

Class: FY BSc

Subject : Financial Mathematics

Chapter: Unit 1 – Chapter 1

Chapter Name: Financial products and Its Cash flows

# Today's Agenda

1. Cash Flow Models
  1. Cash Flow Process
  2. Certainty
2. Financial Products
  1. Zero Coupon Bond
  2. Fixed Interest Security
  3. Index linked security
  4. Cash on deposit
  5. Equity
  6. Annuity Certain
  7. Interest only loan
  8. Repayment Loan (or mortgage)

# 1 Cash flow Models

- Cash flow Process
- Examples
  - Zero – Coupon bond
  - Fixed interest security
  - Index linked security
  - Cash on deposit
  - Equity
  - Annuity Certain
  - Interest only loan
  - Repayment Loan (or mortgage)



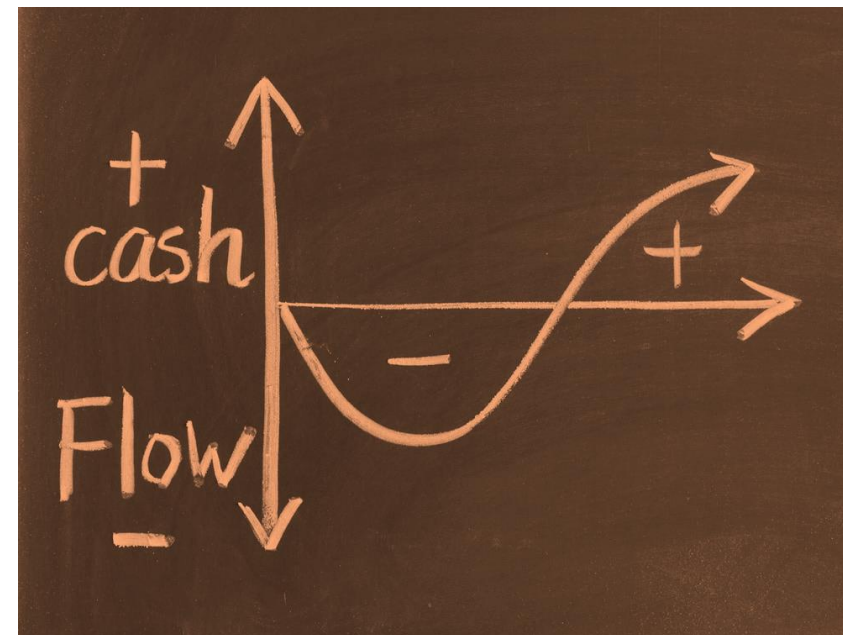
## 1.1 Cash flow Process



Analysis of cash flow involves the mathematical projection of the payments arising from a financial transaction.

- Payments received – income (Positive cash flow)
- Payments paid – outgo (negative cash flow)
- Net cash flow (income less outgo)

Practical Work – Management of cash flows



## 1.1 Cash flow Process

Things to be considered:

- Money – Paid or Received
- Timing – Known or Uncertain
- Amount - Known or Uncertain



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# Case Study

1. Operating a Bridge or a Road
1. Life Insurance Company

## 1.2 Great Certainty = Certain?



What do you think about the certainty of cash flows?

Whether amount is certain, time is certain or both uncertain or something else?

## 1.2 Great Certainty = Certain?

An Employee, gets paid on the last Friday of every month will be certain or almost certain to receive a payment on the last Friday of this month?

And Why?



## 2.1 Zero Coupon Bond



ZCB is a contract to provide a specified lump sum at some specified future date.

For the Investor:

-ve cash flow at point of investment

Single known +ve cash flow on the specified future date

Certainty of +ve cash flow:

Amount – Known

Timing – Known

Payment – Not certain (i.e. Risk of Default)

When Issued by governments of developed countries: Low Risk

When Issued by companies: High Risk

It is like a Loan from investor to the issuer.

No interest payments before redemption.

## 2.1 Zero Coupon Bond - Example

**For example**, an investor who purchases a 3-year ZCB bond at a discount for \$920 will receive \$1,000. The \$80 return, plus coupon payments received on the bond, is the investor's earnings or return for holding the bond.

| Time      | 0       | 3        |
|-----------|---------|----------|
| Cash Flow | - \$920 | + \$1000 |



## 2.1 Zero Coupon Bond

ZCB Value

### Zero Coupon Bond

$$\text{Zero-Coupon Bond Value} = \frac{\text{Maturity Value}}{(1+i)^{\text{Number of Years}}}$$



## Question

1. The principal amount of a bond that is repaid at the end of the loan term is called the bond's:

- A) Coupon
- B) Face value.
- C) Maturity
- D) Yield to maturity.
- E) Coupon rate.

