reaches \$600,000 per annum.
- when the rate of revenue declines to \$600,000 per annum production will stop and the mine will have zero value.

(ii) Determine the overall term of the project.

(iii) Calculate, showing all working, the price that the company should pay in order to earn an internal rate of return (IRR) of 25% per annum effective.

[Ans – (ii) – 20 years, (iii) - \$2,800,210]

23. CT1 April 2016 Q10

The following table gives information concerning a fund held by an investment manager:

Year	2012	2013	2014	2015
Value of fund at 30 June	-	12,700,000	13,000,000	14,100,000
Net cash flow received on 1 July	-	2,600,000	-3,700,000	1,800,000
Value of fund at 31 December	12,000,000	13,500,000	12,900,000	17,200,000

- (i) Calculate, to the nearest 0.1% and showing all working, the annual effective time-weighted rate of return (TWRR) achieved by the fund during the period from 31 December 2012 to 31 December 2015.
- (ii) Show that the annual effective money-weighted rate of return (MWRR) achieved by the fund over the same period is less than the answer obtained in part (i) above.
- (iii) Explain why you would expect the outcome described in part (ii) for this fund.
- (iv) Explain which of the two measures referred to in parts (i) and (ii) is a better indicator of the investment manager's performance over the period.

[Ans – (i) – 13.8%, (iv) – TWRR]

24. CT1 September 2016 Q4

The following table shows the cashflows paid into a fund on three different dates, together with the value of the fund on each date immediately before the cash flow takes place. There were no other cashflows except on the dates shown.

	<i>1 January 2014</i>	<i>1 January 2015</i>	<i>1 January 2016</i>
Value of fund (£m)	112	<i>X</i>	160
Cash flow (£m)	23	43	32

During 2014, the rate of return on the fund was 10% per annum effective.

(i) Calculate *X*.

(ii) Calculate, showing all workings, the annual effective time weighted rate of return on the fund over the two-year period from 1 January 2014 to 1 January 2016.

[Ans: (i) £148.5m, (ii) - 4.132%]

25. CT1 September 2016 Q10

A particular charity invests its assets in a fund on which it has a target rate of return of 8% per annum effective. From time-to-time, the charity also invests in projects that help achieve its charitable objectives whilst providing a rate of return.

Projects that are accepted by the charity must fulfil each of the following criteria:

1. a minimum annual effective internal rate of return of 2% less than the target return on the investment fund.
2. a payback period of no more than ten years.
3. a positive cash flow during the fifth year or earlier.

The charity is considering investing in a social enterprise project that involves providing loans to farmers in low-income countries to help them develop better resilience against

poor weather conditions. The details are as follows:

- The project involves making loans of £1m at the start of each year for three years, the first loan being made at the beginning of 2017.
- The loans will be paid back from the extra income obtained by the farmers from the beginning of 2020.
- The repayments in each year will be through level monthly instalments paid in advance with the rate of payment of the instalments increasing by 1% per year for 10 years after which the payments stop.
- The annual rate of repayment in 2020 will be £495,000.
- The charity will also incur costs at the end of each of the years in which income is received of £50,000 per annum.

- Explain why, in general, the payback period is not an appropriate decision criterion for an investment project.
- Determine which of the three criteria used by the charity are met in this case.

[Ans – (ii) – final criterion]

26. CT1 April 2017 Q8

Two investment funds A and B are administered by different managers. The initial values of the two funds on 1 January 2015 were £1.5 million and £2.3 million, respectively.

The funds received additional net cash flows at the beginning of 2015 and 2016, as follows:

	Fund Net Cash Flows	
	1 January 2015	1 January 2016
<i>Fund A</i>	£300,000	£1,700,000
<i>Fund B</i>	£2,000,000	£200,000

The fund managers achieved the following annual returns during 2015 and 2016:

	Fund Annual Returns	
	2015	2016
<i>Fund A</i>	42%	3%
<i>Fund B</i>	36%	2%

- (i) Calculate the annual effective time weighted rate of return for each fund for the period 1 January 2015 to 31 December 2016.
- (ii) Calculate the annual effective money weighted rate of return per annum for each fund for the period 1 January 2015 to 31 December 2016.
- (iii) Comment on your answers to parts (i) and (ii) by explaining which of the two measures is the better indicator of the comparative performance of the managers for the given two-year period.

[Ans – (i) – $TWRR_A = 20.94\%$, $TWRR_B = 17.78\%$, $MWRR_A = 15.823\%$, $MWRR_B = 17.474\%$,
(ii) – TWRR]

27. CT1 April 2017 Q6

Exactly three months ago an investor purchased an office building for £5.8 million with the intention of renting it out. In three months' time the investor will spend £850,000 on necessary refurbishments and improvements.

A tenant has agreed to lease the building in six months' time for 35 years. The tenant will pay an initial rent of £1.250 million per annum payable monthly in arrear. The rent will be increased at five-yearly intervals at a rate of 4.2% per annum compound. It has further been agreed that at the end of the lease period the tenant will buy the building from the investor for £11.5 million.

