



INSTITUTE OF ACTUARIAL
& QUANTITATIVE STUDIES

Subject: Financial
Mathematics

Chapter: Unit 4

Category: Practice
Questions

1. CT1 April 2018 Q7

A retailer is considering opening a new store as a business venture. The purchase price of the store will be £2 million and there will be a further investment required of £0.5 million 6 months after purchase.

The store will open 12 months after purchase. Revenues less running costs are expected to occur continuously and will be £0.2 million in the first year of operation, £0.25 million in the second year of operation and thereafter increasing at yearly intervals by 4% per annum compound.

Eight years after purchase, a major refit costing £0.8 million will be required. Fifteen years after purchase, it is assumed that the store will be closed and sold for £6.4million.

The retailer requires a rate of return on its investment of 10% per annum effective. (i)

(i) Calculate the net present value of the venture. [8]

It is now assumed that the revenue less running costs will be received mid-way through each year, rather than continuously.

(ii) Explain how your answer to part (i) would change. [2]
[Total 10]

2. CT1 April 2018 Q11

An n -year decreasing annuity is payable annually in arrear where the payment at the end of the first year is n , the payment at the end of the second year is $(n - 1)$, and so on until the final payment at the end of year n is 1.

(i) Show that the present value of this annuity is:

$$\frac{n - a_n}{i}$$

A loan is to be repaid over 25 years by means of annual instalments payable in arrear. The amount of the first instalment is £8,000 and each subsequent instalment reduces by £200.

The effective rate of interest charged by the lender is 5.5% per annum.

- (ii) Calculate the initial amount of the loan. [3]
 - (iii) Determine the interest and capital components of the 10th instalment. [6]
 - (iv) Calculate the total amount of interest payable over the term of the loan. [2]
- [Total 14]

3. CT1 September 2018 Q3

An investment fund is valued at £60m on 1 January 2016 and at £70m on 1 January 2017. Immediately after the valuation on 1 January 2017, £100m was paid into the fund. On 1 July 2018, the value of the fund was £300m.

- (i) Calculate the effective time-weighted rate of return per annum over the whole period. [3]
 - (ii) Explain why the money-weighted rate of return per annum would be higher than the time-weighted rate of return per annum. [2]
- [Total 5]

4. CT1 September 2018 Q8

Two countries have recently signed a free-trade treaty and an insurance company in one of the countries is considering establishing a subsidiary in the other. The country in which the investment will take place currently has a small insurance market, but it is expected to grow slowly over the next ten years and then rapidly thereafter.

The company expects to make investments of £15m in each of the next five years to establish the subsidiary. These costs are assumed to be incurred at the end of each year. The subsidiary will start business immediately. Upon starting business, the following costs and revenues are expected.

- Costs at a rate of £3m per year will be incurred continuously throughout the first 30 years of the subsidiary's life.
- Revenues of £3.1m per year will be received continuously throughout the first 10 years of the subsidiary's life.
- In the 11th year, revenues will be received continuously at a rate of £3.2m. The rate at which revenues will be received is then expected to increase at a rate of 5% per annum from the end of the 11th year to the end of the 30th year with increases occurring at the end of each year from the end of the 11th year.

At the end of the 30th year, the company assumes that it will sell the subsidiary.

- (i) (a) Define the term “payback period”.

(b) State two reasons why the payback period is a poor decision-making criterion in the above circumstances.

[4]

(ii) Calculate the amount for which the company will have to sell the subsidiary at the end of 30 years so that the project breaks even at a rate of interest of 6% per annum effectively. [9]

Some directors are concerned that the project is too risky.

(iii) Suggest two ways in which risk could be taken into account when appraising the project. [2]

[Total 15]

5. CT1 April 2017 Q2

A bank offers two repayment alternatives for a loan that is to be repaid over sixteen years:

Option 1: the borrower pays £7,800 per annum quarterly in arrear.

Option 2: the borrower makes payments at an annual rate of £8,200 every second year in arrear.

Determine which option would provide the better deal for the borrower at a rate of interest of 5% per annum effectively. [5]

6. CT1 April 2017 Q4

An investor borrows money from a bank in order to invest in a business venture. The initial loan is £500,000, with further loans of £250,000 made in 6 months' time and £250,000 made in 12 months' time.

The business venture will provide the investor with an income of £2 million in exactly 10 years' time and £3 million in exactly 15 years' time.