**Answer: B**

## 2.1 Zero Coupon Bond



Watch Video – Zero Coupon Bond

[https://www.youtube.com/watch?v=\\_f\\_eCQLNFQ0](https://www.youtube.com/watch?v=_f_eCQLNFQ0)

## 2.2 Fixed Interest Security



This is form of a loan.

Holder of the bond – receives a lump sum of specified amount at some specified future date together with series of regular level interest payment until the redemption of the lump sum.

Regular level interest payments are called Coupons

## 2.2 Fixed Interest Security

For investor:

-ve cash flow at point of investment

Single known +ve cash flow on the specified future date

Smaller known +ve cash flows on a regular set of specified future dates.

Certainty of +ve cash flow:

Amount – Known

Timing – Known

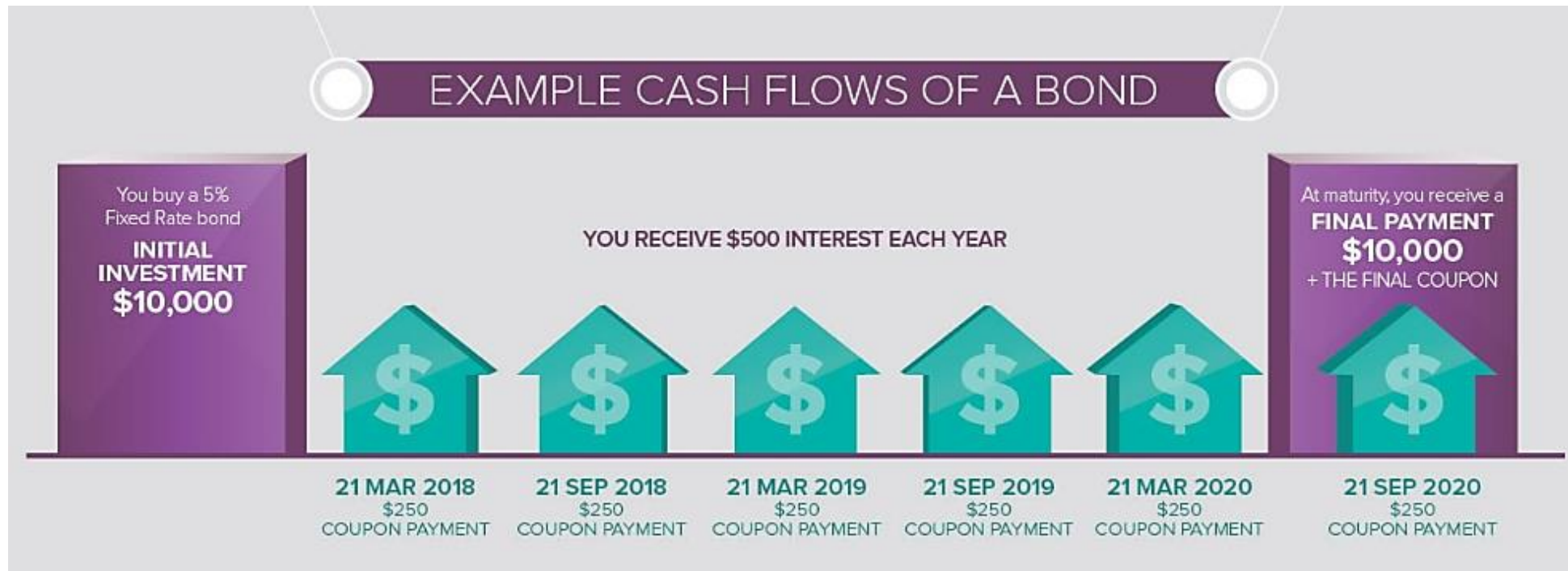
Payment – Not certain (i.e. Risk of Default)

Issued by governments of developed countries: Low Risk

Issued by companies: High Risk

## 2.2 Fixed Interest Security

Example of a Fixed Interest Security – Bond





## 2.3 Index Linked Bond



An index-linked bond is a bond in which payment of interest income on the principal is related to a specific price index, usually the Consumer Price Index (CPI).

Index-linked bonds are issued by governments to help mitigate the impact of inflation, paying a real yield plus accrued inflation. Due to inflation the purchasing power diminishes. Hence investors might be interested in coupons and final redemption amount to be linked with index that reflects the effects of inflation.