The investor pays income tax at a rate of 35% and is expecting a net effective rate of

return of 8% per annum.

Calculate, showing all workings, the net present value of the project to the investor at the time of purchase.

[Ans: £7.9425m]

28. CT1 April 2017 Q4

An investor borrows money from a bank in order to invest in a business venture. The initial loan is £500,000, with further loans of £250,000 made in 6 months' time and £250,000 made in 12 months' time.

The business venture will provide the investor with an income of £2 million in exactly 10 years' time and £3 million in exactly 15 years' time.

The bank offers a force of interest, $\delta(t)$, as a function of time t (measured in years) which is given by:

$$\delta(t) = \begin{cases} 0.04 & 0 \leq t \leq 2 \\ 0.02 + kt & t > 2 \end{cases}$$

- (i) Derive expressions for $v(t)$ which cover all values of t .
- (ii) Determine the minimum value of k that would ensure that the discounted payback period is exactly 10 years.

[Ans – (ii) – $k \leq 0.00975$]

29. CT1 September 2017 Q11

A university offers its students three financing options for a degree course that lasts exactly three years.

Option A

Fees are paid during the term of the course monthly in advance. The fees are £10,000 per annum in the first year and rise by 5% on the first and second anniversaries of the start of the course.

Option B

The university makes a loan to the students which is repaid in instalments after the end of the course. The instalments are determined as follows:

- a) No payments are made until three years after the end of the course.
- b) Over the following 15 years, students pay the university £1,300 per year, quarterly in advance.
- c) After 15 years of payments, the quarterly instalments are increased to £1,500 per year, quarterly in advance.
- d) After a further 15 years of payments, the quarterly instalments are increased to £1,800 per year, quarterly in advance, for a further 15-year period after which there are no more payments.

Option C

Students pay to the university 3% of all their future earnings from work, with the payments made annually in arrears.

A particular student wishes to attend the university. He expects to leave university at the end of the three-year course and immediately obtain employment. The student expects that his earnings will rise by 3% per annum compound at the end of each year for 10 years and then he will take a five-year career break.

After the career break, he expects to restart work on the salary he was earning when the career break started. He then expects to receive salary increases of 1% per annum

compound at the end of each year until retiring 45 years after graduating. The student wishes to take the financing option with the lowest net present value at a rate of interest of 3% per annum effective.

- (i) Calculate the present value of the payments due under option A.
- (ii) Calculate the present value of the payments due under option B.
- (iii) Calculate the initial level of salary that will lead the payments under option C to have the lowest present value of the three options.
- (iv) Comment on whether the student should use the same interest rate to evaluate all three options.

The university is concerned that this scheme exposes it to considerable financial risk.

- (v) Explain three risks which the university faces.

[Ans – (i) - £30,176, (ii) - £30,598, (iii) – £38,047, (iv) – No]

30. CT1 April 2018 Q7

A retailer is considering opening a new store as a business venture. The purchase price of the store will be £2 million and there will be a further investment required of £0.5 million 6 months after purchase.

The store will open 12 months after purchase. Revenues less running costs are expected to occur continuously and will be £0.2 million in the first year of operation, £0.25 million in the second year of operation and thereafter increasing at yearly intervals by 4% per annum compound. Eight years after purchase, a major refit costing £0.8 million will be required. Fifteen years after purchase, it is assumed that the store will be closed and sold for £6.4 million. The retailer requires a rate of return on its investment of 10% per annum effective.

- (i) Calculate the net present value of the venture.

It is now assumed that the revenue less running costs will be received mid-way through each year, rather than continuously.

- (ii) Explain how your answer to part (i) would change.

[Ans – (i) - £0.726m, (ii) - decrease]

31. CT1 September 2018 Q8

Two countries have recently signed a free-trade treaty and an insurance company in one of the countries is considering establishing a subsidiary in the other. The country in which the investment will take place currently has a small insurance market, but it is expected to grow slowly over the next ten years and then rapidly thereafter.

The company expects to make investments of £15m in each of the next five years to establish the subsidiary. These costs are assumed to be incurred at the end of each year.

The subsidiary will start business immediately. Upon starting business, the following costs and revenues are expected.

- Costs at a rate of £3m per year will be incurred continuously throughout the first 30 years of the subsidiary's life.
- Revenues of £3.1m per year will be received continuously throughout the first 10 years of the subsidiary's life.
- In the 11th year, revenues will be received continuously at a rate of £3.2m. The rate at which revenues will be received is then expected to increase at a rate of 5% per annum from the end of the 11th year to the end of the 30th year with increases occurring at the end of each year from the end of the 11th year.
- At the end of the 30th year, the company assumes that it will sell the subsidiary.

- (i) (a) Define the term “payback period”.

- (b) State two reasons why the payback period is a poor decision-making criterion in

the above circumstances.