The bank offers a force of interest, $\delta(t)$, as a function of time t (measured in years) which is given by:

$$\delta(t) = \begin{cases} 0.04 & \text{for } 0 \leq t \leq 2 \\ 0.02 + kt & \text{for } t > 2 \end{cases}$$

(i) Derive expressions for $v(t)$ which cover all values of t . [5]

(ii) Determine the minimum value of k that would ensure that the discounted payback period is exactly 10 years. [4]
[Total 9]

7. CT1 April 2017 Q6

Exactly three months ago an investor purchased an office building for £5.8 million with the intention of renting it out. In three months' time the investor will spend £850,000 on necessary refurbishments and improvements.

A tenant has agreed to lease the building in six months' time for 35 years. The tenant will pay an initial rent of £1.250 million per annum payable monthly in arrear. The rent will be increased at five-yearly intervals at a rate of 4.2% per annum compound.

It has further been agreed that at the end of the lease period the tenant will buy the building from the investor for £11.5 million.

The investor pays income tax at a rate of 35% and is expecting a net effective rate of return of 8% per annum.

Calculate, showing all workings, the net present value of the project to the investor at the time of purchase. [11]

8. CT1 April 2017 Q8

Two investment funds A and B are administered by different managers. The initial values of the two funds on 1 January 2015 were £1.5 million and £2.3 million, respectively. The funds received additional net cash flows at the beginning of 2015 and 2016, as follows:

Fund Net Cash Flows
1 January 2015 1 January 2016

Fund A	£300,000	£1,700,000
Fund B	£2,000,000	£200,000

The fund managers achieved the following annual returns during 2015 and 2016:

Fund Annual Returns

	2015	2016
Fund A	42%	3%
Fund B	36%	2%

- (i) Calculate the annual effective time weighted rate of return for each fund for the period 1 January 2015 to 31 December 2016. [3]
- (ii) Calculate the annual effective money weighted rate of return per annum for each fund for the period 1 January 2015 to 31 December 2016. [8]
- (iii) Comment on your answers to parts (i) and (ii) by explaining which of the two measures is the better indicator of the comparative performance of the managers for the given two-year period. [3]
- [Total 14]

9. CT1 September 2017 Q8

A loan is to be repaid by an increasing annuity. The first payment will be £100 and the payments will increase by £50 per annum. Payments will be made annually in arrear for ten years. The repayments are calculated using a rate of interest of 5% per annum effective.

- (i) Calculate the amount of the loan. [2]
- (ii) Calculate:
- (a) the interest component of the sixth instalment.
- (b) the capital component of the sixth instalment. [4]

Immediately after the sixth instalment, the borrower asks to repay the remaining loan using level annual instalments. The lender agrees, but changes the interest rate at the time of the alteration to 6% per annum effective.

- (iii) Calculate the revised instalment. [3]
- [Total 9]

10. CT1 September 2017 Q11

A university offers its students three financing options for a degree course that lasts exactly three years.

Option A

Fees are paid during the term of the course monthly in advance. The fees are £10,000 per annum in the first year and rise by 5% on the first and second anniversaries of the start of the course.

Option B

The university makes a loan to the students which is repaid in instalments after the end of the course. The instalments are determined as follows:

- ☐ ☐ No payments are made until three years after the end of the course.
- ☐ ☐ Over the following 15 years, students pay the university £1,300 per year, quarterly in advance.
- ☐ ☐ After 15 years of payments, the quarterly instalments are increased to £1,500 per year, quarterly in advance.
- ☐ ☐ After a further 15 years of payments, the quarterly instalments are increased to £1,800 per year, quarterly in advance, for a further 15-year period after which there are no more payments.

Option C

- ☐ ☐ Students pay to the university 3% of all their future earnings from work, with the payments made annually in arrear.

A particular student wishes to attend the university. He expects to leave university at the end of the three-year course and immediately obtain employment. The student expects that his earnings will rise by 3% per annum compound at the end of each year for 10 years and then he will take a five-year career break.

After the career break, he expects to restart work on the salary he was earning when the career break started. He then expects to receive salary increases of 1% per annum compound at the end of each year until retiring 45 years after graduating.

The student wishes to take the financing option with the lowest net present value at a rate of interest of 3% per annum effective.

(i) Calculate the present value of the payments due under option A. [4]

(ii) Calculate the present value of the payments due under option B. [5]

(iii) Calculate the initial level of salary that will lead the payments under option C to have the lowest present value of the three options. [8]

(iv) Comment on whether the student should use the same interest rate to evaluate all three options. [2]

The university is concerned that this scheme exposes it to considerable financial risk.