For investor:

-ve cashflow at point of investment

Single unknown +ve cashflow on the specified future date

Smaller unknown +ve cash flows on a regular set of specified future dates

Certainty of +ve cashflow:

Amount – Unknown &

Timing – Known

Payment – Not certain (i.e. Risk of Default)

## 2.3 Index Linked Bond

Cash flows are said to be known in **real** terms.

Real terms means taking into account inflation, whereas nominal means ignoring inflation

Calculation:



**Indexation**



$$\text{Indexation} = \frac{\text{Value of Good in Given Year}}{\text{Value of Good in Base Year}} \times 100$$



## 2.3 Index Linked Bond



Index linked security – Purchased on 1/1/2015  
Coupons – 600 each year and Redemption payment – 11,000  
(if there had been no inflation)

Inflation Index

| Date  | 1/1/2015 | 1/1/2016 | 1/1/2017 | 1/1/2018 |
|-------|----------|----------|----------|----------|
| Index | 100      | 105      | 108      | 113      |

Calculate the payments actually received by the investor.

## 2.4 Cash on Deposit

Eg: Bank Account (Call deposit)

For investor:

Investor chooses when to invest and disinvest and receive interest during the period of investment  
Interest rate is subject to regular change.

Certainty of +ve/-ve cashflow:

Amount – Unknown

Timing – Unknown

Another type of deposit is term deposit.

## 2.5 Equity/ Shares/ Stock



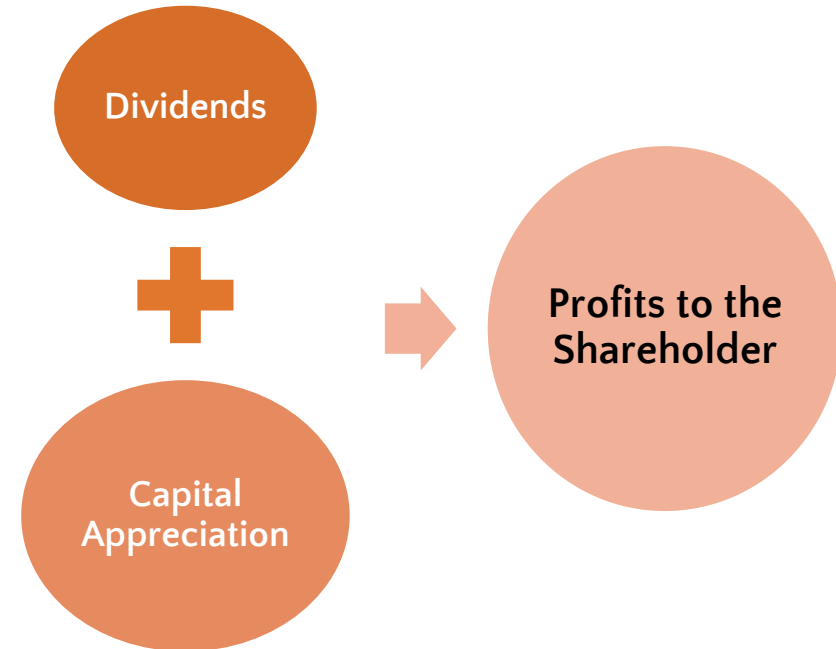
What do you know about Equity and Shares?



A share indicates a unit of ownership of the particular company. If you are a shareholder of a company, it implies that you as an investor, hold a percentage of ownership of the issuing company. As a shareholder you stand to benefit in the event of the company's profits, and also bear the disadvantages of the company's losses.

Small company – few individuals or institutions

Large organisation – many thousands of shareholders



## 2.5 Equity/ Shares/ Stock

Dividend rates are variable (since dependent on company's profit). Relationship between dividends and profits is not a simple one.

Buyback is another way of distributing profits

Equity can be held in perpetuity (hence, dividends too can continue indefinitely)

Risk of failing – Equity holders entitled to any assets which remain after creditors are paid

For investor:

-ve cashflow at point of investment

Amount – Unknown

Timing – Known (in some way)

Payment – Not certain (possibility of future +ve cashflow < Initial invested amount)

## 2.6 Annuity Certain



An annuity certain provides a series of regular payments in return for a single lump sum paid at the outset.

