INSTITUTE OF ACTUARIAL & QUANTITATIVE STUDIES

Subject: **Financial Mathematics**

Chapter: **Unit 2**

Category: **Practice questions 2**

1. CT1 September 2018 Q7

The force of interest, δ(t), is a function of time and at any time t, measured in years, is given by the formula:

$$\delta(t) = \begin{cases} 0.03 & 0 \le t \le 10 \\ 0.003t & t > 10 \end{cases}$$

(i) Calculate the present value of a unit sum of money due at time t = 20. [4]

(ii) Calculate the equivalent constant force of interest from t = 0 to t = 20. [2]

(iii) Calculate the present value at time t = 0 of a continuous payment stream payable at a rate of $e^{-0.06t}$ from time t = 4 to time t = 8. [4]

[Total 10]

2. CT1 April 2018 Q3

An investor pays £80 at the start of each month into a 25-year savings plan. The contributions accumulate at an effective rate of interest of 3% per half-year for the first 10 years, and at a force of interest of 6% per annum for the final 15 years.

Calculate the accumulated amount in the savings plan at the end of 25 years. [6]

3. CT1 April 2018 Q10

The force of interest δ(t) is a function of time, and at any time t, measured in years is given by the formula:

$$\delta(t) = \begin{cases} 0.24 - 0.02t & 0 < t \le 6 \\ 0.12 & 6 < t \end{cases}$$

(i) Derive, and simplify as far as possible, expressions in terms of t for the present value of a unit investment made at any time, t. You should derive separate expressions for each time interval 0≤t ≤6 and 6 ≤t . [5]

(ii) Determine the discounted value at time t = 4 of an investment of £1,000 due at time t = 10. [2]

(iii) Calculate the constant nominal annual interest rate convertible monthly implied by the transaction in part (ii). [2]

(iv) Calculate the present value of a continuous payment stream invested from time t = 6 to t = 10 at a rate of ρ(t) = 20 e0.36+0.32t per annum. [4]

[Total 13]

4. CT1 September 2017 Q9

The force of interest, δ(t), is a function of time and at any time t, measured in years, is given by the formula:

$$\delta(t) = \begin{cases} 0.09 - 0.003t & 0 \le t \le 10 \\ 0.06 & t > 10 \end{cases}$$

(i) Calculate the corresponding constant effective annual rate of interest for the period from t = 0 to t = 10. [4]

(ii) Express the rate of interest in part (i) as a nominal rate of discount per annum convertible halfyearly. [1]

(iii) Calculate the accumulation at time t = 15 of £1,500 invested at time t = 5. [3]

(iv) Calculate the corresponding constant effective annual rate of discount for the period t = 5 to t = 15. [1]

(v) Calculate the present value at time t = 0 of a continuous payment stream payable at a rate of $10e^{0.01t}$ from time t = 11 to time t = 15. [6]

[Total 15]

5. CT1 September 2016 Q2

The nominal rate of interest per annum convertible quarterly is 2%. Calculate the present value of a payment stream paid at a rate of €100 per annum, monthly in advance for 12 years. [4]

6. CT1 September 2016 Q12

The force of interest, δ(t), is a function of time and at any time t (measured in years) is given by:

$$\delta(t) = \begin{cases} 0.03 & \text{for } 0 \le t \le 10 \\ at & \text{for } 10 < t \le 20 \\ bt & \text{for } t > 20 \end{cases}$$

where a and b are constants.

The present value of £100 due at time 20 is 50.

(i) Calculate a. [5]

The present value of £100 due at time 28 is 40.

(ii) Calculate b. [4]

(iii) Calculate the equivalent annual effective rate of discount from time 0 to time 28. [2]

A continuous payment stream is paid at the rate of $e^{-0.04t}$ per annum between t = 3 and t = 7.

(iv) (a) Calculate, showing all workings, the present value of the payment stream.

(b) Determine the level continuous payment stream per annum from time t = 3 to time t = 7 that would provide the same present value as the answer in part (iv)(a) above. [8]

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[Total 19]

7. CT1 September 2014 Q7

The force of interest, δ(t), is a function of time and at any time t, measured in years, is given by the formula:

$$\delta(t) = \begin{cases} 0.03 & \text{for} \quad 0 < t \leq 10 \\ 0.003t & \text{for} \quad 10 < t \leq 20 \\ 0.0001t^2 & \text{for} \quad t > 20 \end{cases}$$

(i) Calculate the present value of a unit sum of money due at time t = 28. [7]

(ii) (a) Calculate the equivalent constant force of interest from t = 0 to t = 28.

(b) Calculate the equivalent annual effective rate of discount from t = 0 to t = 28. [3]

A continuous payment stream is paid at the rate of $e{-}0.04t$ per unit time between t = 3 and t = 7.