(v) Explain three risks which the university faces. [4]

[Total 23]

11. CT1 April 2016 Q5

A loan is to be repaid by a series of instalments payable annually in arrear for 15 years. The first instalment is £1,200 and payments increase thereafter by £250 per annum.

Repayments are calculated using a rate of interest of 6% per annum effective.

Determine:

(i) the amount of the loan. [3]

(ii) the capital outstanding immediately after the 9th instalment has been made. [2]

(iii) the capital and interest components of the final instalment. [2]

[Total 7]

12. CT1 April 2016 Q10

The following table gives information concerning a fund held by an investment manager:

<i>Year</i>	<i>2012</i>	<i>2013</i>	<i>2014</i>	<i>2015</i>
Value of fund at 30 June	–	12,700,000	13,000,000	14,100,000
Net cash flow received on 1 July	–	2,600,000	–3,700,000	1,800,000
Value of fund at 31 December	12,000,000	13,500,000	12,900,000	17,200,000

- (i) Calculate, to the nearest 0.1% and showing all working, the annual effective time-weighted rate of return (TWRR) achieved by the fund during the period from 31 December 2012 to 31 December 2015. [3]
- (ii) Show that the annual effective money-weighted rate of return (MWRR) achieved by the fund over the same period is less than the answer obtained in part (i) above. [2]
- (iii) Explain why you would expect the outcome described in part (ii) for this fund. [2]
- (iv) Explain which of the two measures referred to in parts (i) and (ii) is a better indicator of the investment manager's performance over the period. [2]
- [Total 9]

13. CT1 April 2016 Q12

- (i) Show that:

$$(\bar{Ia})_{\overline{n}|} = \frac{\bar{a}_{\overline{n}|} - nv^n}{\delta}$$

A company is considering the purchase of a gold mine which has recently ceased production.

The company forecasts that:

- ☐ the cost of re-opening the mine will be \$900,000, which will be incurred continuously over the first twelve months.
- ☐ additional costs are expected to be constant throughout the term of the project at \$200,000 per annum, excluding the first year. These are also incurred continuously.
- ☐ after the first twelve months, the rate of revenue will grow continuously and linearly from zero per annum to \$3,600,000 per annum at a constant rate of \$300,000 per annum.
- ☐ when the rate of revenue reaches \$3,600,000 per annum it will then decline continuously and linearly at a constant rate of \$150,000 per annum until it reaches \$600,000 per annum.
- ☐ when the rate of revenue declines to \$600,000 per annum production will stop and the mine will have zero value.

- (ii) Determine the overall term of the project. [2]

(iii) Calculate, showing all working, the price that the company should pay in order to earn an internal rate of return (IRR) of 25% per annum effective. [12]
[Total 18]

14. CT1 September 2016 Q10

A particular charity invests its assets in a fund on which it has a target rate of return of 8% per annum effective. From time-to-time, the charity also invests in projects that help achieve its charitable objectives whilst providing a rate of return.

Projects that are accepted by the charity must fulfil each of the following criteria:

1. a minimum annual effective internal rate of return of 2% less than the target return on the investment fund.
2. a payback period of no more than ten years.
3. a positive cash flow during the fifth year or earlier.

The charity is considering investing in a social enterprise project that involves providing loans to farmers in low-income countries to help them develop better resilience against poor weather conditions. The details are as follows:

- ☐ The project involves making loans of £1m at the start of each year for three years, the first loan being made at the beginning of 2017.
- ☐ The loans will be paid back from the extra income obtained by the farmers from the beginning of 2020.
- ☐ The repayments in each year will be through level monthly instalments paid in advance with the rate of payment of the instalments increasing by 1% per year for 10 years after which the payments stop.
- ☐ The annual rate of repayment in 2020 will be £495,000.
- ☐ The charity will also incur costs at the end of each of the years in which income is received of £50,000 per annum.

(i) Explain why, in general, the payback period is not an appropriate decision criterion for an investment project. [2]

(ii) Determine which of the three criteria used by the charity are met in this case. [12]
[Total 14]

15. CT1 April 2015 Q9

A property development company has just purchased a retail outlet for \$4,000,000. A further \$900,000 will be spent refurbishing the outlet in six months' time.

An agreement has been made with a prospective tenant who will occupy the outlet beginning one year after the purchase date. The tenant will pay rent to the owner for five years and will then immediately purchase the outlet from the property development company for \$6,800,000. The initial rent will be \$360,000 per annum and this will be increased by the same percentage compound rate at the beginning of each successive year. The rental income is received quarterly in advance.

