

Class: FY BSc

Subject: Financial Mathematics

Chapter: Unit 3 – Chapter 2

Chapter Name: Equation of Value

Today's Agenda

- 0. Introduction
- 1. Equation of Value
- 2. Uses
 - 1. Unknown rate of Interest
 - 2. Unknown Time or Number of Payments

0 Introduction

Till Now

- ❖ We studied various examples and exercises involving accumulated values and present values ;
- ❖ Understood equivalent rates of interest and discount ;
- ❖ Learnt about annuity and types of annuity.

Further

- ❖ We will see the value these add when combined in the practical world.
- ❖ Any transaction or project that we invest in includes exchange of money and these components are used to help make better decisions accounting for time value of money.

1 Equation of Value

As a consequence of time value of money , two or more amounts of money payable at different points in time cannot be compared until all the amounts are accumulated or discounted to a common date . This date is called the comparison date and the equation which accumulates or discounts each payment to the comparison date is called the **Equation Of Value**.



What is an equation of value?

An *equation of value* equates the present value of money received to the present value of money paid out:

$$\text{"PV income} = \text{PV outgo"}$$

or equivalently:

$$\text{"PV income} - \text{PV outgo} = 0"$$

1 Example

- ❖ From the prospective of an investor in real-estate project,

INFLOW	OUTFLOW
Periodic rent on the property	Money to buy land
Sale proceeds from the property	Periodic wage to workers
	Expenses of raw materials
	Periodic salary to heads

- ❖ In a transaction of purchase of dividend paying stock,

INFLOW	OUTFLOW
Periodic dividend payments	Purchase price of the stock
Sale proceeds stock (if sold)	

2 Uses

Used to find the unknown quantity:

1. Amount to be invested (PV)
2. Annuity amount
3. Interest rate
4. Term/ Tenure

2.1 Unknown rate of Interest

We understand these concepts better through an example.



Question

At what rate of interest, convertible quarterly, is \$16000 the present value of \$1000 paid at the end of every quarter for five years?

Hint: Make the equation of value for the transaction and then find the one unknown.

Solution

Let $j = i^{(4)} / 4$, so that the equation of value becomes

$$1000a_{\overline{20}|j} = 16,000$$

or

$$a_{\overline{20}|j} = 16.$$

This problem is ideally set up to use a financial calculator. We set

$$\begin{array}{rcl} N & = & 20 \\ PV & = & 16 \\ PMT & = & -1 \end{array}$$

and compute I obtaining

$$I = 2.2262.$$

Thus, we have

$$j = .022262$$

so that

$$i^{(4)} = 4(.022262) = .08905.$$



Question

Find the rate at which $\ddot{s}_{\overline{2}|} = 2.5$

Solution

$$\ddot{s}_{\overline{2}|} = (1+i)^2 + (1+i) = 2.5.$$

Thus, we have a quadratic which simplifies to $i^2 + 3i - .5 = 0$ and applying the quadratic formula

$$\begin{aligned} i &= \frac{-3 \pm \sqrt{(3)^2 + (4)(.5)}}{2} \\ &= \frac{-3 \pm \sqrt{11}}{2}. \end{aligned}$$

Only the positive root is reasonable, so that

$$i = \frac{-3 + \sqrt{11}}{2} = .1583, \text{ or } 15.83\%.$$



Question

Gustavo Larson has saved \$20,000. On 1 January, he purchases a perpetuity that makes end-of-year payments. The perpetuity price is based on an annual effective interest rate of 5%.

What are the annual payments of Gustavo's perpetuity?

Solution

Solution The value on January 1 of a perpetuity with annual end-of-year payments of Q is $Q(\frac{1}{.05}) = 20Q$. Setting this value equal to \$20,000, we find $Q = \underline{\$1,000}$.

2.2 Unknown Time or Number of Payments



Question

Smith wishes to accumulate 1000 by means of semiannual contributions earning interest at a nominal rate of $i^{(2)} = 0.08$. The regular deposits will be of 50 each.

Find the number of regular deposits required and the additional fractional deposit,

- i) If the fractional deposit is made at the time of last regular deposit.
- ii) If the fractional deposit is made six month after the last deposit.

Solution

We solve the relationship $1000 = 50 \cdot s_{\overline{n}|.04}$ for n . Writing this equation as

$$1000 = 50 \times \frac{(1.04)^n - 1}{.04}$$

results in a value of $n = \frac{\ln(1.8)}{\ln(1.04)} = 14.9866$. Thus 14 deposits of the full amount of 50 are required. The accumulated amount on deposit at the time of, and including, the 14th deposit is $50s_{\overline{14}|.04} = 914.60$. If the additional fractional deposit is made at the time of the 14th regular deposit, then it must be $1000 - 914.60 = 85.40$, which is actually larger than the regular semiannual deposit. If the account is allowed to accumulate another half-year, then the accumulated amount in the account six months after the 14th deposit, is $50\ddot{s}_{\overline{14}|.04} = 50(1.04)s_{\overline{14}|.04} = 951.18$. In this case an additional fractional deposit (also called a *balloon payment*) of amount $1000 - 951.18 = 48.82$ is required to bring the amount on deposit to a total of 1000.