(iii) Calculate the present value of the payment stream. [4]

[Total 14]

8. CT1 April 2013 Q.5

The force of interest per unit time at time t, δ(t), is given by:

$$\delta(t) = \begin{cases} 0.1 - 0.005t & \text{for } t < 6 \\ 0.07 & \text{for } t \geq 6 \end{cases}$$

(i) Calculate the total accumulation at time 10 of an investment of £100 made at time 0 and a further investment of £50 made at time 7. [4]

(ii) Calculate the present value at time 0 of a continuous payment stream at the rate £50$e^{0.05t}$ per unit time received between time 12 and time 15. [5]

[Total 9]

9. CT1 September 2013 Q.10

The force of interest, δ(t), is a function of time and at any time t, measured in years, is given by the formula:

δ(t) = 0.05 + 0.002t

Calculate the accumulated value of a unit sum of money:

(i) (a) accumulated from time t = 0 to time t = 7.

(b) accumulated from time t = 0 to time t = 6.

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(c) accumulated from time t = 6 to time t = 7.

[Total 5]

10. CT1 April 2012 Q.8

The force of interest, δ(t), at time t is given by:

$$\delta(t)=\begin{cases}0.04+0.003t^2 & \text{for} \quad 0<t\le 5\\ 0.01+0.03t & \text{for} \quad 5<t\le 8\\ 0.02 & \text{for} \quad t>8\end{cases}$$

(i) Calculate the present value (at time t = 0) of an investment of £1,000 due at time t = 10. [4]

(ii) Calculate the constant rate of discount per annum convertible quarterly, which would lead to the same present value as that in part (i) being obtained. [2]

(iii) Calculate the present value (at time t = 0) of a continuous payment stream payable at the rate of 0.01 100 t e from time t = 10 to t = 18. [4]

[Total 10]

11. CT1 September 2012 Q.8

The force of interest, δ(t), is a function of time and at any time t, measured in years, is given by the formula

$$\delta(t)=\begin{cases}0.03+0.01t & \text{for} \quad 0\le t\le 9\\ 0.06 & \text{for} \quad 9<t\end{cases}$$

(i) Derive, and simplify as far as possible, expressions for v(t) where v(t) is the present value of a unit sum of money due at time t. [5]

(ii) (a) Calculate the present value of £5,000 due at the end of 15 years.

(b) Calculate the constant force of interest implied by the transaction in part (a). [4]

A continuous payment stream is received at rate 0.02 100 t e− units per annum between t = 11 and t = 15.

(iii) Calculate the present value of the payment stream. [4]

[Total 13]

12. CT1 April 2011 Q.1

The force of interest, δ(t), is a function of time and at any time t, measured in years, is given by the formula

$$\delta(t)=\begin{cases}0.04+0.003t^2 & \text{for} \quad 0<t\le 5\\ 0.01+0.03t & \text{for} \quad 5<t\end{cases}$$

(i) Calculate the amount to which £1,000 will have accumulated at 7 t = if it is invested at t = 3. [4]

(ii) Calculate the constant rate of discount per annum, convertible monthly, which would lead to the same accumulation as that in (i) being obtained. [3]

[Total 7]

13. CT1 September 2011 Q.6

The force of interest, δ(t), is a function of time and at any time t, measured in years, is a bt + where a and b are constants. An amount of £45 invested at time t = 0 accumulates to £55 at time t = 5 and £120 at time t = 10.

(i) Calculate the values of a and b. [5]

(ii) Calculate the constant force of interest per annum that would give rise to the same accumulation from time t = 0 to time t = 10. [2]

[Total 7]

14. CT1 April 2010 Q.11

The force of interest δ(t) is a function of time and at any time t, measured in years, is given by the formula

$$\delta(t)=\begin{cases}0.04+0.02t & 0\le t<5\\ 0.05 & 5\le t\end{cases}.$$

(i) Derive and simplify as far as possible expressions for v(t), where for v(t) is the present value of a unit sum of money due at time t. [5]

(ii) (a) Calculate the present value of £1000 due at the end of 17 years.

(b)Calculate the rate of interest per annum convertible monthly implied by the transaction in part (ii)(a). [4]

A continuous payment stream is received at a rate of 10e0.01t units per annum between t = 6 and t = 10. (iii) Calculate the present value of the payment stream. [4]

[Total 13]

15. CT1 September 2010 Q.8

The force of interest, δ(t), is a function of time and at any time t, measured in years, is given by the formula

$$\delta(t)=\begin{cases}0.05+0.001t & 0\le t\le 20\\ 0.05 & t>20\end{cases}$$

(i) Derive and simplify as far as possible expressions for v(t), where v(t) is the present value of a unit sum of money due at time t. [5]

(ii) (a) Calculate the present value of £100 due at the end of 25 years.

(b) Calculate the rate of discount per annum convertible quarterly implied by the transaction in part (ii)(a). [4]

(iii) A continuous payment stream is received at rate $30e^{-0.015t}$ units per annum between $t = 20$ and $t = 25$. Calculate the accumulated value of the payment stream at time $t = 25$. [4]

[Total 13]