Calculate the compound percentage increase in the annual rent required to earn the company an internal rate of return of 12% per annum effectively. [9]

16. CT1 April 2015 Q11

On 1 January 2016, a student plans to take out a five-year bank loan for £30,000 that will be repayable by instalments at the end of each month. Under this repayment schedule, the instalment at the end of January 2016 will be X , the instalment at the end of February 2016 will be $2X$ and so on, until the final instalment at the end of December 2020 will be $60X$. The bank charges a rate of interest of 15% per annum convertible monthly.

(i) Prove that

$$(Ia)_{\overline{n}|} = \frac{\ddot{a}_{\overline{n}|} - nv^n}{i}$$

(ii) Show that $X = £26.62$. [4]

The student is concerned that she will not be able to afford the later repayments and so she suggests a revised repayment schedule. The student would borrow £30,000 on 1 January 2016 as before. She would now repay the loan by 60 level monthly installments of $36X = £958.32$ but the first repayment would not be made until the end of January 2019 and hence the final instalment is paid at the end of December 2023.

(iii) Calculate the APR on the revised loan schedule and hence determine whether you believe the bank should accept the student's suggestion. [5]

(iv) Explain the difference in the total repayments made under the two arrangements. [2]
[Total 14]

17. CT1 September 2015 Q9

A student has inherited £1m and is considering investing the money in two projects, A and B.

Project A requires the investment of the whole sum in properties that are to be let out to tenants. The details are:

- ☐ The student expects to receive an income from rents at an annual rate of £60,000 a year for four years after an initial period of one year in which no income will be received.
- ☐ Rents are expected to rise thereafter at the start of each year at a rate of 0.5% per annum.
- ☐ The income will be received monthly in advance.
- ☐ The project involves costs of £10,000 per annum in the first year, rising at a constant rate of 0.5% per annum.
- ☐ The costs will be incurred at the beginning of each year.
- ☐ At the end of 20 years, the student expects to be able to sell the properties for £2m after which there will be no further revenue or costs.

Project B involves the investment of the whole sum in an investment fund.

- ☐ The fund is expected to pay an income of £60,000 per annum annually in advance and return the whole invested sum at the end of 20 years.

- (i) (a) Calculate the payback period for project B.
 - (b) Show, by general reasoning or otherwise, that the payback period from project A is longer than that from project B. [5]
 - (ii) (a) Define the discounted payback period.
 - (b) Determine the discounted payback period from project B at a rate of interest of 1% per annum effective.
 - (c) Show, by general reasoning or otherwise, that the discounted payback period from project A is longer than that from project B. [5]
 - (iii) Determine the internal rate of return from project B expressed as an annual effective return. [3]
 - (iv) Show that the internal rate of return from project A is higher than that from project B. [10]
 - (v) Discuss which project is the better project given your answers to parts (i)–(iv) above. [3]
- [Total 26]



INSTITUTE OF ACTUARIAL
& QUANTITATIVE STUDIES

18. CT1 April 2014 Q8

An insurance company borrows £50 million at an effective interest rate of 9% per annum. The insurance company uses the money to invest in a capital project that pays £6 million per annum payable half-yearly in arrear for 20 years. The income from the project is used to repay the loan. Once the loan has been repaid, the insurance company can earn interest at an effective interest rate of 7% per annum.

- (i) Calculate the discounted payback period for this investment. [4]
 - (ii) Calculate the accumulated profit the insurance company will have made at the end of the term of the capital project. [5]
- [Total 9]

19. CT1 April 2014 Q10

A loan of £20,000 is repayable by an annuity payable annually in arrear for 25 years. The annual repayment is calculated at an effective interest rate of 8% per annum and increases by £50 each year.

- (i) Calculate the amount of the first payment. [3]
 - (ii) Calculate the capital outstanding after the first three payments have been made. [2]
 - (iii) Explain your answer to part (ii). [2]
 - (iv) Calculate the total amount of interest paid over the term of the loan. [3]
- [Total 10]

20. CT1 September 2014 Q4

A fund had a value of £2.0 million on 1 January 2013. On 1 May 2013, £2.5 million was invested. Immediately before this investment, the value of the fund was £2.1 million. At the close of business on 31 December 2013, the value of the fund was £4.2 million.

- (i) Calculate the annual effective time-weighted rate of return for 2013. [2]
 - (ii) Calculate the annual effective money-weighted rate of return for 2013. [3]
 - (iii) Comment on your answers to parts (i) and (ii). [2]
- [Total 7]

<https://analystprep.com/study-notes/actuarial-exams/soa/fm-financial-mathematics/loans/>



INSTITUTE OF ACTUARIAL
& QUANTITATIVE STUDIES