



Subject: Financial
Mathematics

Chapter: Unit 2

Category: Practice questions

1. CT1 September 2018 Q7

The force of interest, $\delta(t)$, is a function of time and at any time t , measured in years, is given by the formula:

$$\delta(t) = \begin{cases} 0.03 & 0 \leq t \leq 10 \\ 0.003t & t > 10 \end{cases}$$

- (i) Calculate the present value of a unit sum of money due at time $t = 20$. [4]
 - (ii) Calculate the equivalent constant force of interest from $t = 0$ to $t = 20$. [2]
 - (iii) Calculate the present value at time $t = 0$ of a continuous payment stream payable at a rate of $e^{-0.06t}$ from time $t = 4$ to time $t = 8$. [4]
- [Total 10]

2. CT1 April 2018 Q3

An investor pays £80 at the start of each month into a 25-year savings plan. The contributions accumulate at an effective rate of interest of 3% per half-year for the first 10 years, and at a force of interest of 6% per annum for the final 15 years.

Calculate the accumulated amount in the savings plan at the end of 25 years. [6]

3. CT1 April 2018 Q10

The force of interest $\delta(t)$ is a function of time, and at any time t , measured in years is given by the formula:

$$\delta(t) = \begin{cases} 0.24 - 0.02t & 0 < t \leq 6 \\ 0.12 & 6 < t \end{cases}$$

- (i) Derive, and simplify as far as possible, expressions in terms of t for the present value of a unit investment made at any time, t . You should derive separate expressions for each time interval $0 \leq t \leq 6$ and $6 < t$. [5]
 - (ii) Determine the discounted value at time $t = 4$ of an investment of £1,000 due at time $t = 10$. [2]
 - (iii) Calculate the constant nominal annual interest rate convertible monthly implied by the transaction in part (ii). [2]
 - (iv) Calculate the present value of a continuous payment stream invested from time $t = 6$ to $t = 10$ at a rate of $p(t) = 20e^{0.36+0.32t}$ per annum. [4]
- [Total 13]

4. CT1 September 2017 Q9

The force of interest, $\delta(t)$, is a function of time and at any time t , measured in years, is given by the formula:

$$\delta(t) = \begin{cases} 0.09 - 0.003t & 0 \leq t \leq 10 \\ 0.06 & t > 10 \end{cases}$$

- (i) Calculate the corresponding constant effective annual rate of interest for the period from $t = 0$ to $t = 10$. [4]

(ii) Express the rate of interest in part (i) as a nominal rate of discount per annum convertible halfyearly. [1]

(iii) Calculate the accumulation at time $t = 15$ of £1,500 invested at time $t = 5$. [3]

(iv) Calculate the corresponding constant effective annual rate of discount for the period $t = 5$ to $t = 15$. [1]

(v) Calculate the present value at time $t = 0$ of a continuous payment stream payable at a rate of $10e^{0.01t}$ from time $t = 11$ to time $t = 15$. [6]

[Total 15]

5. CT1 September 2016 Q2

The nominal rate of interest per annum convertible quarterly is 2%. Calculate the present value of a payment stream paid at a rate of €100 per annum, monthly in advance for 12 years. [4]

6. CT1 September 2016 Q12

The force of interest, $\delta(t)$, is a function of time and at any time t (measured in years) is given by:

$$\delta(t) = \begin{cases} 0.03 & \text{for } 0 \leq t \leq 10 \\ at & \text{for } 10 < t \leq 20 \\ bt & \text{for } t > 20 \end{cases}$$

where a and b are constants.

The present value of £100 due at time 20 is 50.

(i) Calculate a . [5]

The present value of £100 due at time 28 is 40.

(ii) Calculate b . [4]

(iii) Calculate the equivalent annual effective rate of discount from time 0 to time 28. [2]

A continuous payment stream is paid at the rate of $e^{-0.04t}$ per annum between $t = 3$ and $t = 7$.

(iv) (a) Calculate, showing all workings, the present value of the payment stream.

(b) Determine the level continuous payment stream per annum from time $t = 3$ to time $t = 7$ that would provide the same present value as the answer in part (iv)(a) above. [8]

[Total 19]

7. CT1 September 2014 Q7

The force of interest, $\delta(t)$, is a function of time and at any time t , measured in years, is given by the formula:

$$\delta(t) = \begin{cases} 0.03 & \text{for } 0 < t \leq 10 \\ 0.003t & \text{for } 10 < t \leq 20 \\ 0.0001t^2 & \text{for } t > 20 \end{cases}$$

- (i) Calculate the present value of a unit sum of money due at time $t = 28$. [7]
- (ii) (a) Calculate the equivalent constant force of interest from $t = 0$ to $t = 28$.
- (b) Calculate the equivalent annual effective rate of discount from $t = 0$ to $t = 28$. [3]

A continuous payment stream is paid at the rate of $e^{-0.04t}$ per unit time between $t = 3$ and $t = 7$.