Term of the annuity – Known

Frequency of payments – Known

Payment amount – Level or Vary in line with some Index (eg: Inflation Index)

For investor:

Initial –ve cashflow

Series of smaller +ve cashflows

Certainty of +ve cashflow:

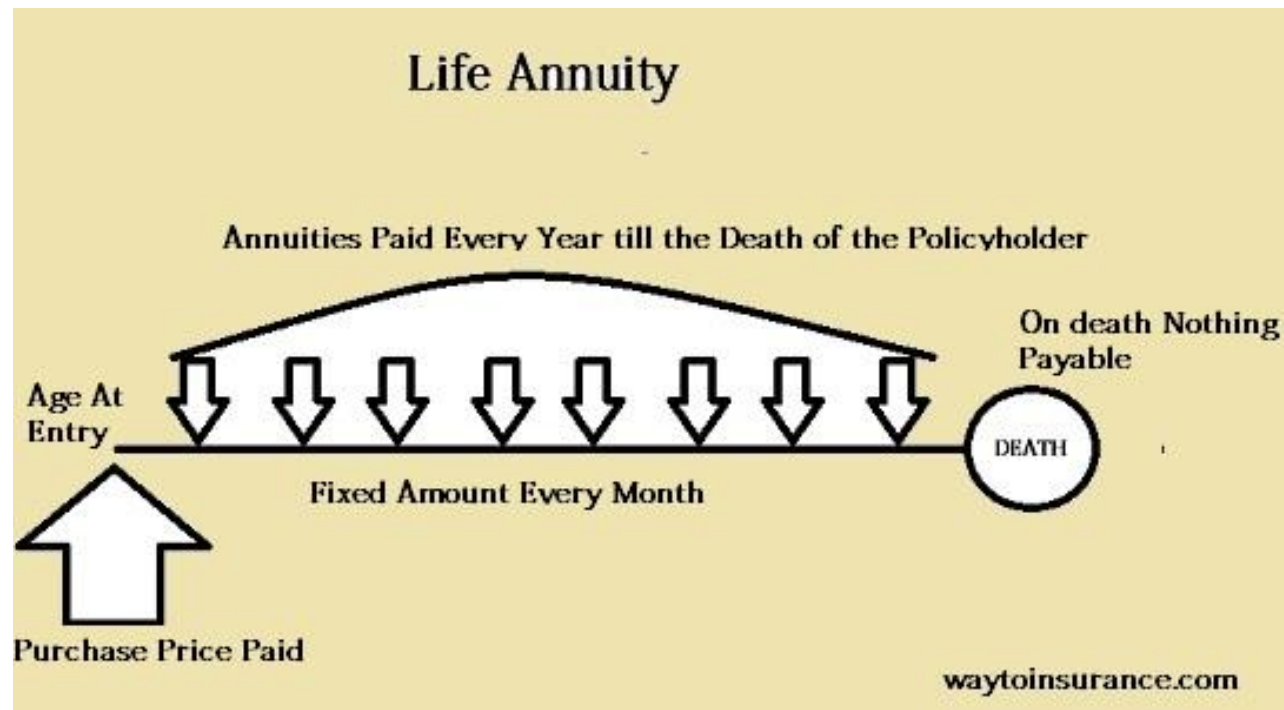
Amount – Known/Unknown

Timing – Known

Cash flows are similar to fixed interest security (but no redemption payment).

## 2.6 Annuity Certain

Example of annuity certain is A Life Annuity



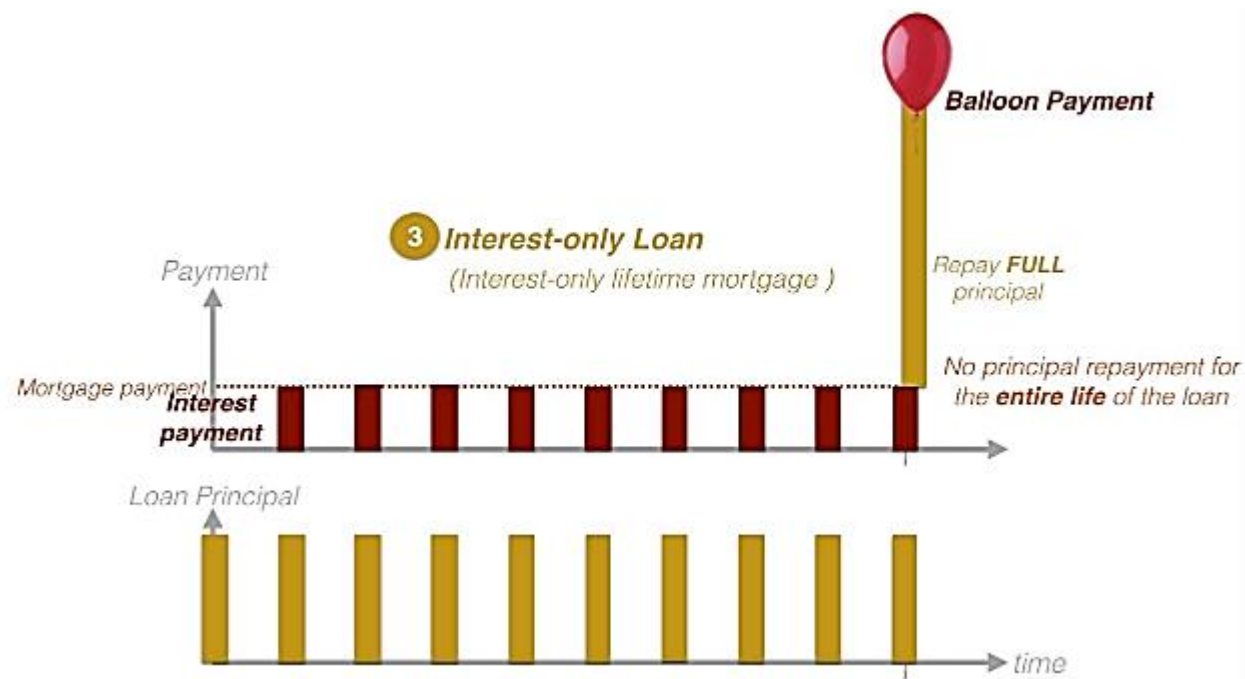


## 2.7 An “Interest Only” Loan



This is a form of a loan. It is repayable by a series of interest payments followed by a return of the initial loan amount.

Note: The Capital amount that is borrowed is owed throughout the term of the loan



## 2.7 An “Interest Only” Loan

It is the reverse of the fixed interest security.

Provider of the loan is buying fixed interest security from the borrower.

For Provider:

Initial -ve cashflow

Series of smaller +ve cashflows

Larger known +ve cashflow at the end of the term

Certainty of +ve cashflow:

Amount – Unknown (Interest rate need not be fixed in advance)

Timing – Unknown (possibility of early repayment)

## Question

1. Given the following information on an interest-only mortgage, calculate the monthly mortgage payment. Loan amount: \$56000, Term: 15 years, Interest rate: 7.5%

- A. \$169.13
- B. \$350
- C. \$519.13
- D. \$4200

Answer: B

## 2.8 Repayment/ Mortgage Loan



It is a form of a loan. This loan is repayable by a series of payments (partial repayment of loan + interest).

Simple form:

Amount – known (interest rate is fixed)

Timing – Known

Can be complicated if prepayment and variable interest rate is considered.

Cash flows similar to those of annuity certain

Breakdown of 'Interest' and 'capital' changes significantly over a period of time.

1<sup>st</sup> Repayment – Almost entirely of Interest (very small capital repayment)

Last Repayment – Very small interest content (almost entirely capital payment)

## 2.8 Repayment/ Mortgage Loan

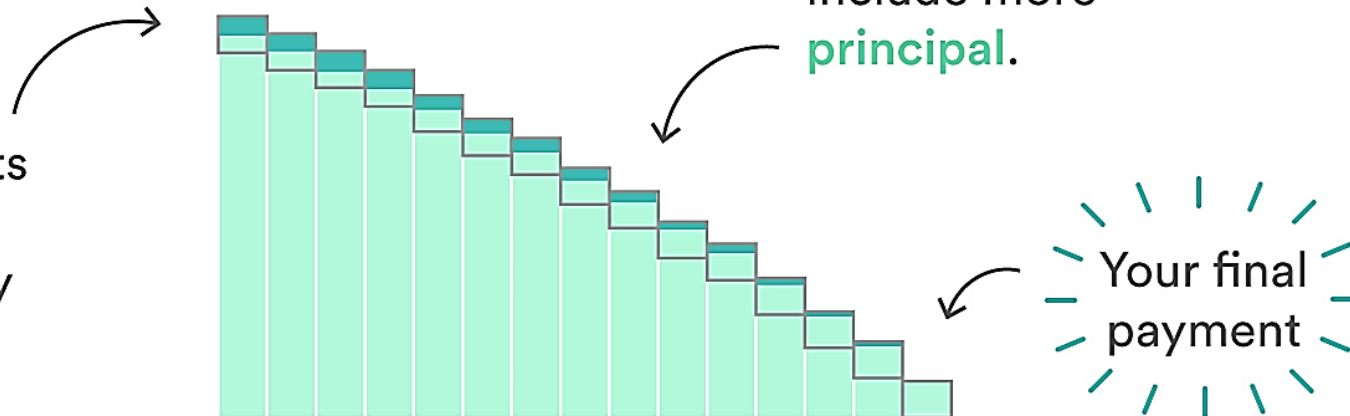
### Structure of Payments

How do payments change over the life of a loan?