(ii) Calculate the amount for which the company will have to sell the subsidiary at the end of 30 years so that the project breaks even at a rate of interest of 6% per annum effective.

Some directors are concerned that the project is too risky.

(iii) Suggest two ways in which risk could be taken into account when appraising the project.

[Ans – (ii) - £289.69 m]

32. CT1 September 2018 Q3

An investment fund is valued at £60m on 1 January 2016 and at £70m on 1 January 2017. Immediately after the valuation on 1 January 2017, £100m is paid into the fund. On 1 July 2018, the value of the fund is £300m.

(i) Calculate the effective time-weighted rate of return per annum over the whole period.

(ii) Explain why the money-weighted rate of return per annum would be higher than the time-weighted rate of return per annum.

[Ans – (i) – 33.49%]

33. CM1A September 2020 Q6

An investment bank borrows \$39.5 million at an effective rate of interest of 8% per annum. The bank uses the money to invest in a capital project that provides an income of \$5 million per annum payable quarterly in arrears for a term of 15 years. This income is used to repay the loan. Once the loan has been repaid, the bank can earn interest on the income at an effective rate of interest of 6% per annum.

- (i) Calculate the discounted payback period for this project.
- (ii) Calculate the accumulated profit the bank will have made at the end of the term.

[Ans – (i) – 12 years 6 months, (ii) - \$14138000]

34. CM1A April 2021 Q9

The Green Investment Company has the opportunity to purchase a factory for \$400,000. The factory is to be leased and two different companies, A and B, are interested in the lease. The two companies have made the following proposals.

Company A

The Green Investment Company will need to spend another \$50,000 refurbishing the factory for Company A. Company A will pay rent annually in advance for 20 years starting immediately. The rent will increase by 3% p.a. compound each year. At the end of 20 years, Company A will purchase the factory from the Green Investment Company for \$450,000.

Company B

Company B will pay rent at an initial level amount of \$44,600 p.a. payable monthly in advance starting immediately. The rent will increase by 50% at the end of the 10th year and remain at this level for the next 10 years. At the end of 20 years, ownership of the factory will pass to Company B at no further cost.

- (i) Calculate the initial annual rent payable by Company A, to give the Green Investment Company an internal rate of return of 9% p.a. effective on the proposal.
- (ii) Demonstrate that the internal rate of return from Company B's proposal would be greater than 9% p.a. effective.

The Green Investment Company does not have the capital available to purchase the factory but can take out a loan at an interest rate of 9.5% p.a. effective. The loan is to be repaid over 20 years in level instalments payable annually in arrears.

The Green Investment Company decides to accept the proposal from Company B, and takes out a loan in order to purchase the factory.

(iii) Calculate the accumulated profit of the investment after 20 years using an effective rate of interest of 9.5% p.a.

[Ans – (i) - \$30027.70 p.a, (ii) – 9%, (iii) - 444,300.21]

35. CM1A September 2021 Q4

A company has agreed to build and operate a ferry service for a regional government.

The company will invest \$10 million at the outset, and a further \$8 million after 1 year.

The ferry will then come into operation and the company will receive payments at the end of each year, the first payment occurring at the end of the second year of the project.

The amount of payment at the end of the second year will be \$4 million, increasing by \$0.5 million in each of the subsequent years until the annual payment is \$7 million, after which the payments will reduce by \$1 million each year. When the payments have reduced to zero, the company's involvement in the project will end.

Calculate the net present value of the project at a rate of interest of 6% p.a. effective.

***Note: You should show your working and determine the present value of income using annuity functions.

[Ans: \$22.1274m]

36. CM1A September 2023 Q5

A car rental company purchases cars at a price of £24,000 per car. The company immediately rents them out to customers on a 4-year contract before selling them at the end of the contract at a price of £9,000.

The rental income is payable continuously throughout the 4-year period and increases by £1,500 at the end of each year.

Calculate, showing all working, the initial annual rental rate that the company must charge for each car in order to achieve an internal rate of return of 12% p.a. effective.

[Ans: £3,645.39 per annum]

37. CM1A September 2024 Q4

An investor has just purchased an office block for £4.8 million. The investor will spend a further £700,000 on refurbishments in 3 months' time.

Company XYZ has agreed to rent the office block 10 months from now. The contract states that Company XYZ will rent the office block from the investor for 20 years and will then purchase the property at the end of the 20-year rental period for £5.7 million.

Rent will be paid monthly in advance and will be increased every year at the rate of 3.5% p.a. compound. The initial rent is set at £800,000 p.a. with the first rental payment due immediately on the date of occupation.

Use an effective rate of interest of 8% p.a. and assume that the investor does not pay tax.

- (i) Calculate, showing all working and using suitable annuity factors, the net present value of the project valued at the time of purchase.
- (ii) Suggest how your answer to part (i) would change if the rent were to be paid continuously. You are not required to do any calculations.

[Ans: (i) £5.625m]