- (iii) Calculate the present value of the payment stream. [4]

[Total 14]

8. CT1 April 2013 Q.5

The force of interest per unit time at time t , $\delta(t)$, is given by:

$$\delta(t) = \begin{cases} 0.1 - 0.005t & \text{for } t < 6 \\ 0.07 & \text{for } t \geq 6 \end{cases}$$

- (i) Calculate the total accumulation at time 10 of an investment of £100 made at time 0 and a further investment of £50 made at time 7. [4]
- (ii) Calculate the present value at time 0 of a continuous payment stream at the rate $£50e^{0.05t}$ per unit time received between time 12 and time 15. [5]

[Total 9]

9. CT1 September 2013 Q.10

The force of interest, $\delta(t)$, is a function of time and at any time t , measured in years, is given by the formula:

$$\delta(t) = 0.05 + 0.002t$$

Calculate the accumulated value of a unit sum of money:

- (i) (a) accumulated from time $t = 0$ to time $t = 7$.
- (b) accumulated from time $t = 0$ to time $t = 6$.

(c) accumulated from time $t = 6$ to time $t = 7$.

[Total 5]

10. CT1 April 2012 Q.8

The force of interest, $\delta(t)$, at time t is given by:

$$\delta(t) = \begin{cases} 0.04 + 0.003t^2 & \text{for } 0 < t \leq 5 \\ 0.01 + 0.03t & \text{for } 5 < t \leq 8 \\ 0.02 & \text{for } t > 8 \end{cases}$$

(i) Calculate the present value (at time $t = 0$) of an investment of £1,000 due at time $t = 10$. [4]

(ii) Calculate the constant rate of discount per annum convertible quarterly, which would lead to the same present value as that in part (i) being obtained. [2]

(iii) Calculate the present value (at time $t = 0$) of a continuous payment stream payable at the rate of $0.01100te$ from time $t = 10$ to $t = 18$. [4]

[Total 10]

11. CT1 September 2012 Q.8

The force of interest, $\delta(t)$, is a function of time and at any time t , measured in years, is given by the formula

$$\delta(t) = \begin{cases} 0.03 + 0.01t & \text{for } 0 \leq t \leq 9 \\ 0.06 & \text{for } 9 < t \end{cases}$$

(i) Derive, and simplify as far as possible, expressions for $v(t)$ where $v(t)$ is the present value of a unit sum of money due at time t . [5]

(ii) (a) Calculate the present value of £5,000 due at the end of 15 years.

(b) Calculate the constant force of interest implied by the transaction in part (a). [4]

A continuous payment stream is received at rate $0.02100te$ units per annum between $t = 11$ and $t = 15$.

(iii) Calculate the present value of the payment stream. [4]

[Total 13]

12. CT1 April 2011 Q.1

The force of interest, $\delta(t)$, is a function of time and at any time t , measured in years, is given by the formula

$$\delta(t) = \begin{cases} 0.04 + 0.003t^2 & \text{for } 0 < t \leq 5 \\ 0.01 + 0.03t & \text{for } 5 < t \end{cases}$$

- (i) Calculate the amount to which £1,000 will have accumulated at $t = 7$ if it is invested at $t = 3$. [4]
 (ii) Calculate the constant rate of discount per annum, convertible monthly, which would lead to the same accumulation as that in (i) being obtained. [3]

[Total 7]

13. CT1 September 2011 Q.6

The force of interest, $\delta(t)$, is a function of time and at any time t , measured in years, is $at + b$ where a and b are constants. An amount of £45 invested at time $t = 0$ accumulates to £55 at time $t = 5$ and £120 at time $t = 10$.

- (i) Calculate the values of a and b . [5]
 (ii) Calculate the constant force of interest per annum that would give rise to the same accumulation from time $t = 0$ to time $t = 10$. [2]

[Total 7]

14. CT1 April 2010 Q.11

The force of interest $\delta(t)$ is a function of time and at any time t , measured in years, is given by the formula

$$\delta(t) = \begin{cases} 0.04 + 0.02t & 0 \leq t < 5 \\ 0.05 & 5 \leq t \end{cases}$$

- (i) Derive and simplify as far as possible expressions for $v(t)$, where $v(t)$ is the present value of a unit sum of money due at time t . [5]
 (ii) (a) Calculate the present value of £1000 due at the end of 17 years.
 (b) Calculate the rate of interest per annum convertible monthly implied by the transaction in part (ii)(a). [4]

A continuous payment stream is received at a rate of $10e^{-0.01t}$ units per annum between $t = 6$ and $t = 10$. (iii) Calculate the present value of the payment stream. [4]

[Total 13]

15. CT1 September 2010 Q.8

The force of interest, $\delta(t)$, is a function of time and at any time t , measured in years, is given by the formula

$$\delta(t) = \begin{cases} 0.05 + 0.001t & 0 \leq t \leq 20 \\ 0.05 & t > 20 \end{cases}$$

- (i) Derive and simplify as far as possible expressions for $v(t)$, where $v(t)$ is the present value of a unit sum of money due at time t . [5]
 (ii) (a) Calculate the present value of £100 due at the end of 25 years.
 (b) Calculate the rate of discount per annum convertible quarterly implied by the transaction in part (ii)(a). [4]
 (iii) A continuous payment stream is received at rate $30e^{-0.015t}$ units per annum between $t = 20$ and $t = 25$. Calculate the accumulated value of the payment stream at time $t = 25$. [4]

[Total 13]



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PRACTICE QUESTIONS