As the **principal** gets smaller with each payment, less **interest** accrues.

Later payments will include more **principal**.

Early payments include relatively more **interest**.



## 2.8 Repayment/ Mortgage Loan

### Example

Consider John, who takes a \$10,000 loan with a 10% annual interest over 10 annual payments. The loan repayment schedule would look as follows:

| End of Year ... | Total Payment      | Principal Payment  | Interest Payment  | Unpaid Balance |
|-----------------|--------------------|--------------------|-------------------|----------------|
| 0               |                    |                    |                   | \$10,000.00    |
| 1               | \$1,627.45         | \$627.45           | \$1,000.00        | \$9,372.55     |
| 2               | \$1,627.45         | \$690.20           | \$937.25          | \$8,682.35     |
| 3               | \$1,627.45         | \$759.22           | \$868.23          | \$7,923.13     |
| 4               | \$1,627.45         | \$835.14           | \$792.31          | \$7,087.99     |
| 5               | \$1,627.45         | \$918.66           | \$708.80          | \$6,169.33     |
| 6               | \$1,627.45         | \$1,010.52         | \$616.93          | \$5,158.81     |
| 7               | \$1,627.45         | \$1,111.57         | \$515.88          | \$4,047.24     |
| 8               | \$1,627.45         | \$1,222.73         | \$404.72          | \$2,824.51     |
| 9               | \$1,627.45         | \$1,345.00         | \$282.45          | \$1,479.50     |
| 10              | \$1,627.45         | \$1,479.50         | \$147.95          | -\$0.00        |
| <b>TOTAL:</b>   | <b>\$16,274.54</b> | <b>\$10,000.00</b> | <b>\$6,274.54</b> |                |



## Question

Nisha takes a Loan of Rs.100,000 at 8.5% p.a. interest (reducing balance rate) for one year (12 months). Calculate the bifurcation of interest and capital repayment for each payment.

# Solution

| Year | Total Payr | Principal   | Interest    | Capital Outstanding |
|------|------------|-------------|-------------|---------------------|
| 0    | 0          | 0           | 0           | 100000              |
| 1    | 8722       | 8013.666667 | 708.3333333 | 91986.33333         |
| 2    | 8722       | 8070.430139 | 651.5698611 | 83915.90319         |
| 3    | 8722       | 8127.595686 | 594.4043143 | 75788.30751         |
| 4    | 8722       | 8185.166155 | 536.8338449 | 67603.14135         |
| 5    | 8722       | 8243.144415 | 478.8555846 | 59359.99694         |
| 6    | 8722       | 8301.533355 | 420.466645  | 51058.46358         |
| 7    | 8722       | 8360.335883 | 361.664117  | 42698.1277          |
| 8    | 8722       | 8419.554929 | 302.4450712 | 34278.57277         |
| 9    | 8722       | 8479.193443 | 242.8065571 | 25799.37933         |
| 10   | 8722       | 8539.254396 | 182.7456036 | 17260.12493         |
| 11   | 8722       | 8599.740782 | 122.2592183 | 8660.38415          |
| 12   | 8722       | 8660.655612 | 61.34438773 | -0.271461875        |



# Recap

- Analysis of cash flow involves the mathematical projection of the payments arising from a financial transaction.
- Zero coupon bond is a contract to provide a specified lump sum at some specified future date.
- In Fixed Interest Security, holder of the security – receives a lump sum of specified amount at some specified future date together with series of regular level interest payment until the redemption of the lump sum.
- An index-linked bond is a bond in which payment of interest income on the principal is related to a specific price index.
- Cash on deposits include bank account deposits, term deposits etc.
- A share indicates a unit of ownership of the particular company. As a shareholder you stand to benefit in the event of the company's profits, and also bear the disadvantages of the company's losses.

# Recap

- An annuity certain provides a series of regular payments in return for a single lump sum paid at the outset.
- An Interest only loan is repayable by a series of interest payments followed by a return of the initial loan amount.
- Mortgage is a form of a loan. This loan is repayable by a series of payments (partial repayment of loan